Study on Numerical Simulation of Three-dimensional Multi-directional Freak Waves Based on OpenFOAM

Cheng CUI1,2,*, Bing YAN1,2 and Shu-hua ZUO1,2

1Tianjin Research Institute for Water Transport Engineering, M.O.T., Tianjin 300456, China
2National Engineering Laboratory for Port Hydraulic Construction Technology, Tianjin 300456, China
*Corresponding author

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Abstract. A three-dimensional numerical model was built by developing the standard solver interFoam in the open source CFD toolbox OpenFOAM. In the developed interFoam, a three-dimensional wavemaker boundary condition based on directional spectra and absorbing boundary conditions are added. The numerical model had been used to simulate multi-directional freak waves. The performance proof of the numerical model was carried out by comparing the numerical results with experiments and target spectra. And then Effects of grid interval and maximum courant number on multi-directional freak wave simulation are analyzed. The freak wave groupiness are examined and compared by the conventional run length approach and smoothed instantaneous wave energy history approach. The following conclusions are drewed. The satisfactory multi-directional freak waves result on the condition of the grid resolution of 0.02m×0.01m×0.02m and the courant number of 0.25. The energy approach is more appropriate for freak wave groupiness description.

Introduction

The freak wave is a type of independently large water wave, which is characterized by strong-nonlinearity, short-duration, focused-energy and serious destructiveness. The field data and study works show that freak waves can suddenly appear on the sea surface all over the world's ocean without any premonition. Such waves are often accompanied by deep troughs and successive large waves occurring either before or after them[1-2]. Therefore freak waves can bring serious damage to vessel and maritime structure, as well as to other facilities in the ocean. The study on freak waves is quite necessary and significant for maritime activity. With the current trend towards deep water and rough sea, freak wave has become an important and pressing subject.

The most common research methods for freak waves are field measurement, physics experiments and numerical simulation. Due to lack of field data, the research results based on field data are fewer than experimental and numerical results. It is widely accepted that the occurrence mechanism of freak wave could be focusing of component waves with different frequencies, self-focusing of wave energy in wave trains based on Benjamin-Feir instability, interaction between sea water and environmental dynamic conditions [1]. In recent years, lots of freak wave simulation works have been implemented on the basis of above-mentioned occurrence mechanism.

Due to Benjamin-Feir instability, the evolution of periodic perturbations on a continuous wave background results in local exponential amplitude growth. As self-attractive nonlinear interaction becomes strong, so freak waves develop. Lu [3] investigated the generation and evolution of the higher-order rational solutions (freak waves) of the nonlinear Schrodinger equation by using the 4th order split-step pseudo-spectral method and analyzed the shapes of such waves. This mechanism reasonably explains the generation of freak waves, but the assumptions of being weakly nonlinear and spectrally narrow bandness cause limitations.

Based on the mechanism of focusing of component waves with different frequencies, the modified Longuet-Higgins model is used to simulate freak waves. Some modified models have been presented [4-6]. Representatively, Krieble [4] controllably simulated freak waves at designed
point in space and time by using a dual superposition model. In the model, an extreme transient wave is embedded into a random wave train based on a partitioning of the total wave energy: with one part of the energy going into the underlying random sea, and another part going into the focused transient wave. It is concluded that a realistic freak wave can be obtained on the condition of 15%-20% wave energy going into focused transient wave train. Due to algebraic solution, this kind of model is simple and efficient, nevertheless the nonlinearity interaction between component waves is ignored. The simulated results can be used for statistic analysis or as boundary conditions of experiment and computational fluid dynamics model.

In some studies, generating simulated freak waves in laboratory and numerical flume taking into account nonlinearity interaction between component waves have been documented [7-18]. Tao [14] and Yang[15] conducted a long time physical modeling for multi-directional irregular wave train, and analyzed the effects of directional spreading parameters on probability distributions and groupiness of freak waves, respectively. She et al [16] examined three dimensional breaking wave kinematics, and found that the degree of angular spreading has great effects on the breaking characteristics. Johannessen and Swan [17] described a laboratory study in which a large number of waves, of varying frequency and propagating in different directions, were focused, and presented that the directionality of the wave field has a profound effect upon the nonlinearity of a large wave event and that an increase in the directional spread of the wave field leads to lower maximum crest elevations and allows larger limiting waves to evolve. Liu et al [18] proposed a method for generating three-dimensional focusing waves using Boussinesq equations and the finite element method, and discussed the effects of the associated wave parameters such as the center frequency, frequency width and the type of frequency spectrum on the focusing waves. From the above it appears that freak waves can be simulated successfully, many such two-dimensional and a few three-dimensional, especially fewer for three-dimensional numerical results, have been proposed.

In this study, a three-dimensional numerical model is built by developing the standard two immiscible fluids solver interFoam in the open source CFD toolbox OpenFOAM, to simulate three-dimensional freak waves taking into account directional spectrum. The grid resolution and courant number effects on simulated results, and groupiness of freak waves have been analyzed. From the present work, the experience gained by the authors in simulating three-dimensional freak waves will be applied to the study on evolution behaviour and wave-structure interaction taking into account directional spreading, in future.

**Mathematical Model**

The governing equations are the Reynolds-averaged N-S (RANS) equations, closed by the two-equation k-ε turbulence model, and the equation discretization is implemented by using Finite Volume method. A VOF (volume of fluid) phase-fraction based interface-capturing approach is used[19].

**Continuity Equation**

\[ \nabla \cdot \mathbf{u} = 0 \]  

**Reynolds-averaged N-S equations**

\[ \frac{\partial \rho \mathbf{u}}{\partial t} + \nabla (\rho \mathbf{u} \cdot \mathbf{u}) = -\nabla p^* - \rho \mathbf{g} \cdot \mathbf{x} + \nabla (\mu \nabla \mathbf{u}) + (\nabla \mathbf{u}) \cdot \nabla \mu \]  

where, \( p \) is the pressure, \( p^* = p - \rho \mathbf{g} \cdot \mathbf{x} \) is the pressure in excess of the hydrostatic, \( \mu \) is the dynamic molecular viscosity, \( \rho \) is the density which varies with the content of air/water in the computational cells, \( \mathbf{g} \) is the acceleration due to gravity, \( \mathbf{u} \) is the velocity field, \( t \) is the time.

**Free Surface**

The free (water-air) interface is captured with the volume-of-fluid (VOF) method. The basic idea is that the two-phases system can be represented as a mixture of the phases in which the
phase-fraction distribution includes sharp yet resolved transitions between the phases. The $a_w$ and $a_b$ are the water and air phase-fraction, respectively, $a_w + a_b = 1.0$, $a_w$ is 0 for air and 1 for water, and any intermediate value is a mixture of the two phases for interface. The local density and the local viscosity of the cell are given by $\rho = a_w \rho_w + (1-a_w) \rho_b$, and $\mu = a_w \mu_w + (1-a_w) \mu_b$.

The complete system of equations for the VOF approach for incompressible two-phase flow are the phase-fraction equation. In order to limit the smearing of the interface and to guarantee boundedness and conservation, the compression term is added [19]. The wave surface height can be obtained from phase-fraction integral along water depth.

$$\frac{\partial \alpha_w}{\partial t} + \nabla (u \alpha_w) + \nabla \left( u \alpha_w (1-\alpha_w) \right) = 0$$  \hspace{1cm} (3)

**Turbulence Model**

Reynolds-averaged turbulence model is chosen. Applying the time averaging procedure to the incompressible Navier-Stokes equations, the equations describe the mean properties of the flow. Consequently, meanwhile, extra six Reynolds stresses terms, $-\rho u_i u'_j$, appear in the time-averaged (or Reynolds averaged) flow equations due to the interactions between various turbulent fluctuations. Using the Boussinesq approximation, Reynolds stresses might be proportional to mean rates of deformation. It is given by:

$$-\rho u_i u'_j = \mu_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \left( \rho k + \mu_t \frac{\partial u_i}{\partial x_j} \right) \delta_{ij}$$  \hspace{1cm} (4)

where, $\mu_t$ is the turbulent viscosity. For $\kappa-\varepsilon$ two-equation turbulence model, the turbulent viscosity is then related to the fluid's turbulent kinetic energy $k$ and its dissipation rate $\varepsilon$, which are governed by their own transport equations. In this study, RNG $\kappa-\varepsilon$ two-equation turbulence model is invoked, which represents the effects of the small-scale turbulence by means of a random forcing function in the Navier–Stokes equation. The RNG procedure systematically removes the small scales of motion from the governing equations by expressing their effects in terms of larger scale motions and a modified viscosity. This model deals well with the flow characterized by high rate of strain and great curve. Therefore it is more appropriate for freak wave simulation than standard $\kappa-\varepsilon$ two-equation turbulence model. The governing equations are given as follows:

$$\frac{\partial \rho k}{\partial t} + \frac{\partial \rho ku_i}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \alpha_k \mu_{\text{eff}} \frac{\partial k}{\partial x_j} \right) + G_k - \rho \varepsilon$$  \hspace{1cm} (5)

$$\frac{\partial \rho \varepsilon}{\partial t} + \frac{\partial \rho \varepsilon u_i}{\partial x_i} = \frac{\partial}{\partial x_i} \left( \alpha_\varepsilon \mu_{\text{eff}} \frac{\partial \varepsilon}{\partial x_j} \right) + \frac{C_\varepsilon}{k} G_k - C_2 \rho \frac{\varepsilon^2}{k}$$  \hspace{1cm} (6)

where, $\mu_{\text{eff}} = \mu + \mu_t$, $C_\mu = 0.0845$, $\alpha_k = a_w = 1.39$, $C_{1\kappa} = 1.40$, $C_{2\kappa} = 1.68$, $\eta_0 = 4.377$, $\beta = 0.012$. The keyword for turbulence model is specified from the turbulenceProperties dictionary.

**Wave-maker Boundary**

The dual superposition model is adopted to calculate wavemaker boundary condition. The three-dimensional freak waves can be generated controllably by extending the Kriebel's model. The developed model can be expressed as:
\[ \eta(x, z, t) = \eta(x, z, t) + \eta_t(x, z, t) = \sum_{m=1}^{M} \sum_{i=1}^{I} \eta_{mi}(x, z, t) + \sum_{m=1}^{M} \sum_{i=1}^{I} \eta_{2mi}(x, z, t) \]

\[ = \sum_{m=1}^{M} \sum_{i=1}^{I} a_{mi} \cos \left[ \omega_m t - k_m(x \cos \theta_z + z \sin \theta_z) + \varepsilon_m \right] + \sum_{m=1}^{M} \sum_{i=1}^{I} a_{2mi} \cos \left[ \omega_m (t - t_0) - k_m(x - x_0) \cos \theta_z - (z - z_0) \sin \theta_z \right] \]

From wave-making theory, the velocity of the wave-maker boundary cell \( U(x_0, y_0, t) \) can be written as:

\[ U(x_0, y_0, t) = \sum_{m=1}^{M} \sum_{i=1}^{I} \omega_m \left( \eta_{mi}(x_0, y_0, t) + \eta_{2mi}(x_0, y_0, t) \right) / T(\omega_m, \theta) \]

where, \( \eta \) is the surface elevation at \((x, z)\), \( x \) and \( z \) are horizontal coordinates, \( y \) is vertical coordinate. \( a_{1mi} \) and \( a_{2mi} \) are the amplitudes of the random and focused wave train, respectively. The energy of the target spectrum \( S(\omega, \theta) \) is divided into \( M \) bands in frequency domain, and directional angles \( \theta_i \) are uniformly distributed in the range \((-\pi/2 ~ \pi/2)\). \( \omega_{mi} \) and \( k_m \) denote the wavenumber and frequency of each component, respectively, satisfying the linear dispersion relationship. \( \varepsilon_{mi} \) is the random wave phase distributed in the range \((0 ~ 2\pi)\). \( \omega_{mi} \) is the representative frequency of the \( mi \)th component wave, which is distributed randomly in the range \((\omega_{m-1} \sim \omega_{m})\) to avoid “phase locking”.

The modified P-M spectrum (ITTC, 1972) is selected as target frequency spectrum[20]. \( H_s \) is the significant wave height; \( T \) is the average period. The directional spreading function is expressed as:

\[ a_{mi} = \sqrt{2p_s S(\omega_m, \theta) \Delta \omega_s \Delta \theta_i} \quad \quad a_{2mi} = \sqrt{2p_s S(\omega_m, \theta) \Delta \omega_s \Delta \theta_i} \quad \quad \omega_{ne} = \hat{\omega}_m - \frac{1}{2} \Delta \omega + (i - 1 + RAN_m) \Delta \omega / I \]

\[ S(f) = Af^{-5} \exp(-Bf^{-4}), \quad A = 0.0177H_s^2T^{-4}, \quad B = 0.4443T^{-4} \]

\[ G(\theta) = C(n) \cos{2n \theta}, \quad |\theta| < \pi / 2, \quad C(n) = \frac{2n!!}{\pi (2n-1)!!} \]

\( T(\omega_m, \theta) \) is the transfer function associated with the \( mi \)th component of propagating wave, calculated by:

\[ T(\omega_m, \theta) = 4 \sinh^2 k_m d l (2k_m d + \sinh 2k_m d) \cos \theta_i \]

**Absorbing Boundary**

To limit the computational domain, absorbing zones are implemented to avoid reflection of waves from outlet boundaries and further to absorb waves reflected internally in the computational domain to interfere with the wave maker boundaries. In the absorbing zone, an attenuation coefficient, \( \alpha_R(x_R) \), is introduced for field data \( \phi \) (velocity, wave surface elevation) as[21]:

\[ a_{\phi}(x_R) = 1 \exp \left( \frac{x_R}{l} \right) \exp \left( \frac{1}{l} \right) \]

\[ \phi = \alpha_R \phi \frac{\alpha_m \rho}{\rho} \]

**Wall Boundary**

For wall boundary, the boundary conditions are given zeroGradient, zeroGradient and fixedValue with uniform \((0 0 0)\) for phase-fraction, press and velocity, respectively.

In order to save computing resources, the wall function technique was used. The kqRwallFunction and epsilonWallFunction nutkWallFunction boundary conditions are assigned to turbulent kinetic energy \( k \), its dissipation rate \( \varepsilon \) and turbulent viscosity \( \mu_t \).

**Simulation Results**

In order to recognize the sensitivity and validity of the numerical model built in this study, the
effects of the mesh resolution and maximum courant number on simulated results are analyzed, and then the optimization setting is used for validity assessment. For the numerical cases, the water depth is set to \( d = 40cm \), the significant wave height to \( H_s = 3cm \) and the peak period to \( T_p = 1.8s \), the number of directional angle to \( I = 30 \), the number of frequency component to \( M = 100 \), the energy ratio of transient wave train to \( P_t = 0.03 \), the focused time to \( t_c = 30s \), the focused position to \( (6, 10) \). The computational domain has a 19m length, a 26m width and a 0.6m height. It is defined as a wave tank, whose lateral boundaries are absorbing zones. Six wave gauges are installed near the focused point. The sketch of computational domain is shown in Figure1. In order to allow locally finer grids to be employed in the regions where the variation of flow variables is expected to be more rapid, the computational domain is discretized with a uniform grid interval in the zone with \( x = 4m \sim 10m, y = 0.3m \sim 0.5m, z = 0m \sim 20m \) (Figure2), and in other zone, a gradually non-uniform grid interval is used.

Effects of Grid Interval on Simulated Results

Five numerical cases with different grid interval were simulated to analyze the effects of grid interval on simulated results. The grid intervals in uniform zone are listed in Table1. The freak wave characteristic parameters \( \alpha_1, \alpha_2, \alpha_3, \alpha_4 \) and relative error between simulated and experimental results are shown in Table2. The comparison of wave surface between numerical and experimental results is described in Figure3. Where, \( \alpha_1 = H_{\text{max}} / H_s, \alpha_2 = H_{\text{max}} / H_{\text{max}-}, \alpha_3 = H_{\text{max}} / H_{\text{max}+}, \alpha_4 = \eta_{\text{maxc}} / H_{\text{max}} \). The \( H_{\text{max}} \) is the maximum wave height, the \( \eta_{\text{maxc}} \) is crest height of the maximum wave, the \( H_{\text{max}} \) and \( H_{\text{max}+} \) are the wave height before and after the maximum wave.

Generally, a wave is defined as a freak wave if the wave height to significant wave height ratio exceeds 2.0[1-2]. In this study, the criterions for a freak wave definition are \( \alpha_1 \geq 2, \alpha_2 \geq 2, \alpha_3 \geq 2 \) and \( \alpha_4 \geq 0.65[22] \), because strong nonlinearity and the relationship between a freak wave and adjacent waves (freak wave is a type of single large wave) can be characterized by the latter definition. In addition, freak wave is a “extreme centre point” in its generation and evolution process which is nearly symmetrical [23-24]. Therefore the latter definition, which can distinguish freak wave from other abnormal waves, is chosen.

![Figure 1. Sketch of computational domain.](image1)

![Figure 2. Sketch of sub-calculation zone.](image2)
The Effects of waves used deviation, for intervals interfered experimental from increases.

For case5, taking the Courant Number is 2, therefore, when \( R = 2 \), for case3, the simulated results are satisfactory. For case5, keeping \( R = 2 \), the mesh resolution is increase further. The horizontal and vertical grid intervals are specified to 0.01m and 0.0005m, respectively. It can be seen from the comparison that for case5 the simulated results are close to case3 except that the crest and period have a little deviation, but the cost in terms of computing resources is too high. Therefore, the mesh resolution used for case3 is appropriate for the present numerical model to simulate three-dimensional freak waves taking into account directional spectrum.

**Effects of Courant Number on Simulated Results**

The courant number is a severe limitation for numerical calculation especially if the free (water-air) interface is captured. Due to the effective down winding used in the compression term in phase-fraction equation, this equation suffers from a time-step restriction beyond the usual Courant...
number constraint to maintain boundedness, especially if a limited second-order scheme without deferred-correction is applied to the normal convection term. As the Courant number decreases, the calculation cost increases. In order to choose appropriate Courant number, The Courant number is set to 0.5 and 0.25 for Case6 and Case7. The comparison between numerical and experimental results is shown in Table 3 and Figure 4 and it can be seen that, the agreement for case7 (Courant number of 0.25) is much better than case6 (Courant number of 0.5).

Table 3. Characteristic parameters and relative error of simulated freak wave.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
<th>$\alpha_3$</th>
<th>$\alpha_4$</th>
<th>$fa_1$</th>
<th>$fa_2$</th>
<th>$fa_3$</th>
<th>$fa_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>experimental</td>
<td>2.33</td>
<td>2.00</td>
<td>2.80</td>
<td>0.69</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Case6</td>
<td>2.33</td>
<td>1.77</td>
<td>2.55</td>
<td>0.63</td>
<td>0.0</td>
<td>0.23</td>
<td>0.25</td>
<td>0.06</td>
</tr>
<tr>
<td>Case7</td>
<td>2.33</td>
<td>2.00</td>
<td>2.93</td>
<td>0.68</td>
<td>0</td>
<td>0</td>
<td>0.13</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Validity Assessment of Model

To validate the numerical model, the comparison of simulated and experimental wave elevations at 6 locations including the presumed focal location (x =6m, z=10m) is presented in Figure 5, and the energy density spectra of simulated sequences and corresponding target spectrum are given in Figure 6. The directional spectrum is calculated by using Bayesian and Fourier transform methods. Figure 7 shows the spatial distribution of three-dimensional freak wave surface and the corresponding contour plot. The comparison of the freak wave characteristic parameters $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$ and relative error between simulated and experimental results are shown in Table 4.

It can be seen that: the good agreement between simulated and experimental results including energy distributions and profile shows the validity of model; and spatial distribution of three-dimensional freak wave surface is characterized by short-crest in accord with the real sea state. Therefore, the numerical model built in the present study can simulate directional freak waves successfully.
Groupiness of Freak Wave Sequences

Generally, large waves do not always come alone, more often several high waves come together, forming a wave group. Freak wave is a type of single large wave with strong nonlinearity, therefore the profile of wave surface does not show groupiness. The average run-lengths of wave sequences including freak wave and conventional random wave sequences are listed in Table 5. It can be seen that the average run-length of freak wave sequences is even smaller than conventional random wave sequences. According to run-length theory, the groupiness of freak wave sequences is not obvious.

Nevertheless, the relationship between freak waves and large wave group is very close. They would like to be accompanied by each other. The Smoothed Instantaneous Wave Energy History (SIWEH) is another important parameter describing the groupiness of wave sequences. Figure 9 shows the comparison of SIWEH between freak wave sequences and conventional random wave sequences. It can be seen that the SIWEH peak value of freak wave sequences is much higher than conventional random wave sequences and so does the Groupiness Factor (GF), exhibiting obvious groupiness.

Therefore, the groupiness of freak wave sequences is not directly exhibited by profile of wave surface, which is more appropriate to be described by energy factors.

Table 4. Characteristic parameters and relative error of simulated freak wave.

<table>
<thead>
<tr>
<th>Case</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$f_{a_1}$</th>
<th>$f_{a_2}$</th>
<th>$f_{a_3}$</th>
<th>$f_{a_4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>experimental</td>
<td>2.59</td>
<td>2.14</td>
<td>3.77</td>
<td>0.66</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>simulated</td>
<td>2.57</td>
<td>2.08</td>
<td>4.05</td>
<td>0.66</td>
<td>0.01</td>
<td>0.03</td>
<td>0.07</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 5. Characteristic run length and Groupiness Factor.

<table>
<thead>
<tr>
<th>Case</th>
<th>average run-length</th>
<th>GF</th>
<th>Case</th>
<th>average run-length</th>
<th>GF</th>
<th>Case</th>
<th>average run-length</th>
<th>GF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Freak wave</td>
<td>1.20</td>
<td>1.03</td>
<td>Random4</td>
<td>1.33</td>
<td>0.74</td>
<td>Random8</td>
<td>1.30</td>
<td>0.65</td>
</tr>
<tr>
<td>random1</td>
<td>1.38</td>
<td>0.76</td>
<td>Random5</td>
<td>1.67</td>
<td>0.71</td>
<td>Random9</td>
<td>1.57</td>
<td>0.62</td>
</tr>
<tr>
<td>Random2</td>
<td>1.28</td>
<td>0.68</td>
<td>Random6</td>
<td>1.50</td>
<td>0.74</td>
<td>Random10</td>
<td>1.05</td>
<td>0.60</td>
</tr>
<tr>
<td>Random3</td>
<td>1.36</td>
<td>0.76</td>
<td>Random7</td>
<td>1.18</td>
<td>0.63</td>
<td>Random11</td>
<td>1.18</td>
<td>0.58</td>
</tr>
</tbody>
</table>

Summary

The conclusions obtained in this study are as follows: The good agreement between simulated and experimental results including energy distributions and wave profile shows the validity of model with the grid resolution of $0.02m \times 0.01m \times 0.02m$ and the courant number of 0.25. The spatial distribution of three-dimensional freak wave surface is characterized by short-crest in accord with the real sea state. The energy approach is more appropriate for groupiness characteristics of freak waves.

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