Two-Magnet Energy Harvesting Devices in Subsea Sensors

Min-Chie CHIU*

Department of Electrical and Power Technology, Chung Chou University of Science and Technology, Chang-Hua County, Taiwan, ROC

*Corresponding author

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Abstract. The subsea environment is difficult to overcome and requires sophisticated tools to monitor such as Autonomous or Remote Underwater Vehicles, submersible sensors, and very skilled manpower. How to supply power indefinitely to stationary sensors at the bottom of the sea becomes an essential issue. In previous work, a one-magnet energy harvester which is installed on the bottom of the sea to generate electricity has been proposed. In order to improve the electrical power, a two-magnet energy harvester is thus proposed to explore in the study. The device is actuated using water current and the output voltage is fed through a voltage rectifier to convert it to the required DC input of the sensors. Simulations results showed that the electrical voltage generated from the two-magnet energy harvester is 400 times than that of the one-magnet energy harvester.

Introduction

Recently, in order to increase the oil and gas production, the Subsea engineering has been prosperously developed. Drilling for oil and gas offshore is very often miles away from the nearest landmass; therefore, underwater facilities are needed for oil and gas production. Because leakage or rupture may occur during and after natural disasters such as earthquakes and/or storms, it is crucial that a monitoring system needs to be ready for detecting leaks in pipelines and production facilities. An approach to this problem is to fix sensors in specific locations undersea and have the underwater vehicle query it whenever needed. The disadvantage of using stationary sensors underwater is that these require a continuous power source such as batteries which cannot provide power for a very long time. To resolve the power supply issue, design of self-generated energy harvester extracted energy from sea current is oblique. Chiu et al. have explored the vibration-based electromagnetic energy harvesters used in air environment [1, 2, 3, 4]. In order to generate the electricity under the sea water, a one-magnet energy harvesting device actuated by the sea current has been designed in previous study [5]. However, the generated electricity is still insufficient for the sensors. Therefore, a thinking of adding the number of magnet to increase the electricity is rising. Here, a two-magnet energy harvester shown in Fig. 1 is then proposed. As indicated in Fig. 1, the energy harvester is composed of springs, magnets, rollers, an electrical storage unit, a device’s housing, a wave regulating tube, an orifice, and a power connection socket.

Figure 1. A two-magnet energy harvester.
Mathematical Model

Current Force

As indicated in Fig. 2, the driving force of the energy harvester is the current force. The current velocity at a height y (from the seabed) is [6, 7, 8, 9]

\[ U_y(t) = E_p \cdot \cos(\Omega t) + U_{ow} \cdot \frac{y}{h} ; \Omega = 2\pi \frac{U_{ow}}{\lambda} ; E_p = \frac{\Omega \cdot H}{2} \cdot \frac{\cosh(k \cdot y)}{\sinh(k \cdot h)} ; \]

\[ U_y(t) = \frac{\Omega \cdot H}{2} \cdot \frac{\cosh(k \cdot y)}{\sinh(k \cdot y)} \cdot \cos(\Omega t) + U_{ow} \cdot \frac{y}{h} ; k = \frac{2\pi}{\lambda} \]  

The oscillating driving force is [10]

\[ f_v(t) = \frac{1}{2} \rho \cdot C_d \cdot U_{ow} \left( \frac{\pi D^2}{4} \right) + A_D \cdot \cos(4\pi f_v \cdot t + \theta_0) ; f_v \approx \frac{S_y y}{hD} U_{ow} \]  

Setting \( C_d = 1 \) and \( \theta_0 = 0 \) yields

\[ f_v(t) = A_1 + A_2 \cdot \cos(\omega_1 \cdot t) + A_3 \cdot \cos(\omega_2 \cdot t) + A_4 \cdot \cos(\omega_3 \cdot t) \]  

Dynamic System Model

A mathematical model of the two-magnet energy harvester is shown on Fig. 3. The dynamic equations of a two-magnet energy harvester using Lagrange equation yields

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{z}_i} \right) - \frac{\partial L}{\partial z_i} + \frac{\partial D}{\partial \dot{z}_i} = Q_i(t) ; \quad \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\varepsilon}_i} \right) - \frac{\partial L}{\partial \varepsilon_i} + \frac{\partial D}{\partial \dot{\varepsilon}_i} + \frac{\partial D}{\partial \dot{\varepsilon}_3} + \frac{\partial D}{\partial \dot{\varepsilon}_5} = Q_3(t) \]  

where \( T = \frac{1}{2} M_{\omega} \dot{z}_1^2 + \frac{1}{2} M_1 \dot{z}_1^2 + \frac{1}{2} M_2 \dot{z}_2^2 ; V = \frac{1}{2} k_1 z_1^2 + \frac{1}{2} k_1 (z_2 - z_1)^2 + \frac{1}{2} k_2 (z_3 - z_2)^2 ; \)

\[ L = T - V = \frac{1}{2} M_{\omega} \dot{z}_1^2 + \frac{1}{2} M_1 \dot{z}_1^2 + \frac{1}{2} M_2 \dot{z}_2^2 - \frac{1}{2} k_1 z_1^2 \]

\[ D = \frac{1}{2} C_1 \dot{z}_1^2 + \frac{1}{2} C_1 (\dot{z}_2 - \dot{\varepsilon}_1)^2 + \frac{1}{2} C_2 (\dot{z}_3 - \dot{\varepsilon}_3)^2 \]  

Rearranging Eq.(4) yields
The equivalent damping coefficient of magnet #1 is

The relative velocity of magnet #1 (M1) with respect to the device’s housing is

The relative displacement of magnet #1 (M1) with respect to the device’s housing is

Rewriting Eqs. (6)-(7) yields

Setting that

Rearrange Eqs. (9)-(10) yields

Simplifying Eq. (11) yields

The displacements of housing and two magnets yield

The relative displacement of magnet #1 (M1) and magnet #2 (M2) with respect to the device’s housing are

The relative velocity of magnet #1 (M1) and magnet #2 (M2) with respect to the device’s housing is

The equivalent damping coefficient of magnet #1 is

where
Electromagnetic Electricity [5]

For the first magnet, the relative magnetic intensity at the point \( p_k \) (in \( \hat{z}_k \hat{k} \) direction) is [5]

\[
\bar{B}_{k(r1)} = B_{k1} \left[ \frac{z_{r1} + \frac{H_{ml}}{2}}{\sqrt{4 \left( z_{r1} + \frac{H_{ml}}{2} \right)^2 + D_{ml}^2}} - \frac{z_{r1} - \frac{H_{ml}}{2}}{\sqrt{4 \left( z_{r1} - \frac{H_{ml}}{2} \right)^2 + D_{ml}^2}} \right]
\]

(18)

The coil is surrounding around magnet #1 with \( N_{c1} \) turns and with layer of \( N_{layer1} \); the electrical voltage induced by the k-th turn coil is

\[
\varepsilon_{k1}(t) = -\frac{\pi D_{ml}^2}{4} \cdot B_{c1} \left[ \frac{D_{ml}^2}{\sqrt{4 \left( z_{k(r1)} + \frac{H_{ml}}{2} \right)^2 + D_{ml}^2}} - \frac{D_{ml}^2}{\sqrt{4 \left( z_{k(r1)} - \frac{H_{ml}}{2} \right)^2 + D_{ml}^2}} \right] \cdot 2_{r1} \cdot N_{layer1}
\]

(19)

Similarly, for magnet #2, the relative magnetic intensity at the point \( p_k \) (in \( \hat{z}_k \hat{k} \) direction) is

\[
\bar{B}_{k(r2)} = B_{r2} \left[ \frac{z_{r2} + \frac{H_{ml}}{2}}{\sqrt{4 \left( z_{r2} + \frac{H_{ml}}{2} \right)^2 + D_{ml}^2}} - \frac{z_{r2} - \frac{H_{ml}}{2}}{\sqrt{4 \left( z_{r2} - \frac{H_{ml}}{2} \right)^2 + D_{ml}^2}} \right]
\]

(20)

Summing up the induced voltage induced by \( N_{c1} \) coil, the induced voltage (at \( \hat{z} \)) for the first magnet is

\[
\varepsilon_{N_{c1}}(t) = \sum_{k=1}^{N_{c1}} \left( -\frac{\pi D_{ml}^2}{4} \cdot N_{layer1} \cdot B_{c1}(t) \right) \cdot \frac{D_{ml}^2}{\sqrt{4 \left( z_{k(r1)} + \frac{H_{ml}}{2} \right)^2 + D_{ml}^2}} - \frac{D_{ml}^2}{\sqrt{4 \left( z_{k(r1)} - \frac{H_{ml}}{2} \right)^2 + D_{ml}^2}} \right) \cdot 2_{r1}
\]

(21)

The root-mean-square electrical power for magnet #1 is

\[
\bar{W}_{[N_{c1}]} = \frac{N_{layer1}}{R_{load}} \sum_{k=1}^{N_{c1}} \left[ \frac{D_{ml}^2}{\sqrt{4 \left( z_{k(r1)} + \frac{H_{ml}}{2} \right)^2 + D_{ml}^2}} - \frac{D_{ml}^2}{\sqrt{4 \left( z_{k(r1)} - \frac{H_{ml}}{2} \right)^2 + D_{ml}^2}} \right] \left[ \sum_{j=1}^{N_{layer1}} \left( \frac{\phi_{inj}}{f_{j2}} - \lambda_{j2} \right)^2 \right]^2
\]

(22)

Similarly, the overall root-mean-square electrical power for both magnet #1 and magnet #2 is
Numerical Optimization

Both the ocean environmental parameters \((\lambda, y, H, h)\) and the energy harvester’s geometric parameters \(\left(D_{m1}, D_{m2}, H_{m1}, M_o, k_o, k_1, k_2, N_{c1}, N_{c2}, N_{layer1}, N_{layer2}, \text{ and } D\right)\) have essential influence to the electrical power. In case of a specific ocean situation having \(h=50 (m), y=5 (m), U_{ow}=3 (m/s), \lambda=3 (m), H=0.05 (m)\), \(R_{load}= 300 (\Omega)\), and Strouhal number \((S_t)=0.212\), and \(C_d=1\), an energy harvester with maximal electrical power is expected. In order to obtain the maximal electrical power, thirteen design parameters \(\left(D_{m1}, H_{m1}, D_{m2}, H_{m2}, M_o, k_o, k_1, k_2, N_{c1}, N_{c2}, N_{layer1}, N_{layer2}, \text{ and } D\right)\) will be optimally adjusted by using an optimizer of Simulated Annealing (SA) [11, 12].

The root-mean-square of the power output is taken as the objective function for the harvester design.

\[
OBJ = \bar{W}_{TT} \left( D_{m1}, H_{m1}, D_{m2}, H_{m2}, M_o, k_o, k_1, k_2, N_{c1}, N_{c2}, N_{layer1}, N_{layer2}, D \right)
\]  \hspace{1cm} (24)

The ranges of the parameters \(\left(D_{m1}, H_{m1}, D_{m2}, H_{m2}, M_o, k_o, k_1, k_2, N_{c1}, N_{c2}, N_{layer1}, N_{layer2}, \text{ and } D\right)\) have been preset and shown in Table 1. To find a maximal value of the objective function \(OBJ\), an SA method using two control parameters, cooling rate \((k_k)\) and iteration number \((iter_{max})\), is adopted. The philosophy of physical simulated annealing is shown in Fig. 4. The optimal design data is searched using the SA parameters \(k_k=(0.91, 0.93, 0.95, 0.97, 0.99)\) and \(iter_{max}=(100, 1000, 2000, 5000, 10000, 20000, 50000, 100000)\).

Table 1. The ranges of the parameters.

<table>
<thead>
<tr>
<th>ranges of the parameters</th>
<th>(D_{m1})</th>
<th>(H_{m1})</th>
<th>(k_o)</th>
<th>(k_1)</th>
<th>(k_2)</th>
<th>(M_o)</th>
<th>(D_0)</th>
<th>(N_{c1})</th>
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</table>

Utilizing Eq.(24) in conjunction with SA optimizer, the optimal result of energy harvester is obtained and shown in Table 2. As indicated in Table 2, the largest root-mean-square electrical power was produced at the 12th set of the solution.

The corresponding optimal values of the design parameters \(D_{m1}, H_{m1}, D_{m2}, H_{m2}, M_o, k_o, k_1, k_2, N_{c1}, N_{c2}, N_{layer1}, N_{layer2}, \text{ and } D\) are obtained as \(0.01 (m), 0.01 (m), 0.01 (m), 0.01 (m), 0.1 (kg), 27 (N/m), 27 (N/m), 27 (N/m), 10 \text{ (turns)}, 10 \text{ (turns)}, 1 \text{ (layer)}, 1 \text{ (layer)}, \text{ and } 0.05 \text{ (m)}, \text{ respectively}. The optimal value of the induced root-mean-square electrical power \((\bar{W}_{TT})\) is reached when the cooling rate \((k_k)\) and iteration \((iter)\) are set at 0.99 and 100000, respectively. Here, the optimal root-mean-square electrical power \((\bar{W}_r)\) was found to be \(0.34 \times 10^{-7} \text{ (watt)}\).
Table 2. Optimization results with respect to various SA control parameters.

<table>
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<tr>
<th>m</th>
<th>r</th>
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<td>0.01</td>
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<td>10</td>
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</tr>
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</table>

Bringing the best design parameter set into theoretical calculation, the electrical power’s spectrum is obtained and plotted in Fig. 5. Both the theoretical electrical voltage spectrum and electrical circuit spectrum also can be calculated and illustrated in Figs. 6 and 7.

![Figure 5. The spectrum of induced electrical power.](image)

![Figure 6. The spectrum of induced electrical voltage.](image)

![Figure 7. The spectrum of induced electrical circuit.](image)

**Result and Discussion**

**Result**

As can be seen in Section 4, the electrical power will reach the maximum when $(D_{m1}, H_{m1}, D_{m2}, H_{m2}, M_0, k_0, k_1, k_2, N_{c1}, N_{c2}, N_{layer1}, N_{layer2}$, and $D) = (0.01, 0.01, 0.01, 0.01, 0.1, 27, 27, 27, 10, 10, 1, 1, 0.05)$. Here, both the diameter and height of magnet #1 and magnet #2 are at 0.01m. The stiffness constants of spring $k_0$, $k_1$, and $k_2$ are all selected at 27 (N/m). The mass of device housing ($M_0$) is at 0.1 (kg). The turns of coil for magnets #1 and #2 are all selected at 10 (turns). Both layers of coil for magnets #1 and #2 are at 1 (layer). As illustrated in Fig.6, the peak value of electrical power is $2.3 \times 10^{-5}$ (watt). The profile of Fig. 6 indicates that the two-magnet vibrational system is under resonance. It means that the external forcing frequency of current force is very close to the natural
frequency of the two-magnet energy harvester system. Moreover, as can be seen in Fig. 7, the peak value of electrical voltage with respect to magnets #1 and #2 are 0.08(V) and 0.005(V), respectively. In addition, as depicted in Fig. 8, the peak value of electrical circuit with respect to magnets #1 and #2 are at 0.00027(A) and 0.00002(A), respectively. Furthermore, the relative displacement and velocity of magnets #1 and #2 with respect to the device housing’s base are calculated and shown in Figs. 9 and 10. As revealed in Fig. 9, the peak value of relative displacement of magnets #1 and #2 are at 2.7(m) and 0.2(m), respectively. As illustrated in Fig. 10, the peak value of relative velocity for magnet #1 and magnet #2 are at 41(m/s) and 3(m/s).

Consequently, The absolute displacement of device housing (M₀), magnet #1 (M₁), and magnet #2 (M₂) with respect to time is calculated and shown in Fig. 11. As illustrated in Fig. 11, the peak values of device housing (M₀), magnet #1 (M₁), and magnet #2 (M₂) are at 2.7(m), 0.1(m), and 3.5(m), respectively.

Discussed

In case of $h=50$ (m), $y=5$ (m), $U_{ow}=3$ (m/s), $\lambda =3$ (m), $H=0.05$ (m), $R_{load} = 300$ (Ω), $\rho_s =1.025\times10^3$ (Kg/m³), and Strouhal number ($S_i$)=0.212, and $C_d=1$, an energy harvester with maximal electrical power has been assessed above. In order to initiate the influence of electrical power with respect to different number of magnets, a one-magnet energy harvester (shown in Fig. 12) under the same ocean environment has been reassessed and compared.
The electrical powers of the one-magnet and the two-magnet energy harvesters are compared and plotted in Fig. 13. Moreover, the comparison of electrical voltage between both one-magnet and two-magnet energy harvesters is also depicted in Fig. 14. As indicated in Fig. 13, the induced electrical power of the two-magnet energy harvester is ten times electrical power of the one-magnet energy harvester. Also, as illustrated in Fig. 14, the peak values of induced electrical voltage with respect to M₁ (the first magnet of the two-magnet energy harvester), M₂ (the second magnet of the two-magnet energy harvester), and M (the magnet of the one-magnet energy harvester) are 0.08(V), 0.007(V), and 0.004(V), respectively. Results reveal that the induced electrical voltage of the two-magnet energy is at least 400 times the electrical voltage of the one-magnet energy.

Consequently, the induced electricity of the two-magnet energy harvester is superior to that of the one-magnet energy harvester.

**Conclusion**

As can be seen above, ocean’s environmental factors (λ (wave length), y (depth of the energy harvester), H (wave amplitude), and h (height of the sea)) have essential influence for the current force which can induce the energy harvester’s electricity. Result reveals that a higher value of H and y and lower value of λ, h will produce more current force. As can be seen in Section 3.2, the geometric parameters (Dₘ₁, Hₘ₁, Dₘ₂, Hₘ₂, Mₒ, kₒ, k₁, k₂, N₁c₁, N₂c₂, N₁layer₁, N₂layer₂, and D) of energy harvester have tremendous influence for the electricity. A numerical optimization of the two-magnet energy harvester using the thirteen design parameters in conjunction with SA method has been performed. Results reveal the two-magnet energy harvester is under resonance and the electrical power reaches the maximum value of 2.3*10⁻⁵ (watt) when design parameter set (Dₘ₁, Hₘ₁, Dₘ₂, Hₘ₂, Mₒ, kₒ, k₁, k₂, N₁c₁, N₂c₂, N₁layer₁, N₂layer₂, and D) is at (0.01, 0.01, 0.01, 0.01, 0.01, 0.1, 27, 27, 27, 10, 10, 1, 1, 0.05). In addition, the peak value of electrical voltage with respect to magnets #1 and #2 are 0.08(V) and 0.005(V). And, the peak value of electrical circuit with respect to magnets #1 and #2 are 0.00027(A) and 0.00002(A). Compared to a one-magnet energy harvester, the induced electrical power of the two-magnet energy harvester is ten times electrical power of the one-magnet energy harvester. Also,
the induced electrical voltage of the two-magnet energy is at least 400 times electrical voltage of the one-magnet energy. Consequently, the increment of magnet in the energy harvester may produce more induced electricity.

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References


