Design of Adaptive Subspace Predictive Controller for Stable Operation

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Abstract. For a class of process which had characteristics of nonlinear time-varying, dynamic characteristic varying significantly and can not be accurately modeled mechanism, this paper proposed a adaptive subspace predictive controller for stable operation. The method compared the prediction error to update prediction model online, and adjuste the length of the scroll window automatically according to the feedback error, which enhanced the adaptability for nonlinear time-varying characteristics, and strengthened its ability to inhibit unpredictable interference. Finally, simulated to verify the effectiveness of the method.

Introduction

Predictive control is a new computer control algorithm in the field of industrial process control in late 1970s with profound engineering background and theoretical significance, and it has been widely used in system control[1-2]. In the last 20 years of twentieth century, there have been a variety of predictive control algorithm applied to the actual industrial process predictive control has many advantages, can easily handle the equipment and security constraints and deal with multi-variables and nonlinear system, it can easily handle large lag and non-minimum phase system; has become a main research direction in the field of control[3-4].

Subspace-based State-space System Identification(4SID for short) since born in 1980s has become an important branch in the field of system identification, and it has been widely used in the field of process control. The traditional subspace predictive controller treats the input and output data as a whole, so it doesn’t conductive to the application of nonlinear time varying process, when the larger unmeasured disturbance occur, then the identification model cannot describe the current characteristics of the system[5-6]. Therefore, in view of the characteristics of this kind of process, this paper proposes an adaptive subspace control strategy for the stable operation, and then simulate to verify the effectiveness of the method.

Predictive Control Method Based on Subspace Identification

Consider the linear time-invariant system:

\[ x_{k+1} = Ax_k + Bu_k + K^f e_k \] (1)

\[ y_k = Cx_k + Du_k + e_k \] (2)

which, \( u_k \), \( y_k \) and \( x_k \) are the input, output and state variables, \( K^f \) is Calman filter gain for system stability, \( e_k \) is unknown new message sequence, and its covariance matrix is

\[ E(e_k e_k^T) = S \] (3)

\( (A,B,C,D) \) is system matrix, \( S \) is covariance matrix for new information, assume the order of the system is \( n \). Given the input / output data set of measurement length is \( k \in \{1,2,\cdots 2i + j - 1\} \), Assume
that the input output variables $u_k$, $y_k$, $k \in \{1, 2, \cdots 2i + j - 1\}$ are measurable in the range of $k \in \{1, 2, \cdots 2i + j - 1\}$, $N$ is scroll window length. Rolling window subspace identification as figure 1.

![Figure 1. Rolling window subspace identification.](image)

Structure input Hankel matrix

$$
U_p = \begin{bmatrix}
    u_1 & u_2 & \cdots & u_f \\
    u_2 & u_3 & \cdots & u_{f+1} \\
    \vdots & \vdots & \ddots & \vdots \\
    u_i & u_{i+1} & \cdots & u_{i+j-1}
\end{bmatrix},
$$

$$
U_f = \begin{bmatrix}
    u_{i+1} & u_{i+2} & \cdots & u_{i+f} \\
    u_{i+2} & u_{i+3} & \cdots & u_{i+f+1} \\
    \vdots & \vdots & \ddots & \vdots \\
    u_{i+j-1} & u_{i+j+1} & \cdots & u_{2i+j-1}
\end{bmatrix}.
$$

$Y_p$, $Y_f$ are also similar defined. The matrix included input $U_p$ and $Y_p$ is defined as $W_p = (Y_p^T, U_p^T)^T$, then the subspace prediction model problem can be described as follows, given the input and output $W_p$ of past and the input $U_f$ of the future, to find the optimal prediction of the future output $Y_f$.

$$
\hat{Y}_f = L_1 W_p + L_2 U_f
$$

Which $L = \begin{bmatrix} L_1 & L_2 \end{bmatrix}$ is the corresponding subspace matrix which can be obtained by solving the least squares problem

$$
\min_{L_1, L_2} \| Y_f - (L_1, L_2) \begin{bmatrix} W_p \\ U_f \end{bmatrix} \|_F^2
$$

Subspace identification using linear algebra tools RQ and SVD decomposition to solve the least squares problem. Optimal predictive value $\hat{Y}_f$ of future output $Y_f$ can be described orthogonal mapping of column spaces from column space $Y_f$ to $W_p$ and $U_f$, as

$$
\hat{Y}_f = Y_f / \begin{bmatrix} W_p \\ U_f \end{bmatrix} = L_1 W_p + L_2 U_f
$$

The numerical implementation of this mapping can be obtained by RQ decomposition.
\[
\begin{pmatrix}
W_p \\
U_f \\
Y_f
\end{pmatrix} =
\begin{pmatrix}
R_{11} & 0 & 0 \\
R_{21} & R_{22} & 0 \\
R_{31} & R_{32} & R_{33}
\end{pmatrix}
\begin{pmatrix}
Q_1^T \\
Q_2^T \\
Q_3^T
\end{pmatrix}
\]
(8)

\[
L = \begin{pmatrix}
R_{31} & R_{32}
\end{pmatrix}
\begin{pmatrix}
R_{11} & 0 \\
R_{21} & R_{22}
\end{pmatrix}\dagger
\]
(9)

Which \dagger is pseudo-inverse can be solved by SVD decomposition. For linear time-varying systems, based on the rolling window data of \( N \), the data set is updated at each sampling time, and the subspace model is solved online.

The objective function of the subspace predictive controller can be expressed as follow

\[
J = (r_f - \hat{y}_f)^\top (r_f - \hat{y}_f) + \Delta u_f^\top (\lambda I) \Delta u_f
\]
(10)

Which, the future output is in the prediction time domain access, the future input increment is in the control time domain. By updating the state space matrix, the optimal output of the future output can be expressed in the future.

\[
\hat{y}_f = \begin{bmatrix}
\hat{y}_{t+1} \\
\vdots \\
\hat{y}_{t+N/2}
\end{bmatrix} = \begin{bmatrix}
C \\
CA \\
\vdots \\
CA^{N/2-1}
\end{bmatrix} x_t + \begin{bmatrix}
D & 0 & 0 \\
CB & D & 0 \\
\vdots & \vdots & \vdots \\
CA^{N/2-1} B & \cdots & D
\end{bmatrix} \begin{bmatrix}
u_t \\
u_{t+1} \\
\vdots \\
u_{t+N/2-1}
\end{bmatrix}
\]
(11)

By mathematical operation, Favoreel & De Moor give the control input as.

\[
u_f = (\lambda I + L_u^\top L_u)^{-1} L_u^\top (r_f - L_u w_p)
\]
(12)

Considering the noise input of the system \( e_t \) is integral noise, the noise is very common in industrial process, so

\[
e_{k+1} = e_k + a_k
\]
(13)

\[
e_k = \frac{a_k}{\Delta}
\]
(14)

Which \( a_k \) is white noise, \( \Delta = 1 - z^{-1} \) is differential device, consider the same as (1) (2) system, the incremental equation can be described as

\[
z_{k+1} = Az_k + B \Delta u_k + K_f a_k
\]
(15)

\[
\Delta y_k = Cz_k + D \Delta u_k + a_k
\]
(16)

Which \( z_k = x_k - x_{k-1} \), the input and output expression of the subspace matrix of the system can be obtained

\[
\Delta y_f = L_w \begin{bmatrix}
\Delta y_p \\
\Delta u_p
\end{bmatrix} + L_u \Delta u_f
\]
(17)

After mathematical transformation, the objective function of the system can be written as

\[
J = (r_f - F - S_{N2,NU} \Delta u_f)^\top (r_f - F - S_{N2,NU} \Delta u_f) + \Delta u_f^\top (\lambda I) \Delta u_f
\]
(18)
Which $F$ is free response of process output, $S_{x2,NU}$ is simple correlation, By means of differentiation (18), and the result of differential is equal to 0 and then the increment input of system is

$$\Delta u_f = (S^T_{x2,x0} S_{x2,Nu} + \lambda I)^{-1} S^T_{x2,x0}(r_f - F)$$ (19)

**Design of Subspace Predictive Controller for Stable Operation**

**Adaptive Subspace Predictive Control Method**

The traditional subspace identification predictive control method used linear model but in industrial process, there are obvious nonlinear and time-varying characteristics, so the linear model predictive control is difficult to achieve the ideal control effect. The time variant characteristic of process model is an important reason for online updating, so by combining adaptive control strategy and subspace prediction method, proposes the rolling window data update model based on online identification, and by judging whether the new model can improve the prediction step error to decide whether to update the control strategy model.

![Figure 2. Chart of adaptive subspace predictive controller.](image)

Calculate the prediction error using the un-updated model.

$$essb = \left\| y_k - \hat{y}_{k|k-1} \right\|$$ (20)

Which $y_k$ is the process output of $k$ moment, $\hat{y}_{k|k-1}$ is the predictive output of $k$ moment by $k-1$ moment.

Calculate the prediction error using the updated model

$$essa = \left\| y_k - \hat{y'}_{k|k-1} \right\|$$ (21)

Which $\hat{y'}_{k|k-1}$ is the predictive output of $k$ moment by $k-1$ moment.

When $essb \leq essa$, The model remains unchanged and still uses the original model to calculate the control action, and when $essb > essa$, shown that the original model has not matched the process, and it needs to update the model.

**Subspace Predictive Controller with Variable Window Length**

In the process of industrial process, due to the variable and unknown disturbance, it is easy to lead to the existence of some dynamic characteristics of the system. In order to better reflect the current characteristics of the system, this paper presents a design method of the subspace predictive controller based on variable window length. When the system parameters change quickly when the system can automatically adjust the adjustment according to the feedback error by reducing the rolling window length, window length, the weight of historical data to accelerate the attenuation,
improve the identification sensitivity, whereas in the system changes slowly, then increase the access method of the window length, increase the amount of information collected, the identification accuracy is improved.

Assume the system prediction error of \( k \) moment

\[
\varepsilon(k) = \left\| y_k - \hat{y}_{k-1} \right\|
\]

Which \( y_k \) is the process output of \( k \) moment, \( \hat{y}_{k-1} \) is the predictive output of \( k \) moment by \( k-1 \) moment.

\[
\eta = \left| \frac{\varepsilon(k) - \varepsilon(k-1)}{\varepsilon(k-1)} \right|
\]

Which \( \eta \) is the error change rate of \( k \) moment, reflects the degree of change of the system, define \( \eta_H \) is the fast threshold of system change, \( \eta_L \) is the slow threshold of system change. Define the allowable error threshold \( \varepsilon_{\text{max}} \) and \( \varepsilon_{\text{min}} \). Define window length change range \([N_{\text{min}}, N_{\text{max}}] \). In stable operation the target of variable length is to update Hankel matrix in order to meet the requirements of the error, at each sampling time, replacing the old data with new data, two times by minimizing the objective function (10) obtained the optimal control law. The length optimization of rolling window process chart can be seen as figure3.

Figure 3. Length optimization of rolling window process chart.

**Realization of Adaptive Subspace Predictive Controller for Stable Operation**

Step by step implementation of adaptive subspace predictive controller for stable operation

Step 1 According to the input and output data of the system, update the Hankel matrix, \( U_f, Y_f \) and \( W_p \), and set the error threshold and error change rate.

Step 2 Solve the subspace matrix \( L_u \) and \( L_y \) by RQ and SVD decomposition online.

Step 3 Construct the optimal objective function and obtain the optimal control law.

Step 4 Compare the error between the current process output and the predictive output, and if it is greater than the set threshold, then update the model to Step3, or go to the next step.

Step 5 Monitoring error rate of change \( \eta \), according to the size of \( \eta \) to adjust the rolling window length.

Step 6 According to the predictive control rolling optimization strategy, at the next sampling time to collect the new input and output data, and go to Step2.
Simulation

Consider the dual input dual output system as follow

\[
\begin{bmatrix}
  y_1(s) \\
  y_2(s)
\end{bmatrix} = \begin{bmatrix}
  12.8e^{-s} & -18.9e^{-3s} \\
  16.7s + 1 & 21.0s + 1 \\
  6.6e^{-7s} & -19.4e^{-3s} \\
  10.9s + 1 & 14.4s + 1
\end{bmatrix} \begin{bmatrix}
  u_1(s) \\
  u_2(s)
\end{bmatrix} + \begin{bmatrix}
  3.8e^{-8s} \\
  14.9s + 1 \\
  4.9e^{-3s} \\
  13.2s + 1
\end{bmatrix} w(s) + \begin{bmatrix}
  e_1(s) \\
  e_2(s)
\end{bmatrix}
\]

(24)

The control target is to let the output tracking the set value change, Output setting is \(S_{pl} = [0 \ 2 \ 4], S_{p2} = [0 \ -1 \ -4]\), jumps occur at the 50th and 500th moments, in two cases of no noise and noise, use n4sid and adaptive subspace predictive controller for stable operation, the simulation results are shown as figure 4.

![Figure 4.-1 n4sid without noise.](image1)

![Figure 4.-2 adaptive subspace predictive controller for stable operation without noise.](image2)

![Figure 4.-3 n4sid with noise.](image3)

![Figure 4.-4 adaptive subspace predictive controller for stable operation with noise.](image4)

Conclusion

It can be seen from the simulation results, compare to n4sid, at the jump moment, the adaptive subspace predictive controller for stable operation can update system model, so the control input amplitude decreased, and the overshoot of the system output is smaller, at stable moment, the control performance is also effective. In the presence of noise, the system can be effectively suppressed, by
changing the length of the window, the sensitivity of the identification is improved, and the system can be effectively suppressed in the presence of noise, and the effectiveness of the algorithm is proved.

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References


