Research on Timetable Optimization for Charging Capacity Reduction in Supercapacitor-powered Urban Rail Transit

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Abstract
For supercapacitor-powered urban rail transit, its fast charging energy supply mode and possible scenario of multiple vehicles charging simultaneously may incur intermittent high-power charging demand to the grid, challenging the safety and economy of the whole system. In this paper, we adjust dwell duration of vehicles at each station to minimize the superposition of charging power resulted from simultaneous stops of vehicles, thereby reducing the capacity of the substation as well as the power stress of the grid. Firstly, in analogy with the related concept in the electric power system, the coincidence factor in urban rail transit is defined to describe the superposition of charging power. Then a timetable optimization model with dwell duration as the decision variable is developed to minimize the maximum coincidence factor and Particle Swarm Optimization is implemented to find the optimal solution. Furthermore, a case study is presented based on the data from Haizhu Line in Guangzhou, China. The result shows that the optimized timetable can reduce the maximum coincidence factor by 20% in comparison with the current used timetable.

Keywords: urban rail transit, timetable optimization, supercapacitor, charging capacity reduction, Particle Swarm Optimization

Nomenclature

**Abbreviation**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>PSO</td>
<td>Particle Swarm Optimization</td>
</tr>
<tr>
<td>EV</td>
<td>electric vehicle</td>
</tr>
<tr>
<td>PEV</td>
<td>plug-in electric vehicle</td>
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</tbody>
</table>

**Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_D$</td>
<td>Headway</td>
</tr>
<tr>
<td>$T_c$</td>
<td>Duration of an operation cycle</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Total dwell duration</td>
</tr>
<tr>
<td>$t_{ai,j}$</td>
<td>Arrival time for vehicle $i$ at station $j$</td>
</tr>
<tr>
<td>$t_{dij}$</td>
<td>Departure time for vehicle $i$ at station $j$</td>
</tr>
<tr>
<td>$t_{ij,i+1}$</td>
<td>Travel duration from station $j$ to station $(j+1)$</td>
</tr>
<tr>
<td>$t_{s_j}$</td>
<td>Dwell duration at station $j$</td>
</tr>
<tr>
<td>$c$</td>
<td>Coincidence factor</td>
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</table>

1. Introduction

In recent years, urban rail transit plays an increasingly important role in the public transportation system due to the advantages of large volume, fast speed, high efficiency and good punctuality [1]. Meanwhile, the application of energy storage system as power source frees urban rail vehicles from the constraint of catenary, making braking energy recovery and visual pollution abatement to be possible [2].

At present batteries and supercapacitors are the two most mature energy storage devices. In consideration of the frequent-stops and short-distance features of urban rail vehicles, supercapacitors are often used as the on-board energy storage system and are charged quickly during the short dwell duration at each station. Obviously, this energy supply mode has intermittent high-power charging demand to the grid. More seriously, the possible scenario of multiple vehicles charging simultaneously will further aggravate the charging power stress on the grid, bringing a new challenge to the safety and economy of the whole system. Therefore, to ensure the long-term development of supercapacitor-powered urban rail transit, much attention should be paid to reducing the charging capacity of the whole system.

In the literatures, charging scheduling for EVs has been heavily investigated [3-4]. Xu et al. proposed a coordinated charging strategy for PEV charging stations to achieve peak shaving based on dynamic time-of-use tariffs [5]. He et al. formulated a globally scheduling optimization problem to minimize the total cost of all EVs...
and proposed a locally optimal scheduling scheme to handle the random arrivals of vehicles [6]. Benetti et al. proposed a real-time control strategy for EV charging processes on the basis of a tight interaction between the scheduling algorithm and the power-flow evaluation procedure [7]. Compared with EVs, urban rail vehicles operate in accordance with timetables and thus have strong regularity. An extensive study on timetable optimization has been made with different objectives. In [8], Yang et al. proposed a cooperative scheduling model to increase the utilization of braking energy by synchronizing the accelerating and braking time of successive trains as much as possible. In [9], an energy-efficient train operation model was established based on the real-time interstation running time monitored by the automatic train supervision system. [10] presented a multi-objective train scheduling model by minimizing the energy and carbon emission cost as well as the total passenger-time.

In terms of reducing charging capacity of supercapacitor-powered urban rail transit, optimizing the timetable is the most direct method on the premise that the configuration of energy storage systems and operation characteristics of vehicles remain unchanged. In this paper, in analogy with the related concept in the electric power system, we define coincidence factor in urban rail transit to describe the superposition of charging power. Then by tuning the dwell duration at each station, the relative positions of multiple vehicles on the time axis are adjusted to minimize the coincidence factor of the line. In this way the peak charging power demand of the whole line to the upper grid is reduced and the operation cost of the whole line can be cut down with lower capacity charge of the substation. What’s more, the stability of the grid is enhanced due to the decrease of the charging power stress.

The rest of the paper is organized as follows. Section 2 defines the coincidence factor in urban rail transit and formulates the problem in the form of matrices. On this basis, a timetable optimization model is developed in Section 3 and then solved by PSO. Section 4 presents a case study based on the data from Haizhu Line in Guangzhou and Section 5 gives the conclusion.

2. Problem formulation

2.1 System description

Considering a public transportation system with $M$ supercapacitor-powered urban rail vehicles running on the line, we define the up direction as Stop 1 to Stop N-1 and the down direction as Stop N+1 to Stop 2N-1. Stop N and Stop 2N are the turn-back stations in the up direction and down direction, respectively. The route map of the above urban rail transit system is shown in Fig.1.

To simplify the model, we make the following assumption:

1. The dwell duration of vehicles at each station is equal to the charging time of on-board supercapacitors.
2. All vehicles follow the same operation strategy and have the same dwell duration at a certain station. The only difference is that the latter vehicle follows the former one with a fixed headway [11].
3. There is only one substation for the whole line.
4. The headway between vehicles is always constant throughout the day.
5. All parameters and variables are assumed to be integers except for the coincidence factor.

2.2 Problem formulation

By analogy to the related concept in the electric power system, we define the coincidence factor $c$ in urban rail transit as follows, which refers to the ratio of the number of vehicles charging simultaneously on the line to the total number of vehicles in the system.

$$c = \frac{m}{M}$$  \hspace{1cm} (1)

Taking the arrival moment of vehicle 1 at Stop 1 as the initial time, i.e. $t^a_{1,1}=0$, we can get the arrival time vector $T_a$ and the departure time vector $T_d$ of vehicle 1 in the first operation cycle, as shown in Eq. (2) and (3).

$$T_a = \left[ t^a_{1,1} \ t^a_{1,2} \ t^a_{i,3} \ \ldots \ t^a_{i,2N} \ t^a_{i,2N+1} \right], T_a \in R^{1\times(2N+1)} \hspace{1cm} (2)$$

$$T_d = \left[ t^d_{1,1} \ t^d_{1,2} \ t^d_{i,3} \ \ldots \ t^d_{i,2N-1} \ t^d_{i,2N} \right], T_d \in R^{1\times2N} \hspace{1cm} (3)$$

Based on the time vectors of vehicle 1 and the timetable of urban rail transit system, the time vectors of vehicle $i$ in the $k$-th cycle can be achieved as follows:
\[ T_{ai} = \left[ t_{i,1}^a, t_{i,2}^a, \ldots, t_{i,2N}^a \right] + (k-1) \times T_e + (i-1) \times T_D, \quad T_{ai} \in \mathbb{R}^{1 \times 2N+1} \]  
(4)

\[ T_{di} = \left[ t_{d,1}^d, \ldots, t_{d,2N}^d \right] + (k-1) \times T_e + (i-1) \times T_D, \quad T_{di} \in \mathbb{R}^{1 \times 2N} \]  
(5)

Subsequently, we divide an operation cycle into \( T_e \) time slots and define the charging-enabled vector \( f_i \) of vehicle \( i \) as follows.

**Definition:** For the \( t \)-th time slot \((1 \leq t \leq T_e)\), if \( p_{i,ij} \leq t \leq p_{i,ij}^\prime \), \( f(t) \) takes value 1, otherwise, it takes value 0.

According to the above definition, we can get the charging-enabled vector \( f_i \) of vehicle 1. By shifting all the elements in the vector to the right by \((k-1) \times T_D\) cells, the charging-enabled vector \( f_i \) of vehicle \( i \) can also be obtained, as shown in Eq. (7) (* denotes a blank cell).

\[ f_i = \left[ \begin{array}{ccccccc} 1 & \ldots & 1 & 0 & \ldots & 0 & 1 & 0 & \ldots \end{array} \right] \quad t_j \quad t_j^\prime \quad \ldots \quad t_{j,p_i} \quad \ldots \]  
(6)

\[ f_i = \left[ \begin{array}{ccccccc} * & \ldots & \ast & 1 & \ldots & 1 & 0 & \ldots \end{array} \right] \quad t_j^\prime \quad t_j^\prime \quad \ldots \quad t_{j,p_i}^\prime \quad \ldots \]  
(7)

Since the operation of urban rail vehicles is cyclic, any complete cycle within the stable operation period of the system can be used for the research. In order to remove the blank cells in the charging-enabled vectors, we reselect the moment when the last vehicle is put into operation, i.e. \( p_{M,i} \) as the beginning.

To ensure the time-homogeneity of all the vectors, the charging-enabled vector of each vehicle is first extended into \( 1 \times 2T_e \), as expressed in Eq. (8). Then we cut it into the one, denoted by \( f_i \), with \( p_{M,i} \) as the initial time and \( p_{M,i}^\prime + T_e \) as the terminal time. Afterwards, the charging-enabled vectors of all the vehicles are piled to get the charging-enabled matrix \( F \) of the whole line within a cycle, as expressed in Eq. (9). Summing each column of matrix \( F \) separately and dividing it by the total number of vehicles, we can get the coincidence factor of the line at each time slot, as shown in Eq. (10).

\[ f_i^{\text{new}} = \left[ f_i, f_i^{(2T_e - \text{size}(f_i))} \right], \quad f_i^{\text{new}} \in \mathbb{R}^{1 \times 2T_e} \]  
(8)

\[ F = \left[ f_1, f_1^{(2T_e - \text{size}(f_1))}, \ldots, f_M, f_M^{(2T_e - \text{size}(f_M))} \right] \]  
(9)

\[ F(t^*) = \left[ c_0, c_1, \ldots, c_{T_e}, c_{T_e} \right] \]  
(10)

### 3. Train timetable optimization model

The goal of this study is to minimize the maximum coincidence factor within an operation cycle. Based on the aforementioned analysis, we formulate the problem as the following model:

\[
\begin{align*}
\text{min} & \quad \max \left( F(t^*) \right) \\
\text{s.t.} & \quad t_{\text{min}}^j \leq t^j \leq t_{\text{max}}^j \\
& \quad \sum_{j=1}^{N-1} t_j^r = T_s
\end{align*}
\]

The first constraint is the dwell duration constraint at each station to meet the requirement of operation efficiency and service quality of the system [12]. The upper bounds and lower bounds are generally determined by the passenger flow and charging schedule. Besides, the total dwell duration within a cycle is restricted to a constant to keep the operation cycle duration unchanged.

As the objective function is non-convex and discontinuous, the traditional optimization algorithms are not suitable for this optimization problem. Therefore, we apply PSO to solve the model in this study. PSO is a heuristic iterative search technique where each particle's movement is influenced by its local best known position, but is also guided toward the best known position visited by the swarm [13]. The parameters of PSO include inertia factor \( w \), acceleration constant \( c_1 \) and \( c_2 \), the number of particles in the swarm \( S \) and the maximum number of iterations \( I \).

### 4. Case study

In this section, a case study is given to verify the validity of the model based on the data from Haizhu Line in Guangzhou which is about 7.7km with 11 operation stations and 2 turn-back stations. The original timetable of the line is shown in Table 1, in which the letters “U” and “D” after the station names denote the up direction
and down direction, respectively. With the current operation timetable and the constant headway of 300s, the maximum coincidence factor of the line is 0.556. Then we keep the headway of the system unchanged and set $t_{\text{min}} = t_j - 5$, $t_{\text{max}} = t_j + 5$, $c = 2$, $c_{\text{max}} = 0.9$, $w_{\text{max}} = 0.5$, $S = 20$, $t = 20$ to implement PSO. The optimized timetable obtained is recorded in Table 2. For the optimized timetable, the objective function value is 0.444, which implies that our approach can reduce the charging capacity of the whole line by 20% without changing the total number of vehicles. Furthermore, through comparing the distribution of coincidence factors in Fig.1 (a) and Fig.1 (b), we can see that the optimized timetable makes the proportion of coincidence factor below 0.2 increase significantly, which indicates from another aspect that the optimized timetable can reduce the charging power stress of the whole system by adjusting the dwell duration of vehicles at every station.

<table>
<thead>
<tr>
<th>Station</th>
<th>Dwell duration (s)</th>
<th>Travel duration (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wanshengwei (U)</td>
<td>30</td>
<td>148</td>
</tr>
<tr>
<td>Pazhou Pagoda (U)</td>
<td>25</td>
<td>117</td>
</tr>
<tr>
<td>Pazhou Bridge South (U)</td>
<td>25</td>
<td>85</td>
</tr>
<tr>
<td>Canton Fair Complex East (U)</td>
<td>25</td>
<td>83</td>
</tr>
<tr>
<td>Canton Fair Complex Middle (U)</td>
<td>25</td>
<td>83</td>
</tr>
<tr>
<td>Canton Fair Complex West (U)</td>
<td>25</td>
<td>80</td>
</tr>
<tr>
<td>Nanfeng (U)</td>
<td>20</td>
<td>109</td>
</tr>
<tr>
<td>Party Pier (U)</td>
<td>20</td>
<td>62</td>
</tr>
<tr>
<td>Pazhou Bridge South (U)</td>
<td>20</td>
<td>91</td>
</tr>
<tr>
<td>Canton Tower East (U)</td>
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<td>63</td>
</tr>
<tr>
<td>Canton Tower (U)</td>
<td>45</td>
<td>43</td>
</tr>
<tr>
<td>Turn-back station (U)</td>
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<td>40</td>
</tr>
<tr>
<td>Canton Tower (D)</td>
<td>45</td>
<td>62</td>
</tr>
<tr>
<td>Canton Tower East (D)</td>
<td>25</td>
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</tr>
<tr>
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<tr>
<td>Canton Fair Complex West (D)</td>
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<td>Canton Fair Complex East (D)</td>
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<tr>
<td>Pazhou Bridge South (D)</td>
<td>25</td>
<td>112</td>
</tr>
<tr>
<td>Pazhou Pagoda (D)</td>
<td>25</td>
<td>142</td>
</tr>
<tr>
<td>Wanshengwei (D)</td>
<td>30</td>
<td>44</td>
</tr>
<tr>
<td>Turn-back station (D)</td>
<td>59</td>
<td>31</td>
</tr>
</tbody>
</table>

**Table 1** Original timetable of Haizhu Line

**Table 2** Optimized timetable of Haizhu Line

<table>
<thead>
<tr>
<th>Station</th>
<th>Dwell duration (s)</th>
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<td>83</td>
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<td>Canton Fair Complex Middle (U)</td>
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<td>Canton Fair Complex West (U)</td>
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<td>Nanfeng (U)</td>
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<td>109</td>
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<td>Canton Tower (U)</td>
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<td>Canton Tower (D)</td>
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</table>
5. Conclusion

The main contribution of this study is to propose a timetable optimization model to reduce the charging capacity of urban rail transit system. Firstly, we put forward the concept of coincidence factor in urban rail transit to describe quantitatively the superposition of charging power resulted from simultaneous stops of vehicles. Then following the formulation of problems in the form of matrices, a timetable optimization model is proposed to minimize the maximum coincidence factor. Afterwards PSO is carried out to find the optimal solution. Last a case study of Haizhu Line demonstrates that the proposed model could significantly reduce the charging capacity of the line by 20% compared with the original timetable.

Generally speaking, this paper just presents a preliminary theoretical discussion on the capacity reduction problem in urban rail transit by means of timetable optimization. Actually, the operation process and charging demand of vehicles are much more complicated in real-world urban rail transit system. In our future research, we will extend our work to a more realistic model considering power supply sections, dynamic headway and so on. Besides, to make the optimization results more convincing, we might use the charging power demand to the grid as the objective function directly rather than the coincidence factor.

Acknowledgement

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Reference


