Combining Mathematical Morphology and Unscented Kalman Filter for Digital Protective Relaying

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ABSTRACT

The paper presents a digital filter algorithm, combining mathematical morphology (MM) and unscented Kaman filter (UKF), for microprocessor-based protective relaying. Firstly, the state and observation equations of UKF are designed based on the state variables of DC-offset component, fundamental angular frequency, and fundamental component of the fault signal. Approximation signals and detail signals are derived from the multi-scale morphological decomposition of the sampling signal. The approximation signals are employed to update the observations of UKF to improve the convergence speed of the filter algorithm, and the detail signals are applied to calculate the measurement noise variance to improve the convergence precision of the filter. The proposed MM-UKF algorithm is evaluated on the data collected from simulation model built in Matlab/Simulink.

Keywords: Multi-scale morphological analysis; Unscented Kalman filter; Protective relaying.

INTRODUCTION

The digital filter algorithm is of great importance in the microprocessor-based protective relaying of power system. Significant amount of publication focus on reducing the transient noise of fault signal and extracting the electrical parameters [1-4]. The classical full-wave and half-wave Fourier filters [5] have been utilized to obtain the fundamental component of fault signal. Nevertheless, the Fourier algorithm fails to remove the DC-offset and non-integer harmonics of the fault signal, which results in the sensitiveness to frequency variation and measurement noise. Besides, the integral calculation of Fourier algorithm leads to high computation cost and slow response speed. The Kalman filter [6-7] has been introduced in the relay protection in order to improve the computation accuracy and convergence speed of the digital filter. In [6-7], the harmonics are regarded as measurement noise, and the decaying DC-offset component is defined as state variable to extract the fundamental component of the fault signal, which can effectively suppress the harmonic and DC-offset components. In addition, the Kalman filter is a recursive algorithm which only requires the present measurement, and it is suitable for real-time implementation with low computation burden.

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However, the linear state-space model of fault signal is adopted in the traditional Kalman filter-based digital relay protection method, while the constant DC-offset component and frequency variation are not taken into account, which reduces the estimation accuracy and adaptability to frequency variation. Additionally, the transient noise of fault signal decreases the convergence rate of the filter algorithm, and the model uncertainty of signal and the inaccuracy of statistic property of measurement noise reduce the convergence accuracy of the filter. Mathematical morphology (MM) can remove the noise and detect the peak point of signal, and it has been applied in the power system signal processing [8-10]. The multi-scale morphological filter decomposes the signal into approximation signals and detail signals at different scales. The contributions of this paper are summarized as follows. In the first place, the nonlinear state-space model of fault signal is established considering the DC-offset component and fundamental angular frequency, and the unscented Kalman filter (UKF) is employed to deal with the nonlinear state estimation issue without model linearization. Besides, the MM is applied to remove the transient noise of the fault signal. The multi-scale decomposition signals obtained from morphological filter are utilized by UKF to estimate the fundamental component and extract the electrical parameters for relaying protection. Case studies are carried out in Matlab/Simulink to testify the proposed MM-UKF algorithm.

MULTI-SCALE MATHEMATICAL MORPHOLOGY

Morphological Filter

Two basic operations of mathematical morphology, dilation and corrosion, are expressed as [8-10]

\[
(f \oplus g)(x) = \max \{f(x-y) + g(y)\}
\]

\[
(f \Theta g)(x) = \min \{f(x+y) - g(y)\}
\]

where \(f(x)\) is one-dimensional signal with the definition domain of \(D_f\), \(g(x)\) is structural element with the definition domain of \(D_g\), \(\oplus\) and \(\Theta\) are dilation and corrosion operators, respectively. Based on (1) and (2), the opening and closing operations are given by

\[
f \circ g = (f \Theta g) \oplus g
\]

\[
f \bullet g = (f \oplus g) \Theta g
\]

where \(\circ\) and \(\bullet\) denote the opening and closing operators, respectively. The opening operation removes the peak noise while the closing operation suppresses the trough noise of the signal. Through the cascade connections of opening and closing operators, the morphological open-close filter OC and close-open filter CO are defined as

\[
OC(f, g) = f \circ g \bullet g
\]

\[
CO(f, g) = f \bullet g \circ g.
\]
The morphological median filter is widely used in practice, and it is the average of OC and CO given by
\[ y(f) = \left[ \text{OC}(f,g) + \text{CO}(f,g) \right] / 2 \]  
where \( y \) is the output.

**Multi-Scale Analysis**

The structural element \( g_\lambda \) of the multi-scale morphological filter at scale \( \lambda \) is defined as
\[ g_\lambda = g \oplus g \oplus \cdots \oplus g. \]  
According to (8), the shape of structural element \( g_\lambda \) varies as the scale parameter \( \lambda \) increases to achieve the multi-resolution decomposition of the input signal. Let \( T \) denotes the morphological operation, and the morphological operation at scale \( \lambda \) is defined as a set of morphological transformations \( \{T_\lambda | \lambda > 0\} \), where \( T_\lambda \) is represented by
\[ T_\lambda = \lambda T(f/\lambda), \quad \lambda > 0. \]  
The multi-scale morphological dilation and corrosion operations are then defined by[8]
\[ (f \oplus g)_\lambda = \lambda[(f/\lambda) \oplus g] = f \oplus \lambda g \]  
\[ (f \Theta g)_\lambda = \lambda[(f/\lambda) \Theta g] = f \Theta \lambda g. \]  
Based on (5), (10), (11), the multi-scale morphological open-close filter is obtained as
\[ \text{OC}_\lambda(f,g) = \lambda \cdot \text{OC}(f/\lambda, g) \]  
\[ = f \Theta \lambda g \oplus \lambda \lambda g \Theta \lambda g. \]  
Similarly, the multi-scale morphological close-open filter can be derived from (6), (10), (11) as
\[ \text{CO}_\lambda(f,g) = f \oplus \lambda g \Theta \lambda \lambda g \oplus \lambda g. \]  
The multi-scale morphological median filter (MSMMF) is adopted to process the signal, and it is obtained from (12) and (13) as
\[ y_\lambda(f) = \left[ \text{OC}_\lambda(f,g) + \text{CO}_\lambda(f,g) \right] / 2. \]  
The input signal \( f \) is decomposed by the morphological filter into scale-space signals. The width of the structure element \( g \) is \( L = 2^{j-1}L_j, \quad j = 1, 2, \ldots, N \), where \( N \) is the maximum level to be processed and \( L_j \) is the initial width of first level. The input signal \( f \) is decomposed into the approximation signal \( a_j \) and detail signal \( d_j \). The length of the structural element is decreased as the decomposition level rises, which makes the morphological filter applicable to remove the small noise of the signal.
UNSCENTED KALMAN FILTER

Principles of the Algorithm

Consider the nonlinear discrete-time system expressed in the general form with the state equation

\[ X_{k+1} = f(X_k) + W_{k+1} \]  \hspace{1cm} (15)

and the observation equation

\[ X_{k+1} = f(X_k) + W_{k+1} \]  \hspace{1cm} (16)

where \( k \) is sampling cycle, \( X_k \in \mathbb{R}^n \) and \( Z_k \in \mathbb{R}^m \) are state and observation vectors, respectively; \( f(\cdot) \) and \( h(\cdot) \) are nonlinear state and observation functions, respectively; \( W_k \in \mathbb{R}^n \) and \( V_k \in \mathbb{R}^m \) are process and measurement noises, respectively. Assume that \( W_k \) and \( V_k \) are zero-mean Gaussian white noises with covariance matrices \( Q_k \) and \( R_k \), respectively. In the UKF algorithm, the unscented transformation is adopted to approximate the system nonlinearities, which is described as follows. A Sigma point set is chosen based on the mean and variance of the probability distribution of system states. The unscented transformation is applied to every Sigma point, and the weighted sums of the transformed points are calculated to obtain the estimations of mean and variance of transferred nonlinearity. The UKF algorithm includes Sigma point sampling, prediction, updating, and the detailed calculation formula can refer to reference [11].

State and Observation Equations

Assuming that the voltage or current signal of the power grid only contains the DC component and fundamental component, which can be expressed in the discrete-time form as

\[ x_k = A_{0,k} + A_{1,k} \sin(\omega_k T + \theta_{1,k}) \]  \hspace{1cm} (17)

where \( T \) is the sampling period, \( A_{0,k} \) is the DC-offset component, \( A_{1,k} \), \( \omega_k \), and \( \theta_{1,k} \) are the amplitude, phase angle, angular frequency of the fundamental component.

Define new states \( x_{3,k} = A_{1,k} \sin(\omega_k T + \theta_{1,k}) \), \( x_{4,k} = A_{1,k} \cos(\omega_k T + \theta_{1,k}) \), \( x_{2,k} = \omega_k \), \( x_{3,k} = A_{0,k} \). Assume that the signal keeps constant between the \( k \)-th and \((k+1)\)-th sampling cycle. Then, we get \( A_{0,k+1} \approx A_{0,k} \), \( A_{1,k+1} \approx A_{1,k} \), \( \omega_{k+1} \approx \omega_k \), \( \theta_{1,k+1} \approx \theta_{1,k} \). At \((k+1)\)-th sampling cycle, the system state variable can be calculated as

\[ x_{3,k+1} = A_{1,k} \sin(\omega_{k+1} T + \omega_{k+1} T + \theta_{1,k+1}) \]
\[ = A_{1,k} \sin(\omega_k T + \omega_k T + \theta_{1,k}) \]  \hspace{1cm} (18)
\[ = x_{3,k} \cos(x_{2,k} T_k) + x_{4,k} \sin(x_{2,k} T_k). \]

Similarly, the state variable \( x_{4,k+1} \) can be approximated as

\[ x_{4,k+1} = -x_{3,k} \sin(x_{2,k} T_k) + x_{4,k} \cos(x_{2,k} T_k). \]  \hspace{1cm} (19)
From (18) and (19), and taking the process noise $W_{k+1}$ into account, the state equation of the fault signal can be expressed by (15) with $X_k = [x_1, x_2, x_3, x_4]^T$.

$$f(X_k) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \cos(x_4 T_k) + x_4 \sin(x_4 T_k) \\ -x_3 \sin(x_4 T_k) + x_4 \cos(x_4 T_k) \end{bmatrix}.$$ 

The actual voltage or current signals also contain the harmonic component and the measurement noise, which are regarded as the observed noise $V_k$. Based on (17), the system observation equation can be represented by (16) with $Z_k = [x_1, \alpha(x_k) = [1 \ 0 \ 1] X_k$.

**PROPOSED MM-UKF ALGORITHM**

**Multi-Scale Morphological Decomposition**

The power grid signal $s(t)$ is decomposed by the MSMMF (14) into $N$-level approximation signals $\{a_1, a_2, \ldots, a_N\}$ and detail signals $\{d_1, d_2, \ldots, d_N\}$. The morphological filter has a low-pass filtering effect, and the measurement noises of approximation signals are reduced after the morphological decomposition. Meanwhile, the higher the decomposition level, the lower the measurement noise. Therefore, the $N$-th level approximation signal $a_N$ is chosen to update the UKF observation to reduce the interference of the transient fault signals. The $N$-th level detail signal $d_N$ is selected to calculate the measurement noise variance in real-time to improve the estimation accuracy. The measurement noise variance is calculated by

$$R_k = \alpha_1 R_{k-1} + \alpha_2 s_{\text{det}}^N \left( s_{\text{det}}^N \right)^T$$

where $\alpha_1, \alpha_2$ are weight coefficients satisfy $\alpha_1, \alpha_2 \geq 0$ and $\alpha_1 + \alpha_2 = 1$.

**Electrical Parameter Calculation**

The state estimations $\hat{X}_k = [\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{x}_4]^T$ can be obtained from the proposed MM-UKF algorithm, where the symbol $\hat{\cdot}$ denotes the estimated value. The DC-offset estimation $\hat{A}_{k, \alpha}$, grid frequency estimation $\hat{f}_k$, fundamental amplitude $\hat{A}_{k, \alpha}$, and fundamental phase angle estimation $\hat{\theta}_{k, \alpha}$ are then calculated by

$$\hat{A}_{k, \alpha} = \hat{x}_1$$

$$\hat{f}_k = \hat{x}_2 / 2\pi$$

$$\hat{A}_{k, \alpha} = \sqrt{\hat{x}_3^2 + \hat{x}_4^2}$$

$$\hat{\theta}_{k, \alpha} = \tan^{-1}(\hat{x}_3 / \hat{x}_4).$$

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Application for Protective Relaying

Taking the phase-A fault for example, and the flow chart of MM-UKF based relay protection scheme is depicted in Figure 1. As seen, the sampling phase-A voltage or current signal is decomposed into approximation signals and detail signals at scale $N$. The observation $Z_k$ of UKF is updated using the $N$-level approximation signal, while the measurement noise variance $R_k$ is calculated using the $N$-level detail signal. With the state estimations provided by UKF, the fundamental components are computed from (21)-(24). Based on the outputs of MM-UKF, conventional relay protection methods, such as overcurrent protection, distance protection, differential protection, etc., are utilized to obtain the electrical parameter $F$ which describes the characteristics of the fault. The fault feature parameter $F$ is then compared with the preset threshold value $F_{set}$ to diagnosis the phase-A fault. Similarly, the protective relaying based on MM-UKF are applied for phase-B and phase-C faults. In addition, the ground fault can be detected by comparing the amplitude of zero-sequence current with the preset threshold, thereby achieving the fault detection and phase selection. The proposed scheme can combine with the current, distance, differential, and other relaying methods to achieve the protection of transmission line, transformer, generator, etc.

CASE STUDY

The two-terminal power system is built in MATLAB/Simulink, in which a short-circuit fault $f_L$ is caused at the transmission line at $t=0.06s$. The parameters of the simulated power grid are: voltage $E_M = 525\angle 0^\circ$ kV and $E_N = 525\angle 30^\circ$ kV, resistances $Z_M = Z_N = 5.74 + j4.193 \Omega$, $Z_f = 5.74 + j4.193 \Omega$. 

Figure 1. Flow chart of the MM-UKF based relay protection scheme.

Figure 2. Two-terminal power system with transmission line faults.
frequency $f_N = f_s = 50\text{Hz}$. The parameters of the simulated transmission line are given as follows: length $L_1 = L_2 = 50\text{km}$, positive-sequence impedance $Z_1 = 0.02083 + j0.2821\Omega/\text{km}$, zero-sequence impedance $Z_0 = 0.1148 + j0.7186\Omega/\text{km}$, line to positive-sequence capacitor $C_1 = 12.94 \times 10^{-3} \mu\text{F/km}$, line to zero-sequence capacitor $C_0 = 5.23 \times 10^{-3} \mu\text{F/km}$. The simulation time is 0.2s, and the sampling frequency is 2000Hz. Taking the phase-A short-circuit fault as example to illustrate the performance of the proposed MM-UKF algorithm.

The measured three-phase currents $i_a$, $i_b$, $i_c$ and zero-sequence current $i_o$ are decomposed using the MSMMF (14), and the decomposition level is selected as $N=3$ to reduce the computation complexity. Take phase-A current $i_a$ for illustration, 20dB Gaussian white noise, together with the pulse noise which exists during $t=0.04$-0.05s and $t=0.10$-0.11s with an amplitude of 4kA, are added to the waveform, as can be seen from Figure 3. The 3-level approximation signal $a_3$ and detail signal $d_3$ derived from the multi-scale morphological decomposition of current $i_a$ are shown in Figure 4 (a) and (b), respectively. As seen, the approximation signal $a_3$ and the original current signal $i_a$ are almost the same, which reveals the excellent noise attenuation ability of the morphological filter. Besides, the filtered noises are distributed in the detail signal $d_3$.

![Figure 3. Waveform of phase-A current with added noises.](image)

![Figure 4. Multi-scale morphological decomposition of phase-A current.](image)
Figure 5 gives the estimated fundamental amplitude $I_{am}$ of the fault signal $ia$ obtained from conventional full-wave Fourier filter algorithm. Figure 6(a) gives the amplitude of fundamental component extracted from the proposed MM-UKF algorithm. Firstly, the convergence accuracy of Fourier and MM-UKF are compared according to Figure 5 and Figure 6(a). It can be observed that the steady-state error of MM-UKF is less than 1%, while the output of Fourier presents small fluctuation at steady state due to the effect of measurement noise. It can be found that the MM-UKF shows higher estimation accuracy than Fourier. In aspect of convergence speed, the output of full-wave Fourier algorithm presents oscillations and reaches to the steady state at about $t=0.12s$, which is influenced by the transient noises such as harmonic and decaying DC components of the fault signal. Nevertheless, the MM-UKF algorithm is able to suppress the transient noise and its convergence process is smooth without oscillations, and the output of MM-UKF converges to steady state at $t=0.08s$, which shows faster convergence speed than Fourier. Moreover, both the Fourier and MM-UKF show reverse mutation component after the short-circuit occurs at $t=0.06s$. Whereas, the output mutation component of MM-UKF is smaller than that of Fourier, which indicates the good filtering performance of MM-UKF. Furthermore, the proposed MM-UKF can not only estimate the fundamental component, but also track the DC-offset $I_{am0}$ and frequency $f_a$ of fault signal $ia$, as can be observed in Figure 6(b) and (c).
The estimated fundamental components of fault signal provided by MM-UKF can be employed by current, distance, and differential protection principles for fault detection and phase selection of transmission line. Take the over-current protective relaying for instance, different types of short-circuit faults occur at the transmission line of two-terminal power system are shown in Figure 2, and the simulation results of MM-UKF are given in Table I. As seen, $I_a$, $I_b$, $I_c$ are the estimations of fundamental amplitude of three-phase currents $i_a$, $i_b$, $i_c$, respectively, $I_0$ is the estimated fundamental amplitude of zero-sequence current $i_0$, and $t_c$ is convergence time of MM-UKF. From Table I we see that, the convergence time obtained under three-phase short-circuit fault is 48ms, and the outputs of MM-UKF obtained under the rest of the fault types can converge to steady-state value within one cycle of 20ms. Additionally, the magnitudes of the fault and non-fault phase currents are huge different, and an appropriate threshold value can be selected to detect and isolate the fault. Meanwhile, the ground fault results in large value of zero-sequence current, and a threshold can be easily chosen to detect the ground fault.

### Table I. Simulation results obtained under various short-circuit faults.

<table>
<thead>
<tr>
<th>Fault Type</th>
<th>$I_a$/[kA]</th>
<th>$I_b$/[kA]</th>
<th>$I_c$/[kA]</th>
<th>$I_0$/[kA]</th>
<th>$t_c$/[ms]</th>
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<tbody>
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<td>AG</td>
<td>11.92</td>
<td>3.797</td>
<td>3.786</td>
<td>3.837</td>
<td>81</td>
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<tr>
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<td>3.796</td>
<td>3.798</td>
<td>80</td>
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<tr>
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<td>3.786</td>
<td>11.49</td>
<td>3.672</td>
<td>81</td>
</tr>
<tr>
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</tr>
<tr>
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<td>14.28</td>
<td>0</td>
<td>83</td>
</tr>
<tr>
<td>ABG</td>
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<td>3.791</td>
<td>3.051</td>
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</table>

### CONCLUSION

This paper has proposed a novel MM-UKF algorithm for digital relay protection. According to the simulation case undertaken in Matlab/Simulink, conclusions can be drawn as follows. The MM-UKF shows smooth convergence process after the short-circuit fault. It is because the multi-scale morphological filter is able to reduce the transient and measurement noises of fault signals. In addition, taking the DC-offset component and angular frequency into account in the modeling of fault signal, and updating observation and measurement noise covariance using the
multi-scale signals of morphological filter, the MM-UKF provides fast convergence speed and high convergence accuracy.

REFERENCES


