A Screw Dipole in Inhomogeneity Interaction with an Elliptical Inhomogeneity Containing a Confocal Crack

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Abstract. The paper is aimed to investigate a theoretical model that a screw dipole inside an elastic elliptical inhomogeneity that containing a confocal line crack under longitudinal shear. The analytic solutions of stress fields of matrix and inhomogeneity, the image force and image torque acting on the center of screw, and the stress intensity factor of line crack tip are obtained by using the complex method of elasticity. Then through the numerical analysis, the effects of the relative shear modulus, the dip angle of screw dipole, the shape of inhomogeneity on image force, image torque and stress intensity factor are discussed in detailed. The result show that both the image force and image torque periodically changes as the dip angle and when the inclusion is hard, the moment of image torque is greatly affected by the material constant and the cracks in the inclusions can enhance the repulsion of the hard matrix to dislocation and weaken the attraction of the soft matrix to dislocations.

Introduction

The dislocation dipole is composed of two Burgers vectors of equal size and opposite direction. The stress field produced by the dislocation dipole is much smaller than that produced by a single dislocation, so it is easier to produce in the material and its influence on mechanical properties of materials is not negligible. The characteristics of the material at the crack tip are the basis of all fracture theories. Cracks occur not only in the matrix and the interface, but also in the small reinforcement phase/inclusions. The study of the interaction between internal crack and dislocation dipole is very important to study the mechanism of strengthening and toughening and the failure effect of the material. Many scholars have studied this issue [1-6]. However, most of researches are concentrated on the interaction of dislocation with single crack or single inclusion, seldom to think over the interaction between them [7-10].

We have study the interaction of a dislocation dipole located in matrix with an elliptic inhomogeneity containing a confocal line crack or a confocal elliptical hole, and obtain serials important results. This paper is devoted to research the situation that the dislocation dipole is located in the elliptical inhomogeneity, and focused on the influence of the relative shear modulus, the dip angle of the screw dislocation dipole, and the shape of inhomogeneity on the image force, image torque.

Model Establishment and Theoretical Formula Deduction

Model Establishment

The theoretical model that a screw dislocation dipole located in an elliptical cylindrical inhomogeneity with a confocal line crack in an infinite matrix under anti-plane stress field is shown in Figure 1. \(s^-\) and \(s^+\) represent the matrix and the inhomogeneity, and they are denoted as subscripts ‘1’
and ‘2’, respectively. The screw dislocation dipole contains two screw dislocations of opposite Burgers vector, and the center of dipole is located at $z_0 = r e^{i \phi}$ in the inhomogeneity. The dipole arm is $2d$ and the dip angle is $\phi$. Adopting the same mapping function [11-12].

$$z = a\zeta = \frac{c}{2} \left( R \zeta + \frac{1}{R \zeta} \right), \quad R \zeta = \frac{z}{c} \left[ 1 + \frac{1 - (z/c)^2}{1} \right]$$

(1)

For anti-plane problem, the displacement $w$, shear stress $\tau_{xz}$, and generalized resultant force $T_j$ can be expressed as [11]

$$\begin{align*}
    w &= \frac{1}{2} \left[ f(\zeta) + \overline{f(\zeta)} \right] \\
    \tau_{xz} - i \tau_{yz} &= \mu \frac{f''(\zeta)}{f'(\zeta)} \omega'(\zeta) \\
    T &= \int_A^B (\tau_{xz} dy - \tau_{yz} dx) = \frac{i}{2} \mu \left[ \overline{f(\zeta)} - f(\zeta) \right]_{A}^{B}
\end{align*}$$

(2)

where $\zeta = \xi + i \eta$, $c = \sqrt{a^2 - b^2}$, $R = \sqrt{(a + b)/(a - b)}$. Then the region $s^-$ and $s^+$ in $z$-plane are conformally mapped into $|\zeta| \geq 1$ and $1/R \leq |\zeta| \leq 1$ in $\zeta$-plane, respectively. And the circle $|\zeta| = 1/R$ refers to the blunt crack, shown Figure 2. The boundary condition at the interface in $|\zeta| = 1$ can be expressed as follows

$$T_1^+(t) = T_2^+(t), \quad w_1^+(t) = w_2^+(t)$$

Theoretical Formula Deduction

The function of $f_1(z)$ and $f_2(z)$ can be expressed as

$$f_1(z) = B \left[ \ln(z - z_1) - \ln(z - z_2) \right] + f_{10}(z) \quad z \in s^+$$

(3)

$$f_2(z) = \Gamma z + f_{20}(z) \quad z \in s^-$$

(4)

where $\Gamma = \left[ \tau_{xz} - i \tau_{yz} \right]/\mu z$, $B = b_c/2 \pi$, $f_{10}(z)$ and $f_{20}(z)$ are holomorphic in definition region $s^+$ and $s^-$, respectively.
Transforming it into $\zeta$-plane, then

\[
f_1(\zeta) = B \left[ \ln(\zeta - \zeta_1) - \ln(\zeta - \zeta_2) + \ln \left(1 - \frac{1}{R^2 \zeta_1 \zeta} \right) - \ln \left(1 - \frac{1}{R^2 \zeta_2 \zeta} \right) \right] + \sum_{k=0}^{\infty} a_k \zeta^{k+1} + \sum_{k=0}^{\infty} b_k \zeta^{-k-1} \quad 1/ R < |\zeta| < 1
\]  \hspace{1cm} (5)

\[
f_2(\zeta) = \frac{c R \Gamma}{2} + f_{20}(\zeta)
\]

where $a_k$, $b_k$ are undetermined complex constant, and $f_{20}(\zeta)$ is holomorphic in $s^-$. By using singularity analysis of stress function and Riemann boundary problem, the complex function in the series form of inhomogeneity is obtained as follows.

\[
f_1(\zeta) = B \left[ \ln(\zeta - \zeta_1) - \ln(\zeta - \zeta_2) + \ln \left(1 - \frac{1}{R^2 \zeta_1 \zeta} \right) - \ln \left(1 - \frac{1}{R^2 \zeta_2 \zeta} \right) \right] + \frac{c R \mu_\zeta (R^2 \zeta + \overline{T}/\zeta)}{2 \mu_\zeta + 2 \mu_\zeta} \left(\mu_1 + \mu_2\right) R^2 + \left(\mu_2 - \mu_1\right)
\]

\[+ \sum_{k=0}^{\infty} (\mu_2 - \mu_1) B \left(\zeta_{1}^{k+1} \zeta_{2}^{-k-1} + \zeta_{2}^{k+1} \zeta_{1}^{-k-1} \right)\zeta^{k+1} \frac{1}{R < |\zeta| < 1} \hspace{1cm} (7)
\]

So the disturbed stress field of $s^-$ can be obtained as

\[
\tau_{x1} - i \tau_{y1} = \frac{2 R \zeta^2 \mu_1}{c \left(R^2 \zeta^2 - 1\right)} \left(\zeta - \zeta_1\right) \left(\zeta - \zeta_2\right) \left(R^2 \zeta \zeta - 1\right)\zeta
\]

\[+ \sum_{k=0}^{\infty} \left(\mu_1 + \mu_2\right) B \left(\zeta_{1}^{k+1} \zeta_{2}^{-k-1} + \zeta_{2}^{k+1} \zeta_{1}^{-k-1} \right)\zeta^{k+1} \frac{1}{R < |\zeta| < 1} \hspace{1cm} (8)
\]

According to the work of Juang and lee [11-12], the image torque and the image force can be deduced as

\[
T = \left(F_{x1} - F_{x2}\right) r \sin \varphi + \left(F_{y2} - F_{y1}\right) r \cos \varphi
\]

\[
F_x - i F_y = F_{x1} + F_{x2} - i \left(F_{y1} + F_{y2}\right)
\]

where $F_{xk}, F_{yk}$ $(k = 1, 2)$ can be obtained by Peach-Koehler equation as $F_{xk} - i F_{yk} = i h k \left(\tilde{\tau}_{xk} - i \tilde{\tau}_{yk}\right)$ and $\tilde{\tau}_{xk}$ and $\tilde{\tau}_{yk}$ is the disturbed stress field of a screw dislocation on $z_k$. And the stress intensity factor of line crack tip can be calculated as
\[ K_3 = \frac{ir\sqrt{\pi \mu_1}}{\sqrt{c}} \left\{ \begin{array}{c} B - \frac{B}{1-R_1} + \frac{1}{1-R_2} - \frac{1}{R_1} \left( \frac{\mu_1 + \mu_2}{R_1} \right)^2 + \frac{cR_1^2 \mu_2}{(\mu_1 + \mu_2)R^2 + (\mu_2 - \mu_1)} \\ - \sum_{k=0}^{\infty} \left( \mu_1 + \mu_2 \right) BR^{k+1} \left[ \left( R_{1-k} - R_{2-k} \right) + \left( \frac{\mu_1}{R_1} - R_{2-k} \right) \right] \\ + \sum_{k=0}^{\infty} \left( \mu_2 - \mu_1 \right) BR^{k+1} \left[ \left( R_{1-k} - R_{2-k} \right) + R^{2k-2} \left( \frac{\mu_1}{R_1} - R_{2-k} \right) \right] \\ + \sum_{k=0}^{\infty} \left( \mu_1 - \mu_2 \right) B \left[ R^{k+1} \left( R_{1-k} - R_{2-k} \right) + R^{-k} \left( \frac{\mu_1}{R_1} - R_{2-k} \right) \right] \end{array} \right\} \]

\[ \Gamma = 0, \text{ and the relative shear modulus is defined as } \mu = \mu_1/\mu_2 \text{ while the short axis ratio of an ellipse as } s = b/a. \text{ The image torque and the non-dimension image force component of } x \text{-direction can be normalized as } F_{x0} = \pi F_x / \mu_2 b_2^2 \text{ and } T_0 = \pi T_x / \mu_2 b_2^2 , \text{ respectively.} \]

\[ T_0 \text{ vs } \varphi \] with different values of relative shear modulus. It can be seen that \( T_0 \) varies periodically with \( \varphi \). \( T_0 \) firstly increase and then decreases with increasing dip angle. When \( u < 1 \) (soft inhomogeneity), the change is not large; when \( u > 1 \) (hard inhomogeneity), \( T_0 \) increases as increasing \( u \). It is obvious that when the inclusion is hard, the moment of image torque is greatly affected by the material constant.

Figure 4 depicts the dependences of normalized image force on dip angle with different relative shear modulus. We can see that \( F_0 \) similarly varies periodically with \( \varphi \). When \( u < 1 \) (soft inhomogeneity), \( F_0 \) is negative attractive force, illustrating that the screw dislocation dipole is attracted by crack and repulsed by hard matrix, and the attracting force increases with decreasing \( u \). When \( u > 1 \) (hard inhomogeneity), \( F_0 \) is positive repulsive force, illustrating that the force of soft inhomogeneity acting on screw dislocation dipole is larger than that of crack.
Conclusions
The problem that a screw dipole in inhomogeneity and a confocal crack in an elliptical inhomogeneity under longitudinal shear is discussed. In summary, the following conclusion can be drawn:
(1) Both the image force and image torque periodically changes as $\phi$. When the inclusion is hard, the moment of image torque is greatly affected by the material constant.
(2) The cracks in the inclusions can enhance the repulsion of the hard matrix to dislocation and weaken the attraction of the soft matrix to dislocations.

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