A State of Charge Estimation Method Based on Adaptive Unscented Kalman Filter for Lithium-ion Parallel-connected Battery System

Simin Peng, Chong Chen, Zhibing Wang, Xiaodong Yang, Zhen Xu
1 School of Electrical Engineering, Yancheng Institute of Technology, Yancheng 224051, China
2 China Electric Power Research Institute, Beijing 100192, China
3 Yancheng electric power economics and technology institute, Yancheng 224005, China

Abstract
Due to state of charge (SOC) is a key parameter for the safe operation and control of a parallel-connected battery system (PBS), it is essential to estimate accurately SOC of a lithium-ion PBS composed of thousands of inconsistent cells when the noise statistics are unknown and/or time-varying, such as model noises and measurement noises. To resolve the problems, an equivalent circuit model of the PBS is firstly presented based on a model parameter regulator that can overcome the influence of cell-to-cell variation. An adaptive SOC estimation method based on adaptive unscented Kalman filter (AUKF), which is a time-varying unscented Kalman filter with a noise statistics estimator, is proposed for the PBS when the noise statistics are unknown. Compared with the UKF and EKF, the accuracy and effectiveness of the SOC estimation method is validated by the simulation results and experimental data.

Keywords: parallel-connected battery system, state of charge, equivalent circuit model, adaptive unscented Kalman filter, noise statistics estimator

1. Introduction
Lithium-ion batteries are developing rapidly and widely applied in the renewable power integration and electric vehicle, because of its many merits, such as high energy densities, long cycles and lower self discharge rate [1]. Due to the voltage and capacity limitation of a lithium-ion cell, such as nominal voltage is 3.2V and nominal energy is 2600mAh, the lithium-ion battery system is generally composited of many of cells in parallel and/or serried to meet higher voltage and energy capacity requirements in the real application. As a core indicator for the battery system, state of charge (SOC) is usually used to illustrate the battery system capacity which is very difficult to be directly measured by the voltage and current sensors [2]. However, it is a challenge problem for the researchers and scientists around the world to correctly estimate the SOC of the battery system because of its electrochemical process with strong nonlinearity and inconsistent cell inside it [3], including the SOC estimation of a parallel-connected battery system (PBS).

A large number of methods have been proposed in previous literature to estimate the SOC accurately. In general [4], the SOC estimation methods can be classified into four groups: looking-up table based...
methods, Ampere-hour integral method, model based estimation methods and data-driven estimation methods. Among these methods, looking-up table based methods including open circuit voltage (OCV) [5] and AC impedance are more suitable for the application in laboratory environment because it needs long rest time and accurate impedance measurement devices. The Ampere-hour integral method can estimate the SOC by integrating the currents flowing into and out of the batteries over time. However, the SOC estimation accuracy using this method is greatly degraded by the measurement errors and accumulated errors. The data-driven estimation methods, such as neural network, support vector machines and uncertainty modeling, can estimate the SOC accurately for the battery system, but are not suitable to use online due to the requirements of an enormous number of samples and computational complexity. The model based estimation methods mainly consists of electrochemical model, electrochemical impedance model and equivalent circuit model (ECM). In these methods, battery models are generally expressed as state equations and the SOC can be estimated accurately by many of nonlinear state estimation methods and adaptive filters, such as Kalman filter (KF), H∞ observer [6], sliding-mode observer, et al. Recently, the Kalman filter based on ECM and its improved method, such as extended Kalman filter (EKF), sigma-point Kalman filter (SPKF), unscented Kalman filter (UKF), are widely applied to accurately estimate the SOC.

However, the SOC estimation accuracy of the above KF methods will be unstable or even divergent when the noise statistics including model and measurement noises are unknown. To overcome these problems, much effort has been done to improve the SOC estimation precision. For example, the adaptive extended filter (AEKF) and adaptive sigma-point Kalman filter (ASPK) are proposed to estimate the SOC when the noise statistics noises are unknown. However, the above adaptive filters suppose the statistical noises are Gaussian white noises which are not suitable to be used in real application. In [7], an adaptive unscented Kalman filter (AUKF) with a noise statistics estimator is proposed to estimate the SOC of battery system, but the SOC estimation accuracy based on the method is degraded because of without considering the cell inconsistent characteristics and time-varying noise statistics.

In this paper, the SOC estimation accuracy of a lithium-ion PBS in two ways: an equivalent circuit model of the PBS is built based on a model parameter regulator considering the cell inconsistent characteristics firstly. On the other hand, an adaptive unscented Kalman filter based on time-varying UKF with a noise statistics estimator is proposed for the PBS when the noise statistics are unknown.

2. The PBS model based on a model parameter regulator

2.1 Prime PBS model

A structure diagram of the researched parallel-connected battery system in this study is shown in figure 1. The battery system consists of $n$ cells connected in parallel.

![Figure 1 Structure diagram of the PBS](image)

Many of battery models have been proposed to present the battery charging and discharging characteristics. In this paper, a PBS model based on two-order ECM is shown in figure 2. The model consists of a controlled voltage source ($U_{b0}$), a resistor ($R_b$), and two resistor-capacitor (RC) networks in series. The $R_{bs}$ and $C_{bs}$ and the $R_{bl}$ and $C_{bl}$ are used to present the short-term transient response and long-term transient response of the battery, respectively. $I_b$ and $U_b$ show the current and the terminal voltage of the battery system, respectively.

![Figure 2 PBS model](image)

In this paper, an approach based on a screening process is applied to build the prime PBS model, which illustrates the performance parameters relation between the battery system and each cell. The relation can be presented as follows [8]:

$$
\begin{align*}
U_{b0}(t) &= U_b(t) \\
R_b(t) &= R(t)/n \\
R_{bs}(t) &= R_s(t)/n \\
R_{bl}(t) &= R_l(t)/n \\
C_{bs}(t) &= nC_s(t) \\
C_{bl}(t) &= nC_l(t) \\
I_b(t) &= nI(t)
\end{align*}
$$

(1)

where $U_b$ is the open circuit voltage of cell, $R$ is the internal resistance of cell, the $R_s$ and $C_s$ and the $R_l$ and $C_l$ are used to present the short-term and the long-term transient response of the cell respectively, and $I$
denotes the current through the cell in series. In general, the static relationship between $U_b$, $R_b$, $C_s$, $R_s$, $C_l$, $R$ and the SOC is intrinsically nonlinear. The nonlinear relationship between the $U_b$, $R_b$, $C_s$, $R_s$, $C_l$, $R$ and the SOC is expressed in Ref [7]. The SOC is expressed as follows:

$$SOC(t) = SOC_0 - \frac{\int \eta I(t)dt}{C_0}$$  (2)

where $t$ is the discharging or charging time, $SOC_0$ is the SOC initial value, $\eta$ is the coulomb efficiency, $C_0$ is the rated capacity.

The functional relationship between the terminal voltage $U_b$ and the current $I_b$ of the PBS can be represented as

$$U_b(t) = U_{b0}(t) - I_b(t) \left( \frac{R_b(t)}{1+R_{bs}(t)jC_{bs}} + \frac{R_b(t)}{1+R_{ls}(t)jC_{ls}(t)} \right)$$  (3)

### 2.2 The PBS model based on a model parameter regulator

To illustrate the influence of inconsistent cells on battery system performance, a model parameter regulator is proposed to build the PBS model. Figure 3 shows the diagram structure of the proposed PBS model based on a model parameter regulator, which mainly is composed of two parts: prime PBS model and model parameter regulator. The prime PBS model is derived from the cell ECM as presented in Section 2.1. The model parameter regulator is applied to attain the SOC error ($\Delta SOC_b$) between the prime PBS model and the actual battery system due to the inconsistent cells.

**Figure 3** Diagram structure of the PBS model

As shown in Figure 3, the model parameter regulator can be described as follows:

1. To attain the cell SOC error at the $k$th step

$$\Delta SOC_{b,k} = (k_p + \frac{k_i}{s}) \cdot [I_{i,k} - I_{b,k}] / (n - m)$$  (4)

where $k_i$ and $k_p$ denote the integral coefficient and proportional coefficient of the $i$th cell respectively; $m$ is the number of cells which arrive the cut-off voltage; $I_{i,k}$ and $I_{b,k}$ denotes the measured current of the $i$th cell and the PBS model current at the $k$th step respectively.

2. To attain the SOC error of the PBS at the $k$th step

$$\Delta SOC_{b,k} = 1/(n - m) \sum_{i=1}^{n} \Delta SOC_{i,k}$$  (5)

3. Update the modified SOC of the PBS at the $k$th step

$$SOC_{b,k} = SOC_{b,k} + \Delta SOC_{b,k}$$  (6)

where $SOC_{b,k}$ is SOC of the PBS at the $k$th step.

### 2.3 Validation of the proposed PBS model

To validate the effectiveness and accuracy of the proposed PBS model, a PBS composed of two cells in parallel was built. The simulation and experimental parameters of cells are listed in Ref.[9]. A test aimed to show the influence of inconsistent cell on the PBS was carried out in pulse load current, and the initial SOC of cell1 and cell2 were set as 0.8 and 0.9 respectively. Figure 4 shows current and voltage profile of the proposed PBS model in the discharging process.
For a nonlinear discrete time system with uncorrelated non-zero-mean Gaussian white noise statistics, the state equation and the measurement equation can be described as

\[
\begin{align*}
    x_k &= f(x_{k-1}, u_k) + w_k = f(x_{k-1}, u_k) + q + \mu_k \\
    y_k &= g(x_k, u_k) + v_k = g(x_{k-1}, u_k) + r + \phi_k
\end{align*}
\]

where \( x \) and \( y \) denote the system state vector and the system measurement vector, respectively; \( f() \) and \( g() \) denote the nonlinear measurement and process models, respectively; \( u \) denotes the system input vector; \( w \) and \( v \) denote the system process noise and measurement noise, respectively; \( q \) and \( r \) denote the mean value of \( w \) and \( v \), respectively; \( \mu \) and \( \phi \) are both uncorrelated zero-mean Gaussian white sequence; and the covariance value of \( w \) and \( v \) can be presented as

\[
\begin{align*}
    \text{cov}(\omega_i, \omega_j) &= Q_k \\
    \text{cov}(v, v) &= R_k
\end{align*}
\]

The steps of the time-varying UKF can be described as follows in detail.

(1) Initialize the mean \((\hat{x})\) and the covariance \((P)\) of the system state

\[
\begin{align*}
    \hat{x}_0 &= E(x_0) \\
    \hat{P}_0 &= E((x_0 - \hat{x}_0)(x_0 - \hat{x}_0)^T)
\end{align*}
\]

(2) Calculate sigma points at time \( k - 1 \)

\[
\begin{align*}
    x_{i,k-1} &= \hat{x}_{k-1} + \sqrt{(N + \lambda)P_{k-1}}, i = 1, 2, ..., N \\
    x_{i,k-1} &= \hat{x}_{k-1} - \sqrt{(N + \lambda)P_{k-1}}, i = N + 1, N + 2, ..., 2N
\end{align*}
\]

where \( N \) is the dimension of the state variable, \( \lambda \) is a scale, \( \lambda = \alpha^2(N + h) - N \), \( \alpha \) is a scale which determines the spread of the sigma point around \( \hat{x} \), and \( h \) is a scaling parameter.

Attain the weighted coefficients

\[
\omega_0^m = \lambda / (N + \lambda) \\
\omega_0^c = \lambda / (N + \lambda) + (1 + \beta - \alpha^2) \\
\omega_i^m = \omega_i^c = 1/2(N + \lambda), i = 1, 2, ..., 2N
\]

where \( \omega^m \) and \( \omega^c \) denote the weight factors, respectively; \( \beta \) denotes a scale.

(3) Time update for the system states

\[
\begin{align*}
    x_{i,k} &= f(x_{i,k-1}), i = 0, 1, ..., 2N \\
    \hat{x}_{i,k} &= \sum_{i=0}^{2N} \omega_i^m f(x_{i,k-1}) + q_k \\
    P_{i,k} &= \sum_{i=0}^{2N} \omega_i^m (x_{i,k} - \hat{x}_{i,k}) (x_{i,k} - \hat{x}_{i,k})^T + Q_k
\end{align*}
\]

(4) Update the measurement states

\[
\begin{align*}
    x_{i,k} &= g(x_{i,k-1}) \\
    \hat{y}_{i,k} &= \sum_{i=0}^{2N} \omega_i^m g(x_{i,k-1}) + r_k
\end{align*}
\]
\[ P_{y,i} = \sum_{i=0}^{2N} \alpha_i^k (x_{1,i,k-1} - \hat{x}_{1,i,k-1}) (x_{1,i,k-1} - \hat{x}_{1,i,k-1})^T + R_k \]  
(17)

\[ P_{x,i} = \sum_{i=0}^{2N} \alpha_i^k (x_{i,k-1} - \hat{x}_{i,k-1}) (x_{i,k-1} - \hat{x}_{i,k-1})^T \]  
(18)

\[ L_k = P_{x,i}^{-1} \]  
(19)

\[ \hat{x}_i = \hat{x}_{i,k-1} + L_k (y_i - \hat{y}_{i,k-1}) \]  
(20)

(5) Calculate the robust adaptive factor \( \gamma_k \) [10]
\[ \gamma_i = \left\{ \begin{array}{ll} 1, & |\varepsilon_i| \leq k_0 \\ k_0 / |\varepsilon_i| & [(k_i - \varepsilon_i) / (k_i - k_0)]^T, \ k_0 < |\varepsilon_i| \leq k_1 \\ 10^{-6}, & |\varepsilon_i| > k_i \end{array} \right. \]  
(21)

Where \( k_0 \) and \( k_1 \) are scale respectively, we assume that \( k_0 = 1.0~1.5, \ k_1 = 3.0~8.0; \ |\varepsilon_i| \) is expressed as \( |\varepsilon_i| = |\varepsilon_i| / \sqrt{Tr(\sum_{i=0}^{2N} \varepsilon_i^2)} \), \( Tr() \) means matrix trace.

(6) Update the measurement states
\[ P_{y,i} = 1/\gamma_i \sum_{i=0}^{2N} \alpha_i^k (x_{1,i,k-1} - \hat{y}_{1,i,k-1}) (x_{1,i,k-1} - \hat{y}_{1,i,k-1})^T + R_k \]  
(23)

\[ P_{x,i} = 1/\gamma_i \sum_{i=0}^{2N} \alpha_i^k (x_{i,k-1} - \hat{x}_{i,k-1}) (x_{i,k-1} - \hat{x}_{i,k-1})^T \]  
(24)

\[ L_k = P_{x,i}^{-1} (P_{x,i}^{-1})^{-1} \]  
(25)

\[ \hat{x}_i = \hat{x}_{i,k-1} + L_k (y_i - \hat{y}_{i,k-1}) \]  
(26)

\[ P_i = P_{x,i}^{-1} - L_k P_{x,i}^{-1} (L_k)^T \]  
(27)

3.2 The noise statistics estimator

To solve the problem which degrades the SOC estimation accuracy of the traditional UKF without knowing the prior noise statistics, a noise statistics estimator based on maximum likelihood estimation and expectation maximization method is applied in this paper. The noise statistics is described as follow [9].

\[ \hat{q}_i = \frac{1}{k} \left\{ (k-1) \hat{q}_{i-1} + \hat{x}_i \sum_{i=0}^{2n} \alpha_i^k \cdot f(x_{1,i}) \right\} \]  
(28)

\[ \hat{Q}_i = \frac{1}{k} \left\{ (k-1) \hat{Q}_{i-1} + \text{diag}[\hat{x}_i \hat{x}_i^T + P_i^{-1} - \hat{q}_i \hat{q}_i^T \sum_{i=0}^{2n} \alpha_i^k \cdot f(x_{1,i}) \cdot x_{1,i}^T \right\} \]  
(29)

\[ \hat{r}_i = \frac{1}{k} \left\{ (k-1) \hat{r}_{i-1} + [y_i - \sum_{i=0}^{2n} \alpha_i^k \cdot g(x_{1,i})] \right\} \]  
(30)

\[ \hat{R}_i = \frac{1}{k} \left\{ (k-1) \hat{R}_{i-1} + \text{diag}[(y_i \hat{y}_i^T - \hat{r}_i \hat{r}_i^T) - \sum_{i=0}^{2n} \alpha_i^k \cdot g(x_{1,i}) \cdot y_i^T \right\} \]  
(31)

3.3 The proposed AUKF

To solve the problems that the UKF is sensitive to the known noise statistics and its time-varying characteristics, an AUKF based on a time-varying UKF and a noise statistics estimator is proposed in this paper. Figure 5 shows flowchart of the proposed AUKF.

![Flowchart of the proposed AUKF](image_url)

Figure 5. Flowchart of the proposed AUKF

4. Simulation and experimental results

To validate the performance of the developed AUKF, a comparison of SOC estimation precision using the three methods (AUKF and UKF and EKF) is carried out by simulation and experimental data. In the test, the PBS is consists of two cells in parallel. The simulation and experimental parameters of cell is listed in Ref. [9]. The initial values of the noise statistics in the three methods are randomly selected as \( \sigma_0 = 0.02, \ T_0 = 0.0005, \ q_0 = 0.001, \ D_0 = 0.03 \). The initial SOC values of the two cells are set as 0.8 and 0.9 respectively, and the reference SOC (SOCr) of the PSB is set as 0.85. The initial SOC of the three methods is uniformly set as 0.8.
Figure 6 shows the SOC estimation comparisons of the PSB using the AUKF, UKF and EKF methods. As shown in Figure 6 (a), the estimated SOC using the proposed AUKF can precisely track the reference SOC in the whole discharging process, and the corresponding SOC absolute error of the AUKF shown in Figure 6 (b) is the smallest one of 0.015 V. Moreover, as listed in Table 1, the mean absolute error (MAE) and root-mean square error (RMSE) of the estimated SOC using the AUKF are the lowest errors, which illustrates that developed AUKF can accurately estimate SOC and show highest SOC accuracy compared to the EKF and the UKF.

Table 1 SOC Error comparison

<table>
<thead>
<tr>
<th>Methods</th>
<th>RMSE</th>
<th>MAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>EKF</td>
<td>0.0553</td>
<td>0.0407</td>
</tr>
<tr>
<td>UKF</td>
<td>0.0372</td>
<td>0.0351</td>
</tr>
<tr>
<td>AUKF</td>
<td>0.0127</td>
<td>0.0122</td>
</tr>
</tbody>
</table>

5. Conclusions

The SOC estimation accuracy of the PBS will be greatly degraded using the EKF and UKF methods when the prior noise statistics are unknown and time-varying. To resolve the problems, an ECM with a model parameter regulator of the lithium-ion PBS and an AUKF based on time-varying UKF and noise statistics estimator are proposed. Simulation and experimental results show that the proposed PBS model can accurately simulate the PBS discharging voltage with low absolute error of 0.05V, and the proposed AUKF not only can precisely track the reference SOC and shows the highest SOC estimation precision with the lowest RMSE of 0.0127 and MAE of 0.0122 compared to the UKF and the EKF when the prior noise statistics are incorrect.

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Reference


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