Fuzzy Proportion and Integral Control of Synchronous Generator
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Abstract. A fuzzy proportion plus integral (FPI) control strategy is investigated for the automatic voltage regulator (AVR) of synchronous generators (SGs). The Takagi-Sugeno-(T-S) fuzzy state observer is employed to solve the SGs state estimation over its full operating range. Based on the obtained observer, an optimal PI controller is proposed to control over the excitation system of a SG. Then, the controller design is formulated as a $H_2$ optimization problem, which can be transformed into eigenvalue problems (EVPs) with linear matrix inequality (LMI) constrains. The EVP is a convex optimization, which can be tackled easily by using MATLAB/LMI toolbox. Simulation results show the effectiveness of the proposed controller.

Introduction

Automatic voltage regulator of SGs is the major voltage regulation of power systems. Over the past two decades, various control strategies have been proposed for AVR design [1-4]. Among these, traditional AVR with traditional PI control (CPI) has been widely adopted in engineering fields for its simple structure, flexibility and ease of implementation [5, 6]. The parameters of the traditional AVR are tuned based on the linearized model at a chosen operating condition [7]. After off-line tuning of the parameters, extensive field testing is done to verify the stability of the closed-loop system. Thus, for the traditional AVR, there must be a compromise between the performance and stability over the full operating range [8].

Moreover, the traditional AVR only yields the optimum moves when the system is operating within a certain range of the point chosen for designing the controller and the disturbance encountered can not be large enough to push the system in a highly nonlinear range. Therefore, the traditional AVR needs approximation methods to ensure the performance over the full operating range. However, for power systems which have wide operating ranges and uncertainties caused by approximations and varying configuration, those uncertainties should be considered in AVR design.

Recently, AVR design with advanced control technologies has been extensively researched [9-11]. However, many of these strategies lack one or more of the three basic and important features that an AVR used in engineering fields should have, e.g., easy implementation, low computation burden and good performance over full operating range. The traditional PI controller has shown its good capability of tracking control for its strong robustness, simplified structure and easy understanding [12].

This paper proposes a fuzzy PI control for a SG. A T-S fuzzy observer is used to solve the state estimation of SG over its full operating range. Based on the obtained observer, an optimal PI control strategy is proposed to control over the excitation system of a SG. Then, the controller design is formulated as a $H_2$ optimization problem, which can be transformed into eigenvalue problems (EVPs) with linear matrix inequality (LMI) constrains.

The rest part of this paper is arranged as follows: the approximation error considered T-S fuzzy observer is employed to solve SG state estimation in Section 2. In Section 3, the proposed controller is used to control the excitation systems of SGs, and simulations in Section 4 demonstrate the effectiveness of the proposed controller. Finally, the conclusions are drawn in Section 5.
T-S Fuzz Observer

In this paper, a third order model of DFIG with respect to a rectangular dq coordinates is used for a good compromise between simplicity and accuracy [13].

SG’s dynamic:
\[
\delta = \omega_r - \omega_0  \\
H_i \dot{\omega}_r = \omega_0 \left[ P_m - P_e - D \left( \frac{\omega_r}{\omega_0} - 1 \right) \right] \\
T_{ao} \dot{E}_q^* = E_{id} - E_q^* - \left( X_d - X_d' \right) I_d
\]

Electric equations:
\[
\begin{align*}
U_d &= X_d I_q \\
U_q &= E_q^* - X_q I_d
\end{align*}
\]

Output equations:
\[
V_i = \sqrt{ \left( E_q^* - X_q I_d \right)^2 + \left( X_d I_q \right)^2 }
\]

By using small single linearization, the linearized form of (1)-(3) can be obtained at a prescribed operating point.
\[
\begin{align*}
\Delta x &= A \Delta x + B \Delta u \\
\Delta y &= C \Delta x
\end{align*}
\]

where subscript “i” denotes the i-th operating point, and N is the number of chosen operating point and state vector \( x = [\delta \quad \omega_r \quad E_q^*]^T \), and manipulated input \( u = E_{id} \), and output vector \( y = V_i \) for the purpose of AVR design.

By using T-S fuzzy modeling to approximate the nonlinear model of SG, the fuzzy system output can be inferred as follows:

Plant Rule i:
If \( z_1 \) is \( F_{i1} \), and ... and \( z_j \) is \( F_{ij} \), then
\[
\begin{align*}
\Delta \dot{x} &= A_i \Delta x + B_i \Delta u \\
\Delta y &= C_i \Delta x
\end{align*}
\]

Observer Rule i:
If \( z_1 \) is \( F_{i1} \), and ... and \( z_j \) is \( F_{ij} \), then
\[ \dot{x} = A_i \dot{x} + B_i u + L_i (y - \hat{y}) \quad \text{for } i = 1, 2, \ldots, N \]  
where \( L_i \) is the fuzzy observer gain, and \( \hat{y} = \sum_{i=1}^{N} h_i C_i \dot{x} \) is the fuzzy observer output.

Similarly, the overall fuzzy observer can be written as

\[ \dot{x} = \sum_{i=1}^{N} h_i \{ A_i \dot{x} + B_i u + L_i (y - \hat{y}) \} = \sum_{i=1}^{N} h_i \{ A_i \dot{x} + B_i u + L_i C_i e \} \]  
where \( e = x - \hat{x} \) is the state estimation error.

By differentiating \( e \), we have

\[ \dot{e} = \dot{x} - \dot{\hat{x}} = \sum_{i=1}^{N} \sum_{j=1}^{N} h_i h_j (A_i - L_i C_j) e \]  

**FPI Controller Design**

The proposed control scheme is shown in Figure 1. For taking the natural advantage of traditional PI controller in tracking control, the proposed fuzzy controller is defined as:

Controller Rule \( i \):

If \( z_1 \) is \( F_{i1} \), and \ldots and \( z_j \) is \( F_{ij} \), then

\[ u = K_{pj} \dot{x} + K_{ji} x_r \quad \text{for } j = 1, 2, \ldots, N \]  
where \( K_p \) and \( K_I \) are proportional and integral gains, respectively, and \( x_r = \int_{0}^{t_f} (r - y) dt \) is the integral of tracking error.

Hence, the fuzzy controller can be obtained as

\[ u = \sum_{j=1}^{N} h_j (K_{pj} \dot{x} + K_{ji} x_r) \]  

By differentiating \( x_r \), we get

\[ \dot{x}_r = r - y = r - \sum_{i=1}^{N} h_i C_i (\dot{x} + e) \]  

By defining the augment variable \( \bar{x} = [\dot{x} \quad e \quad x_r]^T \), the closed loop system can be written as following form according (8), (9) and (12).

\[\begin{pmatrix}
\dot{\bar{x}} = \sum_{i=1}^{N} \sum_{j=1}^{N} h_i h_j \begin{bmatrix}
A_i + B_i K_{pj} & L_i C_j & B_i K_{ji} \\
0 & A_i - L_i C_j & 0 \\
-C_i & -C_i & 0
\end{bmatrix} \begin{bmatrix}
\bar{x} \\
0 \\
0
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
r
\end{bmatrix}
\end{pmatrix}\]  

The compact form of (13) is
\[
\ddot{x} = \sum_{i=1}^{N} \sum_{j=1}^{N} h_i h_j \bar{A}_{ij} \ddot{x} + \varphi
\]  
(14)

where \( \bar{A}_{ij} = \begin{bmatrix} A_i + B_i K_p j & L C_j & B_i K_i j \\ 0 & A_i - L C_j & 0 \\ C_i & C_i & 0 \end{bmatrix} \) and \( \varphi = \begin{bmatrix} 0 \\ 0 \\ r \end{bmatrix} \).

For a AVR design, the cost function can be defined as

\[
J_u = \int_{0}^{T} \left( x^T Q x + u^T R u \right) dt = \int_{0}^{T} \left( \dot{x}^T Q_1 \dot{x} + \dot{e}^T Q_2 \dot{e} + x^T Q_3 x + u^T R u \right) dt
\]  
(15)

where \( Q = Q^T = blk \{ Q_{11}, Q_{22}, Q_{33} \} > 0 \) and \( R = R^T > 0 \) are weighting matrices. Since \( L_i, K_p j \) and \( K_i j \) should be solved from the (15), it is difficult to obtain the optimal solution, and a suboptimal method is chosen to minimize the upper bound of the \( H_2 \) controller.

\[
J_x = \int_{t_0}^{t_f} \left( \ddot{x}^T \bar{Q} \ddot{x} + \varphi^T \bar{P} \varphi \right) dt
\]  
(16)

where \( \bar{Q} = \begin{bmatrix} K_{p j} & 0 & K_{i j} \end{bmatrix}^T \) is the augment gain vector for the fuzzy PI controller. Normally, the reference value of terminal voltage \( r<1.5 \text{pu} \), then we have

\[
r < 1.5 < 1.5 E_{q}^* = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} [\delta \ \omega \ \dot{E}_{q}^*]^T = \varphi^* \ddot{x} = [\varphi^* \ 0 \ 0]^T \bar{x} = \varphi \bar{x}
\]  
(17)

Substituting (17) into (16), and assume that

\[
\bar{A}_{ij}^T P + P \bar{A}_{ij} + \bar{K}_{ij} R \bar{K}_{ij} + \varphi_r^T \varphi_r + Q < 0
\]  
(18)

and we find that based on the (18), (16) equals to

\[
J_x(u) \leq \bar{x}^T (t_0) P \bar{x}(t_0) + \int_{0}^{T} \sum_{i=1}^{N} \sum_{j=1}^{N} h_i h_j \ddot{x}_i^T \left( \ddot{x}_j^T P + P \ddot{x}_j + \bar{K}_{ij}^T R \bar{K}_{ij} + \varphi_r^T \varphi_r + Q \right) \bar{x} dt < \bar{x}^T (t_0) P \bar{x}(t_0)
\]  
(19)

Hence, the upper bound has been optimized. Before we solve the suboptimal problem, the stability of the closed-loop system can be guaranteed. The Lyapunov function for the system of (14) is chosen as follows.

\[
V(\bar{x}) = \bar{x}^T P \bar{x}
\]  
(20)

By differentiating (20),

\[
\dot{V}(x) = \dot{x}^T P \ddot{x} + \ddot{x}^T P \ddot{x}
\]

\[
= \sum_{i=1}^{N} \sum_{j=1}^{N} h_i h_j \left( \bar{A}_{ij} \ddot{x} \right)^T P \ddot{x} + \ddot{x}^T P \sum_{i=1}^{N} \sum_{j=1}^{N} h_i h_j \left( \bar{A}_{ij} \ddot{x} \right) + \bar{r} P \ddot{x} + \ddot{x}^T P \ddot{x}
\]

\[
\leq \sum_{i=1}^{N} \sum_{j=1}^{N} h_i h_j \ddot{x}^T \left( \bar{A}_{ij}^T P + P \bar{A}_{ij} + \varphi_r^T \varphi_r \right) \ddot{x}
\]

According (18), we have

\[
\dot{V}(x) \leq - \sum_{i=1}^{N} \sum_{j=1}^{N} h_i h_j \ddot{x}^T \left( \bar{K}_{ij}^T R \bar{K}_{ij} + Q \right) \ddot{x} < 0
\]

for \( Q > 0 \) and \( R > 0 \).

It can be seen that the stability of the system has been guaranteed by (18). Then, the controller design can be formulated as

\[
\min_p \ \bar{x}^T (t_0) P \bar{x}(t_0) \quad \text{(Subject to: } P > 0 \text{ and } (18))
\]  
(21)
From the above analysis, the most important work of the proposed control problem is to solve the common solution $P^T = P > 0$ from the minimization problem (21). Since (18) is not convex, it is difficult to analytically determine the common solution $P^T = P > 0$. Fortunately, it can be transferred into minimization problems subjected to LMIs constrains called eigen value problems, which are convex optimal problem and can be solved easily by using MATLAB/LMI toolbox [14].

**Simulation**

A generic and simplified multi-machine power system model shown in Figure 2 is used to assess the capabilities of the proposed controller. The model consists of three synchronous generators (SGs), which are equipped with the AVR and the power system stabilizer (PSS). These three SGs are connected via overhead transmission lines of L1-L3, and each line’s distance is 120km.

In this section, a step change with 1s duration of the terminal voltage reference value of SG1 is applied at $t=0+$. The SGs responses with different controllers are shown in Figure 3. The responses shown are output active power ($P_e$), terminal voltage magnitude ($|v|$), and rotor speed ($\omega_r$). From Figure 3, it is seen that the system damping has been considerably improved when the power system stabilizer (PSS) is installed ($P_e$ of Figure 3).

![Multi-machine power system model](image)

Figure 2. Multi-machine power system model.

![Graphs of responses](image)

Figure 3. Responses to the step change of terminal voltage reference of SG1: SGs with AVR plus PSS (dashed line), SG1 with the FPI (solid line).
It is known that the rotor flux of SG is rotated with the rotor itself, and only its magnitude can be manipulated. Since both AVR and PSS manipulates the magnitude of rotor flux, there must be strong coupling between AVR and PSS. It indicates that the PSS loop enhances the damping performance at the cost of the performance of the terminal voltage regulation. It is seen that when the PSS is used, the conventional AVR can not drive the terminal voltage to the desired value ($|v_t|$ of Figure 3 (a)). Since the PSS and AVR loops are intercoupling, tuning PSS should be compromised on damping performance and stability, which demonstrates that the damping performance can be further improved when both the AVR and PSS are used.

However, when the FPI is installed into SG1 as AVR, both the voltage regulation and damping performances have been improved. It is seen that the system damping is enhanced in comparison with the AVR plus PSS. The reason of those improvement can be explained as following aspects. During the step change period, according (15), it can be seen that the proposed H$_2$ controller provides a comprehensive optimization, such as voltage tracking control ($x_r^TQ_{13}x_r$), approximation control ($e^TQ_{22}e$), and state regulation ($x^TQ_{11}x$). The voltage tracking control provides a satisfactory terminal voltage tracking control, while the state regulation ensures the damping performance during the voltage regulation.

**Conclusion**

This paper proposes a T-S fuzzy observer based AVR for a SG, where a T-S observer is employed to approximate the linearized observer of a SG. A H$_2$ controller is employed to achieve specified engineering purposes, of which structure is PI for taking the natural advantage of traditional PI controller. The controller parameter can be solved by using LMI technique. Simulation results are presented and discussed, which demonstrates the capabilities of the proposed controller to improve system damping without degradation of voltage regulation.

**References**


