Abstract
In this paper, a state of charge estimation method for a series connected lithium ion supercapacitor module is carried out. Lithium ion supercapacitor module is modeled by an equivalent circuit model. The state of charge of the cell is estimated based on the extended Kalman filter. Considering the inconsistency of the cells in the module, a module level SOC estimation method is proposed based on a weighting factor strategy. According to the experimental results, the error of the supercapacitor module SOC estimation method is less than 4%.

Keywords: lithium ion supercapacitor; battery pack modeling; extended Kalman filter; SOC estimation

1. Introduction
Lithium ion supercapacitor is a new energy-saving energy storage device. Compared with traditional rechargeable chemical batteries, it has high power density, long charge and discharge cycle life, short charging time, long storage time and high safety. Lithium ion supercapacity (Li-Cap) has been used in electric buses and trams. The state of charge (SOC) of the Li-Cap is an important indicator of the performance of the electric vehicle. By estimating the state of charge, the driver can reasonably arrange the itinerary of the electric vehicles. How to accurately estimate the state of charge is the focus of the design of the vehicle energy storage system. The battery management system measures and monitors battery parameters, and the remaining battery power is an extremely important parameter. The battery can be used more reasonably with an accurate SOC estimation.

There are many SOC estimation methods for lithium-ion batteries. Such as neural network method [1, 2], current integration method [3], open circuit voltage method [4],[5], Coulomb counting method and Kalman filtering method [6], etc. But each method has some drawbacks. The neural network method requires a large amount of data to be collected and a lot of time to train the model. In the Coulomb counting method, noise in the current measurement causes a decrease in the accuracy of the SOC estimation. This effect cannot be ignored. Open circuit voltage method requires very accurate voltage measurements to ensure accuracy. In view of these problems, this paper proposes a method for the SOC of Li-Cap module.

The state of charge estimation method for a Series connected Li-Cap module is based on the extended Kalman filter and a weighting factor strategy. The first section of this paper is the SOC estimation of Li-Cap cell. A first-order RC equivalent circuit model used for the single cell is established. The second section mainly describes the SOC estimation for the Li-Cap module. The influence of the cell inconsistency on module SOC estimation is analyzed. In order to solve this problem, a weighting factor method is introduced to perform SOC estimation of the module. The third section mainly introduces the experimental details. The fourth section analyzes the data obtained from experimental results. The fifth section summarizes the method of this paper.

2. Cell level SOC Estimation for Li-Cap
2.1 Modeling
The first-order RC equivalent circuit model is widely used in SOC estimation for batteries. The first-order RC model has good real-time performance in the estimation of SOC. In this paper, the first-order RC equivalent circuit is used to model the single Li-Cap cell, as shown in Fig.1.

Fig.1. The first-order RC model
The terminal voltage of the first-order RC model can be expressed as:
where \( R_0 \) is the ohmic internal resistance of the capacitor, \( R_p \) and \( C_p \) are the polarized internal resistance and polarized capacitance, \( I \) is the total current, \( U_i \) is the terminal voltage, \( U_{OC} \) is the open circuit voltage.

### 2.2 Parameter identification

In the equivalent circuit model, we need to identify the unknown parameters as ohmic internal resistance, polarization internal resistance and open circuit voltage. The current and voltage data of supercapacitor are obtained by pulse discharge experiment. The parameters are identified by the collected data. Among them, the ohmic internal resistance can be identified by the instantaneous voltage drop when the current jumps, that is:

\[
R_0 = \frac{(U_a - U_b)}{I}
\]

where \( U_a \) is the terminal voltage before the current step and \( U_b \) is the terminal voltage after current step. \( I \) is the total current.

The capacitor will be stationary for a period of time after each pulse test, and the voltage will rise during the stationary process. In this stage, the current \( I \) is 0 and the RC model is zero input response. If time constant \( \tau = C_p R_p \), the zero input response can be expressed as:

\[
U_p = U_p(0) e^{-\frac{I}{\tau}}
\]

where \( U_p(0) \) is the initial voltage of the capacitor \( C_p \).

Eq.3 is solved by least square method, and the polarization resistance \( R_p \) of the Li-Cap is obtained.

Open-circuit voltage (OCV) is the most important parameter of circuit model. The measurement method of OCV is that the battery is discharged to different SOC, and the open-circuit voltage of the battery at steady state is measured as OCV. Through the corresponding SOC value of each OCV, the OCV-SOC curve as shown in Fig.2 is obtained.

### 2.3 SOC Estimation of Single Li-Cap cell

The SOC estimation model of Li-Cap is a non-linear model. The extended Kalman filter is used to estimate SOC. According to the equivalent circuit model of Li-Cap, the state equation and observation equation of the system are as follows:

\[
\begin{align*}
\dot{x}_{k+1} &= Ax_{k} + Bu_{k} \\
y_{k} &= Cx_{k} + Du_{k}
\end{align*}
\]

where \( x_k = \begin{bmatrix} SOC \\ U_p \end{bmatrix} \) is a two-dimensional state vector, \( y_k \) is a one-dimensional observation vector and \( u_k \) is a one-dimensional control vector. \( A = \begin{bmatrix} 1 & 0 \\ 0 & -e^{-\frac{1}{7}} \end{bmatrix} \) is a 2 \times 2 system matrix. \( B = \begin{bmatrix} \frac{1}{3600}C_i \\ -R_p(1-e^{-\frac{1}{7}}) \end{bmatrix} \) is a 2 \times 1 control input matrix. \( C = \begin{bmatrix} dU/dSOC \\ 1 \end{bmatrix} \) is a 1 \times 2 observation matrix and \( D = R_0 \).

The SOC equation of supercapacitor is:

\[
SOC_{k+1} = SOC_k + \Delta t / C
\]

Error covariance:

\[
P_{k+1} = AP_k A^T + Q \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

where \( Q \) is the noise variance matrix of the system.

Kalman gain matrix:

\[
K = P_k C^T \left( C P_k C^T + R \right)^{-1}
\]

Then the update of the state vector \( x_k \) can be expressed as:

\[
x_{k+1} = x_k + K(Y - U_l)
\]

where \( Y \) is the measured end voltage.

Measurement update of error covariance:

\[
P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - KC
\]

### 3. SOC Estimation for Li-Cap Module

The cells in the Li-Cap module are usually series connected in series. Due to the difference of each cell, there is a difference in the SOC of the internal Li-Cap of the module. The supercapacitor module inside the lithium ion capacitor of all cells SOC estimation. This is a large amount of calculation to estimate the SOC of all the cells in the supercapacitor module. Therefore, this paper selects the module with the largest single-cell voltage value and the smallest voltage value in the
module for SOC estimation. By giving a weighting factor, the SOC of the two capacitors is used to characterize the SOC of the capacitor module. This method can reduce the amount of calculation. The SOC estimation formula of Li-Cap module is:

\[
SOC_{\text{pack}}(k) = \omega_1 SOC_{\text{max}}(k) + \omega_2 SOC_{\text{min}}(k)
\]

where, \(SOC_{\text{pack}}\) is the module SOC value, \(SOC_{\text{max}}\) is the SOC estimated by the single voltage maximum data group, \(SOC_{\text{min}}\) is the estimated SOC of the single voltage minimum data set, and \(\omega_1\) and \(\omega_2\) are weighting factors. \(k\) represents the time step.

Because the initial SOC and capacity between capacitors are individual differences. In the process of module charging, when any single capacitor is overcharged, the charging of the module will be stopped. During the discharge of the module, when any one of the cells is overdischarged, the discharge of the module is stopped. So, for this situation, the selection of weighting factors is also divided into different situations.

(1) When one cell in the module that is about to be in the over-discharge state, the module SOC can be defined as 0%:

\[
SOC_{\text{pack}}(k) = SOC_{\text{min}}(k)
\]

At this point, the weighting factor is:

\[
\begin{align*}
\omega_1(k) &= 0 \\
\omega_2(k) &= 1
\end{align*}
\]

(11)

(2) When one cell in the module that is about to be overcharged, the module SOC can be defined as 100%:

\[
SOC_{\text{pack}}(k) = SOC_{\text{max}}(k)
\]

At this point, the weighting factor is:

\[
\begin{align*}
\omega_1(k) &= 1 \\
\omega_2(k) &= 0
\end{align*}
\]

(13)

(3) When no cell in the module is in the state of being overcharged or overdischarged:

\[
SOC_{\text{pack}}(k) = \omega_1 SOC_{\text{max}}(k) + \omega_2 SOC_{\text{min}}(k)
\]

At this point, the weighting factor is:

\[
\begin{align*}
\omega_1(k) &= \left(\frac{1}{SOC_{\text{min}}(k) - SOC_{\text{max}}(k) + 1}\right) \\
\omega_2(k) &= 1 - \omega_1(k)
\end{align*}
\]

(15)

Once the value of the weighting factor is determined, the SOC of the module can be calculated.

4. Experiment

In order to get accurate SOC offline test data. In this section, four experiments are designed, which are the single capacity characteristic test, the capacity characteristic test of the module and the HPPC test of the module, and the FUDS experiment of the actual vehicle condition. The CRRC-60000-P6-3R6 battery was used, and the capacity health of the Li-Cap at room temperature was obtained through experiments. The obtained battery voltage and current data is used for the parameter identification and state estimation.

5. Results and Discussion

Fig 3 is a data plot obtained from a single HPPC experimental voltage. Fig 4 is the result of parameter identification. According to the analysis, Modules are static after charge and static after discharge, and the polarization characteristics of the voltage of the module are not exactly the same. At the end of the static setting, the open circuit voltage of the Li-Cap is also different. It is reasonable to distinguish the module model parameters from the charge and discharge direction, which is consistent with the characteristics of the Li-Cap itself. The obtained voltage data and the identification result error are less than 0.004V. The ohmic internal resistance of the cell is \(R_0=0.0005\Omega\).
5.1 SOC estimation analysis of Li-Cap cell

The sample data was obtained by performing the Li-Cap battery module characteristic experiment and the simulated real vehicle working condition FUDS experiment, and doing offline analysis. The data are processed using the extended Kalman filter method, and the estimated result is shown in Fig 5. Fig 5 is a comparison of the SOC estimation and experimental results obtained by the extended Kalman filter process.

5.2 SOC Estimation Analysis of Li-Cap Module

The accuracy of the single cell SOC and the Kalman filtering method is verified by integrating the current, as showed in Fig 6. Fig 6 is an error curve comparing the estimated results of the extended Kalman filter with the experimental verification results. It can be observed that the error is relatively large when the SOC is close to 1. The maximum SOC estimation error is 3.9%. The accuracy of the SOC estimation of the individual is illustrated and can be used for SOC estimation of the module.
The module SOC estimation result is shown in Fig 10, wherein the red line is the module SOC, which is between the voltage maximum SOC and the minimum SOC. It tends to the maximum SOC before the overcharge phase, and approaches the minimum SOC when overdischarged. By integrating the current, the obtained module SOC and Kalman filtering method are used for accuracy verification, as showed in Fig 11. In the comparison of the two curves, when the SOC approaches 0, the deviation between the two is larger, reaching 4.1%; when the SOC is 0~0.5, the difference between the two is small, only 2.7%. However, the overall error is less than 4%. This result also verifies the feasibility of the proposed module SOC estimation.

6. Conclusions

This paper presents a method for estimating the SOC of Li-Cap module. A first-order RC equivalent circuit model is established for Li-Cap module. Based on the extended Kalman filter method, a weighting factor is introduced to estimate the SOC of Li-Cap module. The feasibility and accuracy of the algorithm are verified by experiments, which can reduce the calculation amount and ensure the real-time performance. The overall error of the algorithm is less than 4%.

Acknowledgement

This work was supported by the National Natural Science Foundation of China (No.51807121).

7. Reference


