An Approach of Trajectory Estimation for High-speed Unmanned Skid-steered Vehicle

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Abstract

With robust structure and high maneuverability, skid-steered vehicles have been widely used in terrain exploration, construction, rescue and relief fields. Inevitable slipping and sliding of the tire that makes the vehicle status difficult to obtain. In this paper, an equivalent differential driven kinematic model is proposed. Additionally, the mapping relationship of slippage coefficient defined in the original 4-wheel unmanned skid-steered vehicle and the wheel location in the equivalent model are discussed. The proposed equivalent wheel location obtains explicit physical significance. The slippage of the tire has also been calculated by estimating the position of instantaneous center of rotation (ICR) of the wheels from multi-sensors. An unscented Kalman filter (UKF) based fusion method is adapted to obtain the vehicle status. A prototype vehicle named DUBHE is adapted to verify the reliability of the informed method. Preliminary experimental results are compared to demonstrate the effectiveness of the method in different scenarios.

Keywords: unmanned skid-steered vehicle, state estimation, unscented Kalman filter, equivalent kinematic model

Nomenclature

Abbreviation

ICR     Instantaneous Center of Rotation
UKF     Unscented Kalman Filter
USSV    Unmanned Skid-steered Vehicle
SSMR    Skid-steered Mobile Robot
EKF     Extended Kalman Filter
GA      Generic Algorithm
IMU     Inertia Measurement Union

Symbols

V      Speed
ϕ      Yaw angular

1 Introduction

Unmanned skid-steered vehicle (USSV) also known as the skid-steered mobile robot (SSMR) [1] is mechanically robust and the extended contact surface (through wheels or tracks) makes the system well adapted for terrain road application. Benefited from the mechanical features, the USSVs perform simpler and more robust than the Ackermann steering vehicles. Additionally, higher traction can be achieved by USSVs rather than the differential steering vehicles [2].

Kinematic model of the skid-steered vehicle has been studied in many research. A traditional approach is to estimate the instantaneous center of rotation (ICR) of the two-side wheels or tracks so that the locomotion states can be calculated.

In research [3], J. L. Martínez discussed a geometric equivalence of instantaneous motion for tracked vehicle under a maximum speed of 4km/h. The proposed model can be tuned by means of off-line parameter identification, which can be derived either from the simulation of a dynamic model or from actual experimental data. Results stated that the asymmetric
ICR model can obtain better performance than the symmetric one. In 2007, Anthony Madow [4] modeled a skid-steered mobile robot to equivalent differential drive kinematic. The proposed kinematics is similar to the one discussed in this paper. Yet, the mapping relationship between the equivalent speed and the original wheel speed wasn’t mentioned in their research. GA algorithm was also adapted in their method to obtain the ICR locations. Implemented in a P3-AT robot, the mean squared error can be reduced to 0.0424. Asymmetric model also performed more accurately than the symmetric model. To our knowledge, uncertain and variable driving conditions call for real-time method to measure the states of ego vehicle. A KF based real-time estimation method was informed by Jingang Yi [5]. They presented a low-cost IMU-based localization and slip estimation scheme for skid-steered mobile robots. They considered the wheel slip while calculating the dead-reckoning that makes the results more reliable. An EKF method was designed to estimate the track ICR in [6]. The researchers didn’t consider the tracks’ relative speed to the body which may reduce the accuracy of the adapted model. Some basic assumptions were applied as low speed, hard and flat terrain. Three kinds of scenarios are designed to validate the effectiveness of the estimation method. In the long-distance traversal scenario, 95% of occurrences was achieved below 1.6m.

Among all the methods mentioned before, the ego vehicle is operating in a relative low speed less than 10 km/h. When the speed of the vehicle accelerates, more critical slippage may occur which will lead to divergence of the estimation algorithm. In fact, a relative high speed is required considering the operation efficiency in industrial implementations. The other hand, rare physical significance of the ICR locations is comprehensively described even though the derived expressions are presented in some researches. In this paper, an equivalent differential driven vehicle kinematic model is derived and discussed elaborately. The mapping relationship between the original kinematic and the equivalent kinematic are analyzed detailed. An UKF-based fusion method is represented combining the encoders, IMU and GPS. An experimental USSV prototype is introduced and some tests are conducted for performance verification under a relative high motion.

2 Modeling of the USSV

2.1 Equivalent motion analysis

A kinematic model of a skid-steered wheeled vehicle maps the wheel velocities to the vehicle velocities and is an important component in the development of a dynamic model. The local frame of the vehicle is assumed to have its origin on the body center of gravity (CG). As most vehicle coordinates, the X-axis aligned with the forward motion direction. The Y-axis aligned with the left direction. Linear motion is relatively simple so that can be described in the later curvilinear motion analysis.

When a skid-steered vehicle is steering left, the locomotion state can be illustrated as figure 1. $ICR_x = (x_1, y_1)$ is the instantaneous center of rotation of the body. $ICR_L = (x_3, y_3)$ and $ICR_R = (x_4, y_4)$ represent the ICR of the contact point of the ground and tires. Based on such a kinematic model, some researches has been done to estimate the wheel slipping and dead-reckoning [7]. However, rarely analysis and experiment have been proposed to identify its physical significance. That may lead to some misunderstanding when some minor transformations are applied in the modeling process. To simplify the modeling process and clarify the physical significance, the equivalent modeling method [3] is put forward in this section.

The locations of equivalent wheels are defined like the ICRs of the two-sides original wheels. Pure rotation of the wheel can be obtained in the equivalent differential driven wheeled skid-steered vehicle model. Generally, the equivalent model is derived as nine conditions as shown in figure 2. In the condition A, the assumption of none lateral speed is given, so that the two-side wheel and $ICR$, stay on a line on the y-axis. Model A-1 represents the same magnitude of two-side wheels’ slipping. Model A-2 represents the more critical slippage occurs at the left wheel. Similarly, the Model A-3 shows the case when the right wheel conduct more significant slipping. Normally, when the skid-steered vehicle is turning, the lateral slippage is about to happen determined by the complex interaction of tire and terrain. So the mentioned scenario can be described as condition B and C. Condition B refers to a situation when the lateral speed is negative, in the opposite direction of the y-axis. Condition C refers to the positive lateral slipping speed in the local body frame.
Also, according to the slipping degree of the two-side wheels, three kinds of model are listed in each condition. Within the discussed equivalent kinematic model, the vehicle motion can be described explicitly and roundly.

2.2 Kinematic modeling

In the local body coordinate, the rotational speed of its left and right wheels governs a skid-steered vehicle. Ideally, direct kinematics of the vehicle can be stated as follows:

\[
(v_x, v_y, \phi) = f(V_l, V_r)
\]

where \( V = (v_x, v_y, \phi) \) is the vehicle's velocity vector with respect to its local frame. And the \( V = (V_l, V_r) \) is the nominal linear velocity of its left and right wheels which can be derived from the rotational speed.

Considering the actual situation, the significant slippage of left or right wheels happens inevitably. A more authentic model can be written as:

\[
(v_x, v_y, \phi) = f(V_l, V_r, y_l, y_r, x_c)
\]

where \( y_l, y_r \) and \( x_c \) are the factors of the locations of the equivalent wheels defined before.

Applying the proposed mapping relationship on a particular USSV, a more concrete kinematic model can be obtained as follows:

\[
\begin{bmatrix}
v_x \\
v_y \\
\phi
\end{bmatrix} = A
\begin{bmatrix}
V_l \\
V_r
\end{bmatrix}
\]

(3)

where \( A \) is calculated as:

\[
A = \begin{bmatrix}
-y_l & y_r \\
y_l - y_r & y_l - y_r \\
-x_c & x_c \\
-1 & 1 \\
y_l - y_r & y_l - y_r
\end{bmatrix}
\]

(4)

To our knowledge, the proposed kinematic model expression agrees well with the model in research [6]. Yet, Jesse P. treated the \( V_l \) and \( V_r \) as the velocity of the tracks relative to the body. It important to notice that the wheel is slipping/sliding significantly or slightly all the time, which is decided by the inherent mechanical characteristics of USSV. So in this research, \( V_l \) and \( V_r \) represent the velocity of the center of left wheel and right wheel respectively. Considering the wheel's slipping and sliding, the longitudinal slipping coefficient [5] need to be defined and expressed as:

\[
\lambda = \frac{r \omega_i - v_i}{r \omega_i}
\]

(5)

where \( r \) is the efficient wheel radius. And \( \omega_i \) is the rotational speed of wheels. So the velocity of wheel center can be obtained as follows:

\[
\begin{bmatrix}
V_l \\
V_r
\end{bmatrix} = \begin{bmatrix}
r(1 - \lambda) & 0 \\
0 & r(1 - \lambda)
\end{bmatrix}
\begin{bmatrix}
\omega_l \\
\omega_r
\end{bmatrix}
\]

(6)

The defined longitudinal slipping coefficient can also be utilized to obtain an explicit physical significance,
combined with the proposed equivalent kinematic model. Similar to the longitudinal slipping coefficient, the locations of the equivalent wheels can be interpreted as the magnitude of the wheel’s slippage. The mapping relationship of the locations of the equivalent wheels and the wheel’s longitudinal slipping coefficient of the original vehicle is written as:

\[
y_i = \frac{r_0 \lambda_i}{\phi} + \frac{B}{2}
y_r = \frac{r_0 \lambda_r}{\phi} - \frac{B}{2}
\]  
(7)

where \(B\) represents the wheel base of the USSV. The longitudinal slipping coefficient is defined to describe the wheel’s motion of the original vehicle so as the location of equivalent wheel is to describe the motion of the equivalent differential driven model. For example, when no slippage happens in all the two-side wheels, the location of equivalent wheel \((y_i, y_r) = (B/2, -B/2)\), that is illustrated as model X-1 in figure 2. And the longitude coordinate of the ICR of equivalent model describes the lateral slippage in original model. Based on that, the linear relationship can be easily comprehended.

3 UKF based fusion method

This section will discuss the formulation of an UKF-based fusion method for the estimation of the kinematic trajectory. In order to obtain the motion states of the USSV, internal sensors like encoders and inertia measurement union (IMU) are placed on the vehicle. Accurate locomotion trajectory of the ego vehicle is required at motion control and planning.

![Diagram of the UKF fusion method](image)

**Figure 3** Diagram of the UKF fusion method

A kind of multi-sensor fusion method is introduced as illustrated in figure 3. Basically, nominal rotation speed \(\omega_i\) of the wheel can be obtained from encoders despite the wheel’s slipping. Combined with the IMU and GPS, a more accurate information is supposed to be obtained. Discretized using the Euler method, the speed of the vehicle in the local coordinate can be written as,

\[
\begin{bmatrix}
    v_{x,k+1} \\
v_{y,k+1} \\
\phi_{k+1}
\end{bmatrix} =
\begin{bmatrix}
    v_{x,k} \\
v_{y,k} \\
\phi_k
\end{bmatrix} +
\begin{bmatrix}
a_x \Delta t \\
a_y \Delta t \\
a_\phi \Delta t
\end{bmatrix}
\]

(8)

where \(\Delta t\) is the time step. \(a_x\), \(a_y\) and \(a_\phi\) are the accelerations aligned with the forward, lateral and z-rotate direction respectively. The noise of the IMU will be considered as a part of predicting process noise in the discussed approach.

The UKF fusion method utilizes the mentioned model Eq. (8), denoted compactly as

\[
\tilde{X}_k = X_{k-1} + Q
\]

(9)

where \(\tilde{X}_k\) is time update variable. \(Q\) donates the prediction noises. As mentioned before, the prediction noises can be written as \(Q = diag(\sigma_1^2, \sigma_2^2, \sigma_3^2)\), \(\sigma_1^2 = a_x^2 \Delta t^2\) is the variances for forward speed prediction related to the IMU measurement, \(\sigma_2^2 = a_y^2 \Delta t^2\) is the variances for lateral speed prediction, \(\sigma_3^2\) is the variances for yaw speed prediction.

Both the encoders and GPS measurement are utilized in the correction process. The measurement model is written as follows

\[
\tilde{Z}_k = H\tilde{X}_k + R
\]

(10)

where \(H = [diag(1,1,1), diag(1,1,1)]^T\). The correction matrix combined with encoders and GPS are written as

\[
Z_k = \begin{bmatrix}
    Z_{encoder} \\
    Z_{GPS}
\end{bmatrix} =
\begin{bmatrix}
    \frac{r_0 \omega_1 + r_0 \omega_2}{2} \\
0 \\
\frac{r_0 \omega_1 - r_0 \omega_1}{B} \\
v_N \sin \phi + v_E \cos \phi \\
v_N \cos \phi - v_E \sin \phi \\
\phi_{imu}
\end{bmatrix}
\]

(11)

where \(v_N\) and \(v_E\) are the velocity aligned with north and east direction measured by GPS. \(\phi\) is the heading angular of the vehicle also can be obtained by GPS.

The key part of the fusion method is the measurement noises tuning. The noise matrix is written as

\[
R = diag(r^2 \sigma_4^2, \sigma_5^2, r^2 \sigma_6^2, \sigma_7^2, \sigma_8^2, \sigma_9^2)
\]

(12)

where \(\sigma_4^2\) and \(\sigma_6^2\) are the variances of encoder. \(\sigma_7^2, \sigma_8^2\) and \(\sigma_9^2\) are the variances of GPS that can be obtained in the product's specification.
The UKF fusion method implemented for the mentioned USSV can be written as a prediction step and a correction step recursively as follows

\[
X_{k|k} = \tilde{X}_{k|k-1} + K_{k|k-1} (Z_k - \tilde{Z}_{k|k-1})
\]

\[
P_{k|k} = P_{k|k-1} - K_{k|k-1} P_{k|k-1} P_{k|k-1}^T
\] (13)

The sigma sampling process and unscented transform process are applied in the same way as research [8]. It’s important to state that the fusion model is linear in terms of the kinematic variables. The sigma sampling process and transform process of UKF that is used to present the nonlinear performance of the locomotion of the USSV.

4 Test platform and experimental validation

4.1 Experimental prototype

As shown in figure 4, the 4-wheel USSV named as DUBHE is designed and manufactured by Special Vehicle lab. in Beijing Institute of Technology. The prototype vehicle measures approximately 1.2m in length, 0.9m in width and 0.5m tall. The distance between the front axle and rear axle is 0.79m.

![Figure 4 Experimental USSV DUBHE](image)

Two DC motors are placed on each side to provide the driving torque. The maximum speed of DUBHE can reach 30km/h. A synchronous belt is designed to guaranteed the same-side wheel driven by equal torque and driving in the same speed constrain. All the equipment like actuators and sensors are powered by a lithium battery with the rated volt of 48v. The vehicle can operate in two modes: remoted control mode and self-driving mode. In self-driving mode, the manipulating commands are computed in the planning computer then published to the vehicle control union (VCU). In this research, the USSV works on the remoted control mode. The manipulating commands are published from a remoted computer directly.

The proposed UKF scheme is implemented in the VCU. The INS/GPS messages are transited from the computer and the encoders’ data transfers through the CAN-bus into the VCU. When finished, the results will go back to the computer for further computing. All the communicating process in computer are programed in ROS architecture.

4.2 Results discussion

All experiments are conducted on the prototype DUBHE. Field-testing are performed on a soft gravel road including three typical scenarios: high-speed curvilinear maneuver, small radius turning maneuver and high-speed turning maneuver.

For the first test presented, to analyze the performance of the proposed estimation method under a relative high angular (yaw) speed and a low forward speed. The kinematic trajectory known as forward speed, lateral speed and yaw speed is shown in figure 5. As seen that, significant gaps between the encoders and GPS are presented among all the three variables. That is to say, the severe longitudinal and lateral slippage appears when driving. As illustrated in figure 5(b), the lateral slipping speed cannot be obtained from encoders for the nonholonomic constrains but the GPS with serious noise. The proposed estimate method can reach a weighted value between the encoder measurement and GPS. To some extent, the estimate method can present the maneuver of the vehicle. In the second scenario, the ego vehicle receives only the command to driving forward. Notice that the forward speed of the USSV is up to 15km/h, much higher than the researches mentioned before. In figure 6(c), it’s seen that yaw speed increases to some degree when the forward speed is changing. The estimate method can obtain a relative steady and accurate value than the encoder and GPS. Also, the forward speed calculated from three methods agree well with each other that means slight longitudinal slippage existing.

![Figure 5 Kinematic trajectory of the USSV at turning maneuver](image)
Figure 6 Kinematic trajectory of the USSV at curvilinear maneuver

Figure 7 Path trajectory of the USSV at turning maneuver

An open-loop implement of the three variable estimation are presented to calculate the estimation path trajectory off-line. In the process, the yaw angular measured by GPS are directly substituted in the trajectory calculation. The estimated trajectory performs well compared to the encoders and GPS trajectory.

5 Conclusion and future work

Wheeled skid-steered robot are used widely for robotic navigation applications. Copious studies have been carried out to discuss the kinematic and dynamic modeling method of the robot for accurate description of its locomotion. The equivalent kinematic model proposed in this paper shows no much newer than the previous work. However, explicit physical significances of all the parameters of the equivalent model are discussed. The consistency also approves the correctness of the proposed kinematic model.

Moreover, the discussed USSV operates at a higher velocity with more severe slippage of the wheels to lead the status of the vehicle harder to obtain accurately. The proposed kinematic trajectory estimation method performs good not only under turning maneuver where severe slippage occurs but also at high driving velocity. Actually, the wheel slippage is still treated as group of compensation factors in the discussed kinematic based estimation method. Afterwards, the estimated results of the three kinematic states can be utilized to obtain the slipping parameters combined the encoder data. Then, the conducted slipping parameters will be substituted into the dynamic models for the analysis of the motion of tires.

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Reference