Mass and Slope Estimation of Electric Vehicles Equipped with AMT Based on Recursive Least Square Method with Multiple Multiple Forgetting Factors

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Abstract

Mass and road gradient influence the dynamic and economic performance of electric city bus equipped with AMT. Therefore, the influence of mass and road gradient must be taken into consideration when formulating the gear shifting rule. In this paper, recursive least square method with multiple forgetting factors is proposed to estimate vehicle mass and road gradient online, and a simulation was carried out based on NEDC working condition, the simulation results show that the proposed algorithm can effectively estimate vehicle mass and road gradient.

Keywords: Electric City Bus, AMT, Mass Estimation, Road Gradient Estimation

Nomenclature

AMT Automated Mechanical Transmission
RLS Recursive Least Square

1. Introduction

Environmental Pollution and Energy Security are great challenge that China must face the development of automobile industry [1], with government's support, electric vehicles are becoming an industry trend as consumers seeking ways to reduce emissions to protect the environment in China.

In terms of powertrain, most electric vehicles are still equipped with the direct drive system, of which performance of climbing and high-speed driving is weaken. In addition, the direct driving system needs to be equipped with high torque motor, it makes the powertrain not only expensive, but also inefficient, and restricts the development of electric vehicles.

Because of its high-mass and complex driving conditions, the powertrain of the electric vehicles should be equipped with AMT as figure 1 to [2] improve the performance of the vehicle and meet the diversified demands of the market.
the situation of cyclic shifting due to insufficient dynamic caused by shifting to high gear; in the process of vehicle driving, the slope needs to be estimated in real time. The general slope estimation method is loading the sensor on the vehicle for direct measurement [3, 4], which increases the equipment cost, the vehicle acceleration can also be calculated according to the speed signal [5, 6], and then the slope can be estimated. The disadvantage of this method is that the estimation of vehicle acceleration has a large error, which leads to a large error of the slope estimation.

To solve the problem of inaccurate parameter estimation, a method based on the recursive least square method with multiple forgetting factors (RLS method) is proposed in this paper to online estimate the unknown parameters: vehicle mass and road slope. Meanwhile, the simulation is carried out based on NEDC working condition, the simulation results show the validity of the proposed method.

2. The recursive least square method

The least square method is a method for solving contradictory equations. The methods of identification and parameter estimation are introduced to estimate the parameters of the difference equation model. The least square method has the advantage of recursive calculation, which is convenient for batch processing of data. It is simple to calculate compared with other algorithms. Therefore, the least square method is the most widely used parameter estimation method in the field of system identification, and it is also the most commonly used method to obtain the best function to match the system data [7].

An n-order MA mathematical model can be expressed as follows.

\[ y(t) = b_0 u(t) + b_1 u(t-1) + \cdots + b_n u(t-n) \]  

(1)

In equation (1), the output \( y(t) \) and input \( u(t), u(t-1), \ldots, u(t-n) \) are both measurable and the parameter \( b_0, b_1, \ldots, b_n \) are unknown to be estimated. Using the matrix to represent the MA model, the following expression can be obtained.

\[ y(t) = \phi^T(t) \theta \]  

(2)

where,

\[ \phi^T(t) = [u(t), u(t-1), \ldots, u(t-n)] \]

\[ \theta(t) = [b_0, b_1, \ldots, b_n] \]  

(3)

If the structure and order of the model are correct and the measured data do not contain noise, the estimation of the unknown is the solution of the first order deterministic equations with \( n+1 \) elements. The unknown parameters can be solved after \( n+1 \) times of measurement on data \( y \) and \( u \). All the equations can be expressed as follows.

\[ y(n+1) = b_0 u(n+1) + b_1 u(n) + \cdots + b_n u(1) \]

\[ y(n+2) = b_0 u(n+2) + b_1 u(n+1) + \cdots + b_n u(2) \]

\[ \vdots \]

\[ y(2n+1) = b_0 u(2n+1) + b_1 u(2n) + \cdots + b_n u(n+1) \]  

(4)

It can be written in matrix form.

\[ Y^T = \Phi \theta \]

where \( Y \) is the output vector, \( \Phi \) is the observation matrix, and \( \theta \) is the parameter vector. However, noise is inevitably contained in the observation matrix and the output vector, and the equation (5) becomes a contradiction equation.

Considering the error vector \( E \) in the equation, the system model is changed to the MA differential model:

\[ Y^T = \Phi \theta + E \]

(5)

Model residual is as follows:

\[ Y = y(t) - \phi^T(t) \theta \]

(6)

From the least squares method, the estimated unknown parameter \( \hat{\theta}(t) \) is obtained to minimize the sum of squares of residuals, that is:

\[ J = \sum_{i=1}^{N} \varepsilon^2(i) = \sum_{i=1}^{N} [y(i) - \phi^T(i) \hat{\theta}]^2 \]

(7)

\[ J = \sum_{i=1}^{N} \varepsilon^2(i) = \sum_{i=1}^{N} [y(i) - \phi^T(i) \hat{\theta}]^2 \]

(8)

The cost function \( J \) should be minimized, where \( N \) is the total number of measurements. The \( \hat{\theta} \) which minimizes the cost function \( J \) is the least squares estimation value \( \hat{\theta}(t) \) of the parameter to be estimated. The following formula can be obtained by

\[ \left. \frac{\partial J}{\partial \theta} \right|_{\theta=\hat{\theta}} = 0 \]

\[ \hat{\theta} = [\Phi^T \Phi]^{-1} \Phi^T Y \]

(9)

Equation (9) is the general form of least squares estimation. However, in vehicle parameter estimation, real-time calculation and identification are necessary and least squares recursive calculation should be implemented in the process. The update form of recursive least squares method is as follows:
\[
\hat{\theta}(k) = \hat{\theta}(k-1) + L(k) ( y(k) - \phi^T(k) \hat{\theta}(k-1) ) \\
L(k) = \frac{P(k-1) \phi(k) }{ 1 + \phi^T(k) P(k-1) \phi(k) } \\
P(k) = P(k-1) - P(k-1) L(k) \phi^T(k) 
\]

Where \( k \) is the number of steps in the recursive calculation, \( P(k) \) is the covariance matrix, and \( L(k) \) is the gain matrix. The figure below is the flow of the recursive RLS algorithm.

**Figure 2** Flowchart of the recursive RLS algorithm

Equation (10) can make a valid estimation when the parameters to be estimated remain unchanged, but the vehicle parameters, especially the road gradient parameters, may be time-varying. At this time, the conventional RLS method will lose the ability to estimate the parameters. The RLS method with forgetting factor is introduced to deal with the problem of time-varying parameter estimation. The collected data are weighted by the forgetting factor, that is each term of the sum is multiplied by an exponential weighting coefficient \( \lambda^{k-1} \) in the performance index function. The essence of the forgetting factor is the weighting of the collected data. As the sampling time going on, weight of the collected data will decrease at the beginning and the data are gradually "forgotten", so that the newly collected data play a role in the estimation and identification. The unknown parameter to be estimated is derived from this as follows:

\[
\hat{\theta}(k) = \hat{\theta}(k-1) + L(k) ( y(k) - \phi^T(k) \hat{\theta}(k-1) ) \\
L(k) = \frac{P(k-1) \phi(k) }{ \lambda + \phi^T(k) P(k-1) \phi(k) } \\
P(k) = ( P(k-1) - P(k-1) L(k) \phi^T(k) ) / \lambda 
\]

When multiple unknown parameters need to be estimated at the same time, and the rate of change of each parameter to be estimated varies greatly, it is necessary to introduce different forgetting factors for different parameters to be estimated, which is called RLS method with multiple forgetting factors. In this paper, two unknown parameters, vehicle mass and road gradient, should be estimated in real time. The expressions of the two parameters to be estimated are:

\[
\dot{\hat{\theta}}(k) = \hat{\theta}(k-1) + K(k) ( y(k) - \phi(k) \hat{\theta}(k-1) - \phi(k) \hat{\theta}(k-1) ) \\
\dot{\hat{\theta}}(k) = \hat{\theta}(k-1) + M(k) ( y(k) - \phi(k) \hat{\theta}(k-1) - \phi(k) \hat{\theta}(k-1) ) \\
K(k) = \frac{ P(k-1) \phi(k) }{ \lambda_1 + \phi^T(k) P(k-1) \phi(k) } \\
M(k) = \frac{ P(k-1) \phi(k) }{ \lambda_2 + \phi^T(k) P(k-1) \phi(k) } \\
P_1(k) = ( P(k-1) - P(k-1) K(k) \phi^T(k) ) / \lambda_1 \\
P_2(k) = ( P(k-1) - P(k-1) K(k) \phi^T(k) ) / \lambda_2 
\]

According to the above formula, real-time online estimation of two time-varying parameters of vehicle mass and road gradient can be realized.

### 3. Online identification of vehicle mass and road gradient

In order to facilitate the use of the RLS method for parameter estimation of vehicle mass and road grade, according to the vehicle dynamics equations:

\[
\delta \cdot \frac{du}{dt} = \left( \frac{C_m \cdot u_m \cdot \cos \alpha}{r} - \frac{C_m \cdot u_m^2}{2 \cdot 11.5} \right) \cdot \frac{1}{m} \cdot \frac{1}{\cos \alpha_f} \cdot \sin(\alpha + \alpha_f) 
\]

In equation (13), \( \tan(\alpha_f) = f \).

The matrix form can be written as:

\[
y(t) = \phi^T(t) \Theta, \phi = [\phi_1, \phi_2]^T, \Theta = [\theta_1, \theta_2]^T 
\]

In the equation, \( \Theta \) is the estimated parameter.

\[
\theta = \left[ \frac{1}{m}, \sin(\alpha + \alpha_f) \right] 
\]

\[
y = \delta \cdot \frac{du}{dt} \Theta_1 = \left( \frac{C_m \cdot u_m \cdot \cos \alpha}{r} - \frac{C_m \cdot u_m^2}{2 \cdot 11.5} \right) \cdot \frac{1}{\cos \alpha_f} \cdot \sin(\alpha + \alpha_f) 
\]

(16)

All parameters of the equation (16) are known. The longitudinal acceleration \( du/dt \) of the vehicle can be obtained by the differential of the vehicle speed, and the output torque \( T_m \) of the driving motor can be calculated by the accelerator pedal opening angle and the vehicle speed table.

In this section, the RLS method with multiple forgetting factors is used to estimate the vehicle mass and road gradient online. Two forgetting factors are introduced, \( \lambda_1 \) corresponds to mass \( m \), and \( \lambda_2 \) corresponds to road gradient \( \sin(\alpha + \alpha_f) \). Since the vehicle mass remains basically unchanged for a period of time, its corresponding forgetting factor can take a large value, taking 0.99; and in order to maintain the
sensitivity of the road grade, the corresponding 
forgetting factor should be small, take 0.5. For the 
estimation of the unknown parameters, the initial value 
setting of the identification parameters will affect the 
convergence condition of the parameters to be 
estimated, so the initial value setting of each parameter 
is very important. In the recursive calculation, the initial 
value of the unknown parameter is set to a sufficiently 
small real number, and its covariance is taken as a 
larger real number. The initial values are set as follows:
\[ \hat{\theta}_1(0) = 0.01, \hat{\theta}_2(0) = 0.01, P_1(0) = 1000, P_2(0) = 1000 \] 

(17)

4. Simulation results

Online parameter estimation is based on the S 
Function function constructed by the recursive least 
squares method in the Simulink model, as shown in 
Figure 3. The vehicle simulation model can be seen in 
Figure 4. The simulation condition is NEDC working 
condition, and the simulation vehicle mass is 16500kg. 
In addition, Gaussian white noise with a variance of 
0.001 is superimposed on the vehicle acceleration 
signal during simulation.

Figure3 Simulation module for online parameter 
estimation for vehicle quality and road gradien

Figure4 The vehicle simulation model

The simulation results for vehicle mass and road 
grade are shown in Figure 5 and Figure 6. It can be 
seen from the simulation results that the online 
estimation of the vehicle mass is different from the 
actual mass by 1.1%. The road grade estimation is 
basically consistent with the actual value, and the 
change of the road gradient can quickly and timely 
response. Using the recursive least squares method 
with multiple forgetting factors is effective for vehicle 
mass and road grade estimation.

Figure5 Estimation simulation results of vehicle mass

Figure6 Estimation result of road gradient

5. Conclusions

In this paper, a recursive least square (RLS) 
algorithm with multiple forgetting factors was proposed 
to estimate the vehicle mass and road slope online. 
The simulation results show that the proposed 
algorithm can effectively estimate the vehicle mass and 
road slope. However, the paper didn’t carry out further 
work to optimize the shifting rule according to the 
estimated parameters. The subsequent work would be 
carried out in the future: optimizing the shifting rule 
according to the algorithm proposed in the paper, and 
conducting simulation based on the time-varying mass 
and gradient condition to verify the effectiveness of the 
proposed algorithm.

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Reference