Research on the Crack Initiation in Precision Cropping Based on Energy Release Rate Theory

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Abstract. In the low-stress precision technology, the initiation of crack at the V-shaped notch tip is a complicated process. It is of great significance to improve the quality of the bar cross-section through obtaining the conditions that influence the initiation angle and the critical loading force of the bar. Due to the complexity of the force at V-shaped notch tip, the parameter model of the V-shaped notch tip is established and the stress field of the composite crack is analyzed, thus obtaining the stress component formula of the V-notch tip. The energy release rate theory is used to calculate the crack initiation angle and the critical loading force in the low-stress cropping. The formula for calculating the energy release rate at the maximum initiation angle is established. The calculated results show that when the critical loading force is 1500N, the micro-cracks at the V-shaped notch tip first occur at the initiation angle of 20.28°, and the corresponding energy release rate is 7.539×10⁴ Pa.m². The radial blanking fatigue experiments also prove that when the critical loading force was 1500N, the bifurcate micro-cracks are generated at the tip of V-shaped notch.

Introduction

Low stress cropping uses the crack technology to crop bars. First, artificially prefabricate a ring-like approximate crack on the bar surface. And then, exert a certain force to the bar with the circumferential surface crack to separate it in the form of low-stress brittle fracture [1]. Since V-notch crack initiation and propagation is a complicated process, in order to get the best cropping effect, it is necessary to establish a crack model at the V-notch tip; investigate its crack initiation and propagation; determine the bar V-notch tip stress field during the cropping process; and, finally, obtain V-notch tip crack initiation conditions. Under different loading modes, the crack initiation conditions at the bar V-notch tip are quite different. In the high-speed centrifugal low-stress cutting process that this paper mentioned, the crack initiation condition at the bar V-notch tip has a very important practical significance in frequency control of the cropping machine.

Parametric Model and Stress Field Analysis of V-shaped Notch Tip

The loading model of the bar is shown in Figure 1. In figure 1, one end of the bar is fixed in the fixture, and the other end is subjected to a varying loading force. In this way, the V-shaped notch tip of the bar is subjected to axial tensile stress and shear stress, thus creating favorable conditions for the crack initiation at the V-shaped notch tip. This crack is a combination of open crack and slip crack (called composite crack), and the calculation of composite crack is based on the linear elasticity theory. Therefore, the solution of composite crack can be based on the overlay principle by firstly calculating the stress fields of crack tip under the single stress condition, and then adding together the two crack stress fields. According to the linear elastic theory [2], the parameter model of the crack tip
can be regarded as the result of the combined effect of tensile stress and shear stress, which is shown in figure 2. For this reason, the crack of V-shaped notch tip is decomposed into open-type cracks and slip-type cracks, which is studied one by one. Finally, the stress field of V-notch crack tip is obtained.

\[ \rho \text{- bottom corner radius; } q \text{- depth of V-notch notch; } a \text{- radius of the bar on the section of V-notch notch} \]

Figure 1. Bar force model.

(a) Open crack tip parameter diagram  
(b) Slip crack tip parameter diagram

Figure 2. Crack tip parameter model.

**Calculation of Crack Initiation Condition**

At present, there are mainly three theories which study the crack initiation and the crack propagation of elastic materials, i.e., the maximum circumferential tensile stress theory, the energy release rate theory and the strain energy density factor theory [3]. Among them, the energy method [4] is a very effective method to analyze the crack initiation process of the composite crack of the elastic materials. Therefore, this paper studies the crack initiation conditions of V-shaped notch tip using the theory of energy release rate. In order to analyze the crack initiation moment, the crack initiation model of V-notch tip is established firstly as shown in figure 3.
Figure 3. Crack initiation model.

According to the stress function of the crack tip in the crack initiation model [5], the stress component under the polar coordinate is obtained, as shown in Eq. (1).

\[
\begin{align*}
\sigma_r &= \frac{K_1}{2\sqrt{2\pi r}}(3 - \sin \theta \cos \frac{\theta}{2}) + \frac{K_{\Pi}}{\sqrt{2\pi r}} \sin \frac{\theta}{2} (3 \cos \theta - 1) \\
\sigma_\theta &= \frac{K_1}{2\sqrt{2\pi r}} \cos \frac{\theta}{2} (1 + \cos \theta) - \frac{3K_{\Pi}}{2\sqrt{2\pi r}} \sin \theta \cos \frac{\theta}{2} \\
\tau_r^\theta &= \frac{K_1}{2\sqrt{2\pi r}} \sin \theta \cos \frac{\theta}{2} + \frac{K_{\Pi}}{\sqrt{2\pi r}} \cos \frac{\theta}{2} (3 \cos \theta - 1)
\end{align*}
\]

(1)

According to Palaniswamy's theory, when the energy release rate is maximized, crack initiation occurs [6], i.e., the crack begins to occur at the tip of the V-shaped notch and propagate under the influence of nominal stress. The formula of crack energy release rate [4] is expressed as follows.

\[
\bar{G} = \frac{1}{E} (\bar{K}_1^2 + \bar{K}_{\Pi}^2)
\]

(2)

Where, \( \bar{G} \) -- Crack energy release rate; \( \bar{K}_1 \) -- Stress intensity factor of type I crack at bifurcate crack tip; \( \bar{K}_{\Pi} \) -- Stress intensity factor of type II crack at bifurcate crack tip.

In the crack initiation model, assuming that the angle between the crack propagation direction and the x-axis is \( \theta_0 \). Then, the stress \( \bar{\sigma}_\theta \) and \( \bar{\tau}_\theta \) at the V-notch tip in the propagation direction are obtained. Therefore, the formula of stress intensity factor is written as follows.

\[
\begin{align*}
\bar{K}_1 &= \lim_{r \to 0} \sqrt{2\pi r \bar{\sigma}_\theta} \\
\bar{K}_{\Pi} &= \lim_{r \to 0} \sqrt{2\pi r \bar{\tau}_\theta}
\end{align*}
\]

(3)

The values of \( \bar{\sigma}_\theta \) and \( \bar{\tau}_\theta \) are expressed by the polar coordinates and then substituted into formula (3).

\[
\begin{align*}
\bar{K}_1 &= \frac{1}{2} [K_1 (1 + \cos \theta_0) - 3K_{\Pi} \sin \theta_0] \cos \frac{\theta_0}{2} \\
\bar{K}_{\Pi} &= \frac{1}{2} [K_1 \sin \theta_0 + K_{\Pi} (3 \sin \theta_0 - 1)] \cos \frac{\theta_0}{2}
\end{align*}
\]

(4)

So the energy release rate at the crack initiation is expressed as follows.

\[
\bar{G}_{\theta_0} = \frac{1}{E} (\bar{K}_1^2 + \bar{K}_{\Pi}^2)
\]

(5)
By using the inverse derivation method, the angle of crack initiation is assumed as $\theta = \pi - \theta_0$, and the energy release rate at this direction reaches the threshold value of crack initiation $G_c$, i.e., $G_c = G_0$. By calculating, the angle $\theta_0$ should satisfy the following relationship.

$$\frac{-2}{3} \left. \frac{\partial \sigma_\theta}{\partial \theta} \right|_{\theta = \pi - \theta_0} = \tau_{\theta \theta} \bigg|_{\theta = \pi - \theta_0} = 0$$  \hspace{1cm} (6)

It can be seen from formula (1) and formula (3) that the maximum crack initiation angle is a directional angle with the maximum value of $\sigma_\theta$, i.e., the direction which the energy release rate reaches the maximum value is also the direction which the circular direction stress $\sigma_\theta$ reaches the maximum value, with $K_{II} = 0$. So the energy release rate equation on the maximum crack initiation angle is as follows.

$$G_0 = \frac{1}{2} K_1 f\left(\frac{q + a}{a}\right) \cos \theta \left(1 + \cos \theta\right) - \frac{3}{2} K_{II} f\left(\frac{q + a}{a}\right) \sin \theta \cos \frac{\theta}{2}$$  \hspace{1cm} (7)

Where, $K_1$, $K_{II}$ -- stress intensity factor of type I and type II crack at the tip of bifurcate crack,

$$f\left(\frac{q + a}{a}\right)$$ -- The correction coefficient of stress intensity factor, and it is given by [7]:

$$f\left(\frac{q + a}{a}\right) = \frac{3}{8} \left[1 - \frac{\frac{1}{2} q + a}{q + a} + \frac{3}{8} \left(\frac{\frac{1}{2} q + a}{q + a}\right)^2 + \frac{5}{16} \left(\frac{\frac{1}{2} q + a}{q + a}\right)^3 + \frac{35}{128} \left(\frac{\frac{1}{2} q + a}{q + a}\right)^4 + 0.537 \left(\frac{\frac{1}{2} q + a}{q + a}\right)^5\right]$$

On the basis, formula (2) and formula (4) are programed in MATLAB software. And the initiation angle $\theta_0$ of the energy release rate can be obtained as 1.217rad, so $\theta = \pi - \theta_0 = 20.28^\circ$.

Then substituting the initiation angle $\theta_0$ into the formula (7) to get the energy release rate curve changing with the loading force as shown in figure 4. By comparing the energy release rate at the V-shaped notch tip with the threshold value of the initiation of cracking, the intersection of the energy release rate surface and the initiation threshold surface is obtained. It is found that when the critical loading force is about 1500N (the grid matrix is separated by 50 dimensions), the corresponding crack initiation angle energy release rate is $7.539 \times 10^4$Pa.m$^2$.

![Figure 4. Comparison of the energy release rate at the bar V-notch tip with the threshold of initiation.](image-url)
Experiment Radial Forging Punch

Using the fatigue feeding machine with high-speed radial forging [8], as shown in Figure 5, 45 # steel bars were tested to verify the critical loading force and the crack initiation angle of V-notch tip. The experimental results show that several bifurcate microcracks occur at the V-shaped notch tip under the critical loading force of 1500N. After these microcracks are polymerized, macroscopic cracks occur after the initial cracks are formed and expanded, until the bar breaks.

Figure 5. Fatigue feeding device with high-speed radial forging punch.

Results and Discussion

The angle of V-shaped notch is $2\beta = 60^\circ$, so $\beta = 30^\circ$. According to the geometry characteristics of V-shaped notch tip, the formula for calculating the crack initiation angle can be obtained:

$$\theta = |\theta_2| + \beta - |\theta_1|$$

Where, $\theta_1$ is the extrapolation of the free surface angle at the V-shaped notch tip; $\theta_2$ is the extrapolation of the incision fracture angle at the V-shaped notch tip.

According to the formula (8), the crack initiation angle with 45 # steel bar is measured and calculated, and the result is shown in figure 6. The left of V-shaped notch tip incision fracture angle $\theta_2$ is $25.8^\circ$, and the left of V-shaped notch tip free surface angle $\theta_1$ is $32.5^\circ$, so the left of V-shaped notch tip initiation angle $\theta$ is $23.3^\circ$. Similarly, the right of V-shaped notch tip initiation angle $\theta$ is $24.8^\circ$. Thus, the crack initiation angle obtained by the cropping experiments and the theoretical calculation of the cracking angle are generally consistent.

Figure 6. Calculating results of tip crack initiation angle of 45 steel V-groove.

Summary

This paper proposes a method for calculating the crack initiation angle and the critical loading force in precision low-stress cropping based on energy release rate theory. And the formula for calculating the energy release rate at the maximum crack initiation angle is established. The calculated results show that when the critical loading force is 1500N, the micro-cracks in the V-shaped notch tip begin to occur at the angle $\theta$ of $20.28^\circ$ and the energy release rate at the initiation angle is $7.539 \times 10^4$Pa.m$^2$. The cropping experiment shows that when the critical loading force is 1500N, the initiation angle at
the left of 45 # steel V-shaped notch tip is 23.3°, the initiation angle $\theta$ of the right of 45 # steel V-shaped notch tip is 24.8°. Thus, the crack initiation angle obtained by the experiments and the theoretical calculation of the cracking angle are generally consistent.

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