Research on the Impact of Price’s Co-opetition on Stability of the Enterprise Willingness to Cooperate Based on Game Analysis

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Abstract: The paper established cournot duopoly competition model with price’s co-opetition, introduced degree of willingness to cooperate, and analyzed the impact of price’s co-opetition on the degree of willingness to cooperate. the overall profit and individual differences are compared under two cases which are both partial cooperation, one partial cooperation and the other betrayal, revealing the characteristics of decision-making behavior. as co excess profits do not represent the individual's excess profits, betrayal is the optimal choice. reasonably efficient profits allocation mechanisms and contractual constraints, are the condition of oligopolistic stable cooperation.

1. Introduction

Although competition is the basic form of the organization, but because of the ability of enterprises between the endowment, the resources are not the same, can still create a "harmony" between the possible of space. for a region or an industry in the oligopoly business, the competition is reflected in the profits of the competition. Because the profit competition, the enterprises can not form the production of cooperation, and also led to the overall profit and individual profits of the conflict. selfish and altruism are two controversial extreme assumptions about the behavior goal of actors. for the selfish assumption, the classic prisoner's dilemma in game theory shows that the behavior from the (individual) interest can not really achieve the ultimate benefit of the individual \cite{1}, for the altruistic assumption, (completely) the behavior of the interests of others ultimately can not really achieve the best interests of others, also do not accord with the essence of the subject to pursue their own interests \cite{2}, many of which have been studied selfish and altruism in many ways.

The literature \cite{3} considered the production of homogeneous products by an N-vendor Cournot model. In this context, each game is equally concerned with the profits of himself and its competitors, which is a sufficient condition for the GR equilibrium set to be consistent with the set of strategic combinations with collective Pareto efficiency. the literature studied the effect of alternative parameters with product on R & D cooperation. this paper \cite{5}, discusses the existence and stability of Nash equilibrium and the complex dynamical phenomena such as branch, chaos and singular attractor are found by numerical simulation.

2. Model parameter setting and thesis ideas

Assuming that the two companies in a region or a certain kind of product market for oligopoly competition, are rational economic people, the information between the two is complete, the marginal cost of each unit product is $c_i$ oligopoly enterprises were 1,2,enterprise $i$ production is $q_i$, the inverse demand function is :

$$\pi_i = (p_i - c_i) * q_i = (m_i - q_1 + b_i * q_2 - c_i) * q_i.$$ \hspace{1cm} (1)

Where $b_i$ represents the Price's Co-opetition of a product for another product $-1 \leq b_i \leq 1$, if $b_i$ greater than 0, indicating that the product to improve the price of another product, that is complementary. when $b_i$ is less than 0, indicating that the product to weakening the price of another product,
To analyze the effect of price’s co-petition $b_1$ on the cooperation willing of the two parties. assume $v_1 = v_2$, $m_1 = m_2$, $c_1 = c_2$.

3. In the case of partial cooperation, the best cooperative willingness analysis

3.1 In the case of partial cooperation, the best degree of cooperation and production, income

Most of the literature, the perfect competition for the subject profit function and profit decision model are

$$
\pi^1_c = \pi^1_p(\alpha, \beta) = (p_1 - c_1) \cdot q_1 = (m_1 - q_1 + b_2 \cdot q_2 - c_1) \cdot q_1. 
$$

$$
\pi^2_c = (p_2 - c_2) \cdot q_2 = (m_2 - q_2 + b_1 \cdot q_1 - c_2) \cdot q_2.
$$

$$
\pi = (p_1 - c_1) \cdot q_1 + (p_2 - c_2) \cdot q_2 = (m_1 - q_1 + b_2 \cdot q_2 - c_1) \cdot q_1 + (m_2 - q_2 + b_1 \cdot q_1 - c_2) \cdot q_2.
$$

In the case of partial cooperation, set the degree of cooperation is $\alpha, \beta (-1 \leq \alpha, \beta \leq 1)$, and set the letter $p$ represent part cooperation, and the two sides simultaneously decide the output and the willingness degree of cooperation. then the subject 1, 2 in the cooperative willingness the production decision model is:

$$
\pi^1_p = (p_1 - c_1) \cdot q_1 + \alpha \cdot (p_2 - c_2) \cdot q_2 = (m_1 - q_1 + b_2 \cdot q_2 - c_1) \cdot q_1 + \alpha \cdot (m_2 - q_2 + b_1 \cdot q_1 - c_2) \cdot q_2.
$$

$$
\pi^2_p = (p_2 - c_2) \cdot q_2 + \beta \cdot (p_1 - c_1) \cdot q_1 = \beta \cdot (m_1 - q_1 + b_2 \cdot q_2 - c_1) \cdot q_1 + \beta \cdot (m_2 - q_2 + b_1 \cdot q_1 - c_2) \cdot q_2.
$$

$$
\frac{\partial \pi^1_c}{\partial q_1} = 0; \frac{\partial \pi^2_c}{\partial q_2} = 0.
$$

$$
q^1_p = \frac{(c - m) \cdot (2 + b_1 + \alpha \cdot b_1)}{b_2 \cdot b_1 + \alpha \cdot b_1^2 + \beta \cdot b_2^2 + \alpha \cdot \beta \cdot b_1 \cdot b_2 - 4}.
$$

$$
q^2_p = \frac{(c - m) \cdot (2 + b_1 + \beta \cdot b_2)}{b_2 \cdot b_1 + \alpha \cdot b_1^2 + \beta \cdot b_2^2 + \alpha \cdot \beta \cdot b_1 \cdot b_2 - 4}.
$$

Both sides in the production decision-making, are to maximize their own profits as the goal, to determine the best willing degree of cooperation, so part of the cooperation after their own income is:

$$
\pi^1_p = \pi^1_p(\alpha, \beta) = (p_1 - c_1) \cdot q_1 = (m_1 - q_1 + b_2 \cdot q_2 - c_1) \cdot q_1 = \frac{-(c - m) \cdot (2 + b_2 + \alpha \cdot b_1) \cdot (\alpha \cdot b_1 - b_2 + \alpha \cdot b_1^2 + \alpha \cdot \beta \cdot b_1 \cdot b_2 - 2)}{b_2 \cdot b_1 + \alpha \cdot b_1^2 + \beta \cdot b_2^2 + \alpha \cdot \beta \cdot b_1 \cdot b_2 - 4^2}.
$$

$$
\pi^2_p = \pi^2_p(\alpha, \beta) = (p_2 - c_2) \cdot q_2 = (m_2 - q_2 + b_1 \cdot q_1 - c_2) \cdot q_2 = \frac{-(c - m) \cdot (2 + b_1 + \beta \cdot b_2) \cdot (\beta \cdot b_2 - b_1 + \beta \cdot b_2^2 + \alpha \cdot \beta \cdot b_1 \cdot b_2 - 2)}{b_2 \cdot b_1 + \alpha \cdot b_1^2 + \beta \cdot b_2^2 + \alpha \cdot \beta \cdot b_1 \cdot b_2 - 4^2}.
$$
\[ \frac{\partial \pi^{11}_p(\alpha, \beta)}{\partial \alpha} = 0, \frac{\partial \pi^{21}_p(\alpha, \beta)}{\partial \beta} = 0. \] (12)

Solution of the equations:

\[ \alpha_i = \frac{b_i^2 + b_j}{b_i - b_i * b_j + 2}, \beta_i = \frac{b_j^2 + b_i}{b_j - b_i * b_j + 2}. \] (13)

The yield of the best cooperative willingness and poor yield are:

\[ q_p^1(\alpha^*, \beta^*) = \frac{(c - m) * (b_j - b_i * b_j + 2)}{4 * (b_j * b_i - 1)}. \] (14)

\[ q_p^2(\alpha^*, \beta^*) = \frac{(c - m) * (b_i - b_i * b_j + 2)}{4 * (b_i * b_i - 1)}. \] (15)

\[ q_p^1(\alpha^*, \beta^*) - q_p^2(\alpha^*, \beta^*) = \frac{(c - m) * (b_j - b_i)}{4 * (b_j * b_i - 1)}. \] (16)

\[ \pi^{11}_p(\alpha^*, \beta^*) = (c - m)^2 \frac{(b_j - b_i * b_j + 2) * (b_i + 2) * (b_j + 2)}{16 * (1 - b_i * b_j)}. \] (17)

\[ \pi^{21}_p(\alpha^*, \beta^*) = (c - m)^2 \frac{(b_i - b_i * b_j + 2) * (b_j + 2) * (b_i + 2)}{16 * (1 - b_i * b_j)}. \] (18)

The sum and the margin of the proceeds of the subject 1 and 2 is

\[ \pi_p(\alpha^*, \beta^*) = \pi^{11}_p(\alpha^*, \beta^*) + \pi^{21}_p(\alpha^*, \beta^*) = (c - m)^2 \frac{(b_i - b_i * b_j + 2) * (b_j + 2) + (b_j - b_i * b_j + 2) * (b_i + 2)}{16 * (1 - b_i * b_j)}. \] (19)

\[ \pi^{11}_p(\alpha^*, \beta^*) - \pi^{21}_p(\alpha^*, \beta^*) = \frac{(b_j - b_i) * (c - m)^2 * (b_i + b_j - b_i * b_j + 4)}{16 * (1 - b_i * b_j)}. \] (20)

When \( b_i > b_j \), the revenue of the subject 1 is smaller than the income of the subject 2. When \( b_i < b_j \), the revenue of the subject 1 is greater than the income of the subject 2. When \( b_i = b_j \), both sides of the same income.

### 3.2. Analysis on the Impact of Price’s Co-petition on Cooperative Willingness

For subject 1, when \( b_2 > 0, \alpha^* > 0 \), or \( b_2 < 0, \alpha^* < 0 \) When \( b_2 = 0, \alpha^* = 1 \). the greatest degree of cooperation. when \( b_2 = -1 \) or \( b_2 = 0 \), so \( \alpha^* = 0 \). for subject 2, when \( b_1 > 0, \beta^* > 0 \) when \( b_1 < 0, \beta^* < 0 \), when \( b_1 = 0, \) or -1, \( \beta^* = 0 \). it can be seen that when the other side of the price to one's own product price have improved, there is a good desire for cooperation; when the other side of the price to one's own products are competitive, the willingness to cooperate is negative, there will be acts of selfishness , such as malicious increase in product output.

When \( b_2 = b_1, \alpha^* = \beta^* \). when the Price’s Co-petition of both products are the same, the cooperation willingness of the two is the same. when \( b_2 > 0, \) and \( b_1 < 0, \alpha^* \) larger, when the other side of the product itself is highly complementary, and their products on the other's highly competitive, the greatest degree of cooperation.
When \( b_2 = b_1 = -1 \), or \( b_2 = b_1 = 0, \alpha = \beta = 0 \), that is, when product price for the two sides have full competition or both product prices are not relevant, the willingness to cooperate is 0. \( b_2 = b_1 = 1, \alpha = \beta = 1 \), that is, when the two sides of the product price is complementary, the highest degree of cooperation.

\[
\left. \frac{\partial \alpha^*}{\partial b_1} \right| = \frac{(b_2^2 + b_2) \times (b_2 - 1)}{(b_1 - b_2 \times b_1 + 2)^2}.
\]

(21)

when \( b_2 > 0, \frac{\partial \alpha^*}{\partial b_1} < 0 \); this shows that if the other side of the product to enhance their own products, then the other side of the product price complement each other, the lower the degree of cooperation. when \( b_2 < 0, \frac{\partial \alpha^*}{\partial b_1} > 0 \); if the other product is competitive with one's own products, the greater the complementarities of the other party's product price, the higher the degree of cooperation.

4 Conclusion

This paper breaks through the scope of competing and fully cooperative production of homogeneous products based on the Cournot model, which is concentrated by most oligopoly production research institutes. Research on Production Competition and Cooperation Mechanism under Asymmetric Price Competition. This paper analyzes the differences between the oligarchs in partial cooperation and partial cooperation, and the difference between the overall and individual profits in the two cases and reveal the characteristics of the decision-making behavior. It is a stable condition for the formation of the cooperation between the oligopoly enterprises and the effective contract of excess profits.

References