Reliability Evaluation and Selection in Multi-State System

Yishuang Hu¹, Zhoubin Liu² and Hongjie Gu²

ABSTRACT

The multi-state system is an essential tool to present engineering system and previous researches have been focusing on the reliability evaluation of multi-state system recently. The Analytical algorithm and Monte Carlo simulation are the major reliability evaluation methods. However, it is inefficient to apply these algorithms to find the optimal multi-state system with greatest reliability in numbers of systems, whose computational time may last for hours. Hence, this paper proposes a new method based on ordinal optimization algorithm and Monte Carlo simulation applied in reliability evaluation. Calculation methods concerning the characteristics of system are defined, which can dramatically and effectively shorten the computational time. An effective and convenient precision and efficiency verification according to the proposed method is illustrated.

Keywords: Multi-state, ordinal optimization, Monte Carlo simulation, computational time.

INTRODUCTION

The multi-state system is an essential tool to present engineering system and the reliability of elements should be considered together with the desired demand when the whole system is estimated [1]. Previous researches have been focusing on the reliability evaluation of multi-state system recently [2-6]. As time went by, the commonly used evaluation method of multi-state system consists of two algorithms, Analytical algorithm and Monte Carlo simulation [7-8]. Compared with Analytical algorithm, Monte Carlo simulation is more suitable in large-scale system. Thus, in this paper, we choose Monte Carlo simulation as the basic reliability evaluation method.

However, there is a difficulty that if we want to find the system with greatest reliability among many large-scale multi-state systems by Monte Carlo simulation, whose computing time may last for hours. Therefore, this paper introduces a new model consisted of Monte Carlo simulation and ordinal optimization algorithm [9-10] to shorten the computing time in finding the greatest system among a great number of multi-state series-parallel systems.

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In the proposed model, firstly, we need to classify the systems into several sorts and choose the smallest series-parallel system with least components in each sort, calculate its precise reliability by Monte Carlo simulation. Then calculate the remaining systems’ rough reliabilities one based on the parallel components by ordinal optimization and choose the top g systems. Fourthly, based on the parallel components one and parallel components two, we use Blind Picking selection rule (BP) to calculate rough reliability of two of the top g element and select the top s system. Finally, we calculate the precision reliability for the selected top s systems to find the optimal system by Monte Carlo simulation.

This paper is organized as follows. Section 2 gives the process of the proposed method. Section 3 illustrates a practical case and comparisons are made in section 3. Section 4 draws the conclusion of this paper.

THE PROPOSED NEW MODEL (MO MODEL)

Assumption

a) Assume every component in system only has two states: fully function and failure.

b) Assume parameters of components in system are statistically identical, including capacity, mean time to failure and mean time to repair.

Definitions

a) Ordinal Optimization method

Reference [10] explained a viewpoint for the search problem using ordinal optimization. In this paper, we use equation (1) (BP selection rule) to calculates the value of s.

\[
\varphi% = \sum_{i=1}^{\min(g,s)} \left( \frac{C N-g}{i (g-i)} \right)
\]

(1)

where g is number of systems in the first selection, s is number of systems in the second selection, N is the size of design space in OO method and \(\varphi%\) is defined alignment probability.

b) Sort

Based on the number of series components which possesses the parallel components, we can divide the systems into several sorts. For example, the two systems in Figure 1 can be distinguished into two sorts, as they hold different series-parallel components. System two has one series component with parallel element while system one has no series-parallel component. Moreover, this series-parallel system represents the constituent parts in power system, which are generation, transformation, transmission and distribution parts.

Figure 1. Three series-parallel system examples.
c) Smallest system
In one sort, if the number of total components in one system is the least, then this system is named as smallest system in this sort. For example, in Figure 2, as system one has 6 components, while system two has 5, thus system two is the smallest system.

![Figure 2. Two series-parallel system examples in one sort.](image)

Figure 2. Two series-parallel system examples in one sort.


d) The parallel components
According to the structure of smallest system, for the remaining system in one sort, if there are more than one added parallel components parallel to the same series component, then the other added parallel components except for the first one is called as the parallel component two, else we name it as parallel component one. For example, system one in Figure 3 is the smallest system, then system two has 1 parallel component two while system three has 1 parallel component one and 1 parallel component two.

![Figure 3. Three series-parallel system examples in one sort.](image)

Figure 3. Three series-parallel system examples in one sort.

**Process of the MC model**

The application of MC model is to find the optimal system with greatest reliability among numbers of systems with less computing time compared with former methods. Steps can be seen as below.

Classify the systems into several sorts. Choose the smallest series-parallel system and apply Monte Carlo simulation method to obtain the reliabilities.

Compared with the smallest system in one sort, define \( R' \) as the reliability of new system, \( R \) is reliability of smallest system and \( r \) is reliability of component. If the system in the same sort adds one parallel component, the reliability can refer to equation (2). Then we can get the rough reliability one of all systems--calculation method one.

\[
R = 1 - (1 - R)(1 - r)
\]

(2)
Rank all systems’ rough reliabilities in decrease order and choose the top g systems. Use BP evaluation scheme to get the value of s.

Compared with the smallest system in one sort, if the added parallel component is a parallel component two, the new system reliability can be changed by equation (3). If the added parallel component is a parallel component one, the new equation is equation (2) --calculation method two.

\[ R = (1-(1-R)(1-r))^*r \]  \hspace{1cm} (3)

Based on the step before, we choose the top s systems according to the rough reliabilities and calculate the precision reliabilities by Monte Carlo simulation to find the optimal system with greatest reliability--calculation method three.

CASE STUDIES

Classify the design space

Assume there are 15 series-parallel multi-state systems which can be separated into 5 sorts. The simplified reliability block diagrams are presented in Figure 4. Moreover, this series-parallel system represents the constituent parts in power system, which are generation, transformation, transmission and distribution parts.
Sort three as 3-1, 2, 3

Sort four as 4-1, 2, 3

Sort five as 5-1, 2, 3

Figure 4. The five system sorts.

Depending on the definition of smallest system, we choose system 1-1, 2-1, 3-1, 4-1 and 5-1 as the smallest system. Assume the reliability of one component is 0.98, thus, the precise reliabilities of smallest systems are 0.8904, 0.9142, 0.9293, 0.9511 and 0.9605 separately.

**Calculation method one**

Based on equation (2), we can obtain the rough reliability one of each system and choose the top 9 systems, which can be seen from Table 1.

<table>
<thead>
<tr>
<th>system</th>
<th>reliability</th>
<th>system</th>
<th>reliability</th>
<th>system</th>
<th>reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3</td>
<td>0.99999991232</td>
<td>3-2</td>
<td>0.9999994344</td>
<td>4-3</td>
<td>0.9999996088</td>
</tr>
<tr>
<td>2-2</td>
<td>0.99999993136</td>
<td>3-3</td>
<td>0.9999999998</td>
<td>5-2</td>
<td>0.9999999684</td>
</tr>
<tr>
<td>2-3</td>
<td>0.99999993132</td>
<td>4-2</td>
<td>0.9999996088</td>
<td>5-3</td>
<td>0.9999999684</td>
</tr>
</tbody>
</table>

Table 1. Rough reliability one for all the systems.
Calculation method two

We define $\%$ equal to 95%, $N$ equal to 15 and $g$ equal to 9. With the help of equation (2), the value of $s$ is equal to 5. Based on equation (3), we can get the rough reliabilities two of the chosen 9 systems, which can be seen in Table 2. The top 5 systems are 2-2, 3-2, 4-2, 5-2 and 5-3.

<table>
<thead>
<tr>
<th>system</th>
<th>reliability</th>
<th>system</th>
<th>reliability</th>
<th>system</th>
<th>reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-3</td>
<td>0.9799991407</td>
<td>3-2</td>
<td>0.999994344</td>
<td>4-3</td>
<td>0.960396244</td>
</tr>
<tr>
<td>2-2</td>
<td>0.999993136</td>
<td>3-3</td>
<td>0.941919998</td>
<td>5-2</td>
<td>0.999999684</td>
</tr>
<tr>
<td>2-3</td>
<td>0.979993269</td>
<td>4-2</td>
<td>0.999996088</td>
<td>5-3</td>
<td>0.9799996903</td>
</tr>
</tbody>
</table>

Calculation method three

We apply Monte Carlo simulation method to precisely evaluate the reliability of top 5 systems, which are:

<table>
<thead>
<tr>
<th>system</th>
<th>reliability</th>
<th>system</th>
<th>reliability</th>
<th>system</th>
<th>reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-2</td>
<td>0.9145</td>
<td>5-2</td>
<td>0.9665</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3-2</td>
<td>0.9307</td>
<td>5-3</td>
<td>0.9642</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4-2</td>
<td>0.9540</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

From Table 3, we can see system 5-2 has the greatest reliability among all the systems which is the optimal system we are looking for. We can see from this case that OO method with BP selection rule can effectively reduce the evaluation time, like this case, we only need to assess the precision reliabilities of 5 systems instead of 15, which is nearly one third of the total number.

Validation

To demonstrate the accuracy and efficiency of proposed model, we make two comparisons.

a) Comparison with maximum reliability

Applied calculation method three, we can get the accurate results for all the 15 systems in Table 4.

<table>
<thead>
<tr>
<th>system</th>
<th>Reliability</th>
<th>system</th>
<th>reliability</th>
<th>system</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>0.8904</td>
<td>2-2</td>
<td>0.9145</td>
<td>3-3</td>
<td>0.9244</td>
</tr>
<tr>
<td>1-2</td>
<td>0.8944</td>
<td>2-3</td>
<td>0.9144</td>
<td>4-1</td>
<td>0.9411</td>
</tr>
<tr>
<td>1-3</td>
<td>0.8968</td>
<td>3-1</td>
<td>0.9293</td>
<td>4-2</td>
<td>0.9540</td>
</tr>
<tr>
<td>2-1</td>
<td>0.9214</td>
<td>3-2</td>
<td>0.9307</td>
<td>4-3</td>
<td>0.9532</td>
</tr>
<tr>
<td>5-1</td>
<td>0.9505</td>
<td>5-2</td>
<td>0.9665</td>
<td>5-3</td>
<td>0.9642</td>
</tr>
</tbody>
</table>

From Table 4, we can see system 5-2 has greatest reliability which is the same result as the proposed model. This comparison demonstrates the accuracy of MC model.

b) Comparison of computing cost with former reliability evaluation method

In order to better embody the innovation and advancement of this model, we compared this new approach with the traditional algorithm which is evaluating all the system by Monte Carlo simulation of CPU operation time. The result can be expressed in Table 5.
Table 5. The CPU time for two models.

<table>
<thead>
<tr>
<th>The number of components</th>
<th>methods</th>
<th>CPU time (second)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-20</td>
<td>Monte Carlo</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>simulation</td>
<td>MC model</td>
</tr>
<tr>
<td>20-30</td>
<td>Monte Carlo</td>
<td>623</td>
</tr>
<tr>
<td></td>
<td>simulation</td>
<td>MC model</td>
</tr>
<tr>
<td>30-40</td>
<td>Monte Carlo</td>
<td>3045</td>
</tr>
<tr>
<td></td>
<td>simulation</td>
<td>MC model</td>
</tr>
<tr>
<td>40-50</td>
<td>Monte Carlo</td>
<td>3782</td>
</tr>
<tr>
<td></td>
<td>simulation</td>
<td>MC model</td>
</tr>
</tbody>
</table>

From this table, we can indicate that the computational time for searching the multi-state series-parallel system with greatest reliability by the proposed method is much less than the former method, which is nearly one third as much as the latter algorithm, i.e. the MC model combining Monte Carlo simulation and OO algorithm can shorten the computation time effectively, and its efficiency can be more obvious if the number of systems is larger.

SUMMARY

Multi-state system plays a significant role in power system. Searching the optimal multi-state system is beyond calculation by former reliability evaluation methods, especially for the large-scale system. In this paper, we propose a combined model which can reduce the computing time in searching for the system with largest reliability among many multi-state systems. The defined calculation methods can guarantee the accuracy of result and effectively shorten the computational time by the proposed two selections. From the presented case and the two comparisons, we can indicate that the proposed method applied in selection in multi-state series-parallel systems can reduce the scope of design space. And the verification of the three defined reliability evaluation methods can be applied into practical cases with highly efficiency.

REFERENCES