Portfolio Selection Based on BP Neural Network and Black-Litterman Model

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ABSTRACT

With the revolution of technology, the rapid development of machine learning has brought new impetus to the development of finance. Combining machine learning technology with traditional investment portfolio theoretical models can effectively reduce investment risks and increase returns. The Mean-Variance model proposed by the pioneer of contemporary portfolio theory, Markowitz, defines risk as the volatility of the rate of return. For the first time, the method of mathematical statistics is applied to the study of portfolio selection. This method makes the multi-objective optimization of returns and risks achieve the best balance effect. However, because this model is too sensitive to the input value, this article will use the Black-Litterman model and use the BP neural network to make predictions for the BL model's view matrix. The empirical results show that the improved Black-Litterman model obtains excess returns compared to the Mean-Variance model and other models, which optimizes the investment portfolio.

KEYWORDS

Mean-Variance Model, Black-Litterman Model, BP Neural Network.

INTRODUCTION

In recent years, great changes have taken place in the total assets and structure of Chinese residents. Personal property has doubled and ordinary households have more disposable income. As deposit income continues to decrease, residents hope that investment will guarantee the quality of future life. Therefore, financial management needs continue to rise.

Robo-advisor, as an emerging investment method, uses algorithms to provide digital financial advice based on information from clients about their financial situation and future goals through an online survey. It is growing rapidly in the US market. According to the survey of the A.T.Kearney, which has a high degree of acceptance in the industry, 20% of the 4,002 banking wealth management clients have learned about it, 48% of clients of the traditional investment advisory are interested in the Robo-advisor and 69% of the traditional investment advisory clients are willing to try the services of Robo-advisor. The market size of Rob-advisor continues to expand.

A core technology of Robo-advisor is asset allocation. Thorough research on asset allocation models can effectively improve the returns of investors. Under the background of the new round of technological revolution, the rapid development of technology, such as artificial intelligence and machine learning,
has also provided a new impetus for the development of portfolio methods. Combining traditional theories with machine learning methods can reduce the risks and increase returns to some extent. This paper will mainly study various portfolio theories and focus on the Markowitz model, the Black-Litterman model and the Black-Litterman model modified by BP neural network to optimize the portfolio.

The remainder of this paper is organized as follows. Section 2 provides a literature review of the Black-Litterman model. Section 3 describes the Black-Litterman model, BP neural network and BL-BP model in detail. Section 4 provides empirical analysis. Section 5 is a conclusion.

LITERATURE REVIEW

Literature Review of Black-Litterman Model

In 1990, the Black-Litterman model was first officially proposed by Fisher Black and Robert Litterman in Goldman Sachs internal magazines and was widely promoted and applied after it was published in the FAJ (Financial Analyst Magazine) in 1992. The model is based on the prior distribution of market equilibrium excess returns, adding the investor's subjective view and using Bayes' formula to find the posterior distribution of expected returns of assets.

Black and Litterman's subsequent research further pointed out that investors' confidence in the subjective opinion of the expected return of assets may not be 100% which means there is an error in the opinion[1]. Investors can adjust the degree of deviation in the expected return and the equilibrium return of the market by adjusting the level of confidence in the views. Too high or too low confidence levels will affect the model's effectiveness.

Since then, many scholars and financial practitioners have conducted extensive and in-depth research on the Black-Litterman model.

Research on Model Parameters:

Bevan and Winkelmann (1998) added an information ratio (generalized Sharpe ratio) measurement to adjust the confidence level in the empirical analysis process of global asset allocation and set \( \tau \) at 0.025-0.05[2].

He and Litterman (1999) believed that the uncertainty of the subjective point of view and the market uncertainty are proportional to each other and set the error matrix of subjective opinion matrix \( \Omega \) to \( \Omega = \text{diag}(P' \cdot \Sigma \cdot P) \), where \( \tau \) is a small value which was set to 0.05[3].


Research on model improvement:

Meucci (2006) uses the Copula function to improve the Black-Litterman model which allows the market equilibrium return to follow a non-normal distribution and made the model more widely used and more realistic[5].

Christodoulakis (2002) made a detailed mathematical derivation of the Black-Litterman model and gave the mathematical form of the model[6].
INTRODUCTION OF MODELS

Mean-Variance Model

Markowitz's theory is based on the following assumptions:

- When investors measure each investment choice, they are based on the probability distribution of return within a certain holding time.
- The investor estimates the risk of the portfolio based on the expected return.
- Investors' decisions are only based on the risk and return.
- With a certain level of risk, investors want the maximum return; the corresponding is that with a certain level of income, investors want the minimum risk.

Based on the above assumptions, Markowitz established the calculation method of the expected return and risk of the portfolio and the efficient frontier theory and established the Mean-Variance model of the optimal allocation of assets. According to Hypothesis 4, investors can calculate the optimal asset ratio.

Black-Litterman Model

ORIGINAL BLACK-LITTERMAN MODEL

The Black-Litterman model is an extension of the Mean-Variance model. And its main contribution is to provide a theoretical framework that can integrate market equilibrium return and personal perspectives to re-estimate a more reliable expected return rate and obtain the optimal asset allocation.

The framework of the Black-Litterman model is as follows.

Step1: Starting from the market equilibrium assumption, obtain the prior distribution of the expected return of assets

Assume a portfolio of \( N \) assets which follow normal distribution:

\[
\mathbf{r} \sim N(\mathbf{\mu}, \mathbf{\Sigma})
\]  

(1)

Then assume that the prior distribution of the expected return of the assets, \( \mathbf{\mu} \), is a normal distribution.

\[
\mathbf{\mu} \sim N(\mathbf{\pi}, \tau \mathbf{\Sigma})
\]  

(2)

\( \mathbf{\Sigma} \) represents the covariance matrix of the return of each asset. \( \tau \) represents the adjustment factor for the covariance of equilibrium returns which is usually 0.025-0.05 and the value in this paper is 0.05.

We can use

\[
\max (\mathbf{\omega}^T \mathbf{\pi} - \frac{1}{2} \mathbf{\omega}^T \tau \mathbf{\Sigma} \mathbf{\omega})
\]

subject to \( \sum_{i=1}^{N} \omega_i = 1 \)  

(3)

to calculate the equilibrium returns of market.
\( \delta \) represents the risk aversion index under market equilibrium. The value of \( \delta \) is generally 1-3.

We can get the expression of \( \pi \) by inverse optimization

\[
\pi = 2 \delta \omega
\]  

(4)

Step 2: Construct investor perspectives and conditional distribution of expected asset returns

Construct expressions to describe the views of investors:

\[
P \mu - N(\nu, \Omega)
\]  

(5)

\( P = \begin{bmatrix} p_{11} & \cdots & p_{1N} \\ \vdots & \ddots & \vdots \\ p_{m1} & \cdots & p_{mN} \end{bmatrix} \) is pick matrix consisting of \( N \) assets and \( m \) views.

\( \nu = [p_{1}, p_{2}, \ldots, p_{m}]^{T} \) is the expected return vector of the views. \( \Omega \) represents the error matrix of the investor's opinion which means the investor's confidence in the opinion. Meucci gave a way to set \( \Omega \), which is,

\[
\Omega = \frac{1}{c} P \Sigma P^{T}
\]  

(6)

where \( c \in (0, \infty) \) is the overall confidence in opinions.

Step 3: Use Bayesian theory to solve the posterior distribution of asset returns after combining subjective views

By using Bayes formula, we can get the posterior of \( \mu \)

\[
\mu \mid \nu; \Omega - N(\mu_{bl}, \Sigma_{bl})
\]  

(7)

and the expected return and covariance of \( \mu \) are:

\[
\mu_{bl} = [(\tau \Sigma)^{-1} + (P^{T} \Sigma^{-1} P)^{-1}]^{-1} \left[(\tau \Sigma)^{-1} \pi + P^{T} \Sigma^{-1} \nu\right]
\]  

(8)

\[
\Sigma_{bl} = [(\tau \Sigma)^{-1} + (P^{T} \Sigma^{-1} P)^{-1}]^{-1}
\]  

(9)

In order to get the distribution of return, \( r \), we introduce \( z \sim N(0, \Sigma) \) and express \( r \) as \( r = \mu + z \). Then, we get

\[
r \mid \nu; \Omega - N(\mu_{bl}, \Sigma_{bl})
\]  

(10)

Since \( \mu \) and \( z \) are independent of each other,

\[
\Sigma_{bl} = \Sigma + \Sigma_{bl}
\]  

(11)

We write the parameters as follows to facilitate the calculation.

\[
\mu_{bl} = \pi + \tau P^{T} (\pi P \Sigma P^{T} + \Omega)^{-1} (\nu - P \pi)
\]  

(12)
\[ \Sigma_{bl} = (1 + \tau)\Sigma - \tau^2 \Sigma P^T (\tau P \Sigma P^T + \Omega)^{-1} P \Sigma \]  

(13)

CONTRADICTION OF OPINION CONFIDENCE IN TWO EXTREME CASES

Firstly, when the investor has no confidence in the subjective views, \( \mu \), the posterior distribution should be equal to the prior distribution, namely, \( r - N(\pi, \Sigma) \). However, in the original Black-Litterman model, when \( \Omega \to \infty \), (10) becomes

\[ r - N(\pi, (1 + \tau)\Sigma) \]  

(14)

The above formula is equivalent to the prior distribution only in the case of \( \tau = 0 \). And \( \tau = 0 \) represents that investor has full confidence in the estimate of \( \mu \) which is not equivalent to having full confidence in \( r \).

Secondly, when investor has full confidence in \( \mu \), the posterior distribution should be

\[ r | \nu - N(\mu | \nu, \Sigma | \nu) \]  

(15)

Where

\[ \mu | \nu = \pi + \Sigma P^T (P \Sigma P^T)^{-1}(\nu - P \pi) \]  

(16)

\[ \Sigma | \nu = \Sigma - \Sigma P^T (P \Sigma P^T)^{-1} P \Sigma \]  

(17)

However, in original Black-Litterman model, when \( \Omega \to 0 \), the posterior distribution is

\[ r - N(\mu_{bl}^{\Omega=0}, \Sigma_{bl}^{\Omega=0}) \]  

(18)

Where

\[ \mu_{bl}^{\Omega=0} = \pi + \Sigma P^T (P \Sigma P^T)^{-1}(\nu - P \pi) \]  

(19)

\[ \Sigma_{bl}^{\Omega=0} = (1 + \tau)\Sigma - \tau^2 \Sigma P^T (P \Sigma P^T)^{-1} P \Sigma \]  

(20)

Obviously, (18) is not equal to (15)

The cause of these two contradictions is that in the above BL model modeling was performed for \( \mu \), so the confidence of the viewpoint only affects \( \Sigma_{bl} \) in \( \Sigma_{bl} \) and has no effect on the part of market fluctuations.

MARKET BLACK-LITTERMAN MODEL

To resolve the above contradiction, we can directly model \( r \)

\[ r - N(\pi, \Sigma) \]  

(21)

Investor's subjective view is \( \nu = Pr \), subject to condition-al distribution.
\[ \nu \mid R \sim N(Pr, \Omega) \]  

where \( \Omega \) represents uncertainty of opinion. Similarly, we can get the posterior distribution is

\[ r \mid \nu, \Omega \sim N(\mu_{BL}, \Sigma_{BL}) \]

Where

\[ \mu_{BL} = \pi + \Sigma_{\pi}^{-1}(\nu - P\pi) \]

\[ \Sigma_{BL} = \Sigma - \Sigma_{\pi}^{-1}(P\Sigma P^T + \Omega)^{-1}P\Sigma \]

It can be verified that the market Black-Litterman model resolves contradictions in extreme cases.

Finally, as long as the following function is maximized, the optimal asset portfolio can be solved.

\[
\max(\omega^T \mu_{BL} - \frac{1}{2} \delta \sigma^T \Sigma_{BL} \delta) \\
\text{s.t. } \sum_{i=1}^{N} \omega_i = 1
\]

**BP Neural Network Model**

BP neural network is a kind of multilayer feed-forward neural network. Its main characteristics is that the signal is propagated forward and the error is propagated backward. Specifically, for the following neural network model with only one hidden layer, the process is mainly divided into two stages as shown in Figure 1. The first stage is the forward propagation of the signal from the input layer through the hidden layer and finally to the output layer. The second stage is the back propagation of the error from the output layer to the hidden layer and finally to the input layer in order to adjust the weight and bias.

**Figure 1.** BP neural network of 3 layers.

\( a) \) Step1: Initialize the network
Assume that the number of nodes in the input layer is $n$, the number of nodes in the hidden layer is $q$, and the number of nodes in the output layer is $m$. The weight from the input layer to the hidden layer is $\omega_{ij}$, the weight from the hidden layer to the output layer is $\omega_{jk}$, and the bias from the input layer to the hidden layer is $a_j$. The bias from hidden layer to output layer is $b_k$. The learning rate is $\eta$ and the activation function is $g(x)$, which is a Sigmoid function. It has the form:

$$g(x) = \frac{1}{1+e^{-x}}$$  \hspace{1cm} (27)

**(b) Step2: Output of the hidden layer**

First, normalize the original data to get the input value $x = (x_1, x_2, \ldots, x_n)$, then the output of the hidden layer $H_j$ is:

$$H_j = g(\sum_{i=1}^{n} \omega_{ij} x_i + a_j)$$  \hspace{1cm} (28)

**(c) Step3: Input of output layer**

$$O_k = \sum_{j=1}^{q} H_j \omega_{jk} + b_k$$  \hspace{1cm} (29)

**(d) Step4: Calculation of error**

Assume $Y_k$ is the expected output. Then the error is

$$E = \frac{1}{2} \sum_{k=1}^{m} (Y_k - O_k)^2$$  \hspace{1cm} (30)

where $Y_k - O_k = e_k$

$$E = \frac{1}{2} \sum_{k=1}^{m} e_k^2$$  \hspace{1cm} (31)

**(e) Step5: Update of weight**

Derivation through mathematics

$$\begin{align*}
\omega_{ij} &= \omega_{ij} + \eta H_j (1-H_j) x_i \sum_{k=1}^{m} \omega_{jk} e_k \\
\omega_{jk} &= \omega_{jk} + \eta H_j e_k
\end{align*}$$  \hspace{1cm} (32)

The following is the derivation process of the formula.

This is the process of error back propagation. Our goal is to minimize the error function, which is $\min E$. We apply the gradient descent method.

$$\frac{\partial E}{\partial \omega_{jk}} = \sum_{k=1}^{m} (Y_k - O_k)(- \frac{\partial O_k}{\partial \omega_{jk}}) = (Y_k - O_k)(-H_j) = -e_k H_j$$  \hspace{1cm} (33)

The update formula of weight is
(34)

\[ \omega_k = \omega_k + \eta H_k e_k \]

The weight of the input layer to the hidden layer is updated to

\[ \frac{\partial E}{\partial \omega_i} = \frac{\partial E}{\partial H_i} \frac{\partial H_i}{\partial \omega_i} \]  

(35)

Where

\[
\frac{\partial E}{\partial H_i} = (Y_i - O_i)(-\frac{\partial O_i}{\partial H_i}) + \ldots + (Y_m - O_m)(-\frac{\partial O_m}{\partial H_i})
\]

\[ = -(Y_i - O_i)\omega_i \ldots -(Y_m - O_m)\omega_m = -\sum_{k=1}^{m} (Y_k - O_k)\omega_k = - \sum_{k=1}^{m} \omega_k e_k \]  

(36)

\[
\frac{\partial H_i}{\partial \omega_j} = g'(\sum_{i=1}^{n} \omega_j x_i + a_j)
\]

\[ = g'(\sum_{i=1}^{n} \omega_j x_i + a_j)[1-g(\sum_{i=1}^{n} \omega_j x_i + a_j)] \frac{\partial (\sum_{i=1}^{n} \omega_j + a_j)}{\partial \omega_j}
\]

\[ = H_i(1-H_i)x_i \]  

(37)

Therefore, the weight update formula is

\[ \omega_j = \omega_j + \eta H_j(1-H_j)x_i \sum_{i=1}^{m} \omega_j e_k \]  

(38)

\textbf{f) Step6 : Update of bias}

Bias is

\[
\begin{cases}
  a_j = a_j + \eta H_j(1-H_j) \sum_{i=1}^{m} \omega_j e_k \\
  b_k = b_k + \eta e_k
\end{cases}
\]  

(39)

The following is the derivation process of the formula.

Update of bias from hidden layer to output layer

\[
\frac{\partial E}{\partial b_k} = (Y_k - O_k)(-\frac{\partial O_k}{\partial b_k}) = -e_k
\]  

(40)

The update formula for the bias is

\[ b_k = b_k + \eta e_k \]  

(41)

Update of bias from input layer to hidden layer
\[
\frac{\partial E}{\partial a_j} = \frac{\partial E}{\partial H_j} \frac{\partial H_j}{\partial a_j}
\] (42)

Where

\[
\frac{\partial E}{\partial a} = (Y - O_j)(-\frac{\partial O_j}{\partial H_j}) + \ldots + (Y - O_n)(-\frac{\partial O_n}{\partial H_j})
\]
\[
= -(Y- O_j)\omega_{jk} - \ldots - (Y - O_n)\omega_{jn}
\]
\[
= -\sum_{i=1}^{m}(Y - O_i)\omega_{jk} = -\sum_{i=1}^{m}\omega_{jk}e_i
\] (43)

\[
\frac{\partial H_j}{\partial a_j} = g(\sum_{i=1}^{n}\omega_{ji}x_i + a_j)
\]
\[
= g(\sum_{i=1}^{n}\omega_{ji}x_i + a_j)[1 - g(\sum_{i=1}^{n}\omega_{ji}x_i + a_j)] \frac{\partial(\sum_{i=1}^{n}\omega_{ji}x_i + a_j)}{\partial a_j}
\]
\[
= H_j(1 - H_j)x_j
\] (44)

The update formula for the bias is

\[
a_k = a_k + \eta H_j(1 - H_j)\sum_{i=1}^{m}\omega_{jk}e_i
\] (45)

g) Step7 : Determine the end of algorithm iteration
Determine whether the specified number of iterations has been reached and determine whether the error is less than the specified value.

**BL-BP Model**

Using the above theory, an optimized BL-BP model can be constructed. First, use historical data and BP neural network to predict future returns. Then use the predicted returns to construct a views matrix. Use the Black-Litterman model to calculate posterior distributions and solve optimization problems. Finally, substitute the actual rate of return into the investment portfolio for back testing.

**EMPIRICAL ANALYSIS**

First of all, I select 50 stocks from A share market with large circulation and large market value as follows:

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<tr>
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<tr>
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</table>
Building a Neural Network

SELECT SAMPLE DATA

I select 731 samples from January 5, 2016 to December 28, 2018 as the training set, and 16 trading days from January 1, 2019 to January 23, 2019 as the test set. And use the closing price to calculate the percentage return, that is,

$$R_n = \frac{s_n - s_{n+1}}{s_{n-1}}; \text{ where } s_n \text{ is the } n \text{ day's closing price.}$$

PREPROCESSED SAMPLE DATA

Because the daily return rate of stock has a lot of noise, I use the second-order difference of the return rate and normalize the differentiated data.

In forecasting, the choice of analysis period is very crucial. Usually, the period of analysis is 5 days, 10 days, 20 days, etc. This article selects 5 days as the analysis period to predict the return, which means the second-order difference of the return rate of the previous five days are used to predict the second-order difference of the return rate of the next trading day. The function can be expressed as

$$\Delta^2 R_n = f(\Delta^2 R_{n-1}, \Delta^2 R_{n-2}, \Delta^2 R_{n-3}, \Delta^2 R_{n-4}, \Delta^2 R_{n-5})$$

BUILD A NEURAL NETWORK

It is proved that the three-layer BP network can arbitrarily approximate a non-linear continuous function. In addition, it is easy to perform as the calculation amount is small. Since it cannot make a great improvement to the optimization of the result if we increase the number of layers, this paper uses a three-layer neural network structure. Regarding the selection of hidden layer nodes, there is no clear analytical determination. The common method is generally trial. Namely, first, set fewer hidden layer nodes to train the network; then gradually increase the number of hidden layer nodes. The initial number of hidden layer neurons can be roughly referred to

$$m = \sqrt{n + l + a}, \text{ where } m \text{ is the number of hidden layer nodes; } n \text{ is the number of input layer nodes; } l \text{ is the number of nodes in the output layer and } a \text{ is a constant between 1-10. After trying many times, I decide to use 12 neurons. Using the nntool toolbox in Matlab, the tan-sigmoid function is used as the activation function and the adaptive lr gradient descent method is the training function.}$$

PREDICTED PRICE

After building the neural network, use the test set to test. Some inspection results are as Figure 2:
BUILD PORTFOLIO

First, by using Mean-Variance Model, we can use Monte Carlo method to stimulate benefits and risks under of investment portfolios. Then, plot the efficient frontier to get the portfolio of largest Sharpe ratio. We can use the plotFrontier() in Matlab to complete this process as shown in Figure 3.

![Efficient Frontier of Mean-Variance Model](image)

The optimal portfolio selected by Mean-Variance model is as shown in Table II:

**TABLE II. OPTIMAL PORTFOLIO BY MEAN-VARIANCE MODEL**

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</tr>
</tbody>
</table>
After obtaining the expectations returns of the 50 stocks, we can construct the pick matrix $P$ and the expectation matrix of rate of return $\mu$, where $P$ is $50 \times 50$, $\mu$ is $50 \times 1$, each row of which corresponds to the predicted rate of return of a stock. According to Idzorek's article [4].

The value of $\tau$ is taken as 0.05. Applying (24) and (25) we can get the expected rate of return and covariance of BL-BP model. Finally, the function of Matlab is used to obtain the optimal portfolio for 16 trading days from January 2, 2019 to January 23, 2019.

In order to compare the optimization effect of BL-BP model, we also construct another three portfolios.

First, we invest all assets in equal proportions.

Second, we get the standard deviation of each stock and their reciprocal, then calculate the weight to construct the stock. To express it mathematically, assume $\sigma_i$ is the standard deviation of $i$-th stock, then the proportion of the stock is $\frac{\sigma_i}{\sum_{i=1}^{50} \sigma_i}$.

Third, we classify 50 stocks according to market value and industry as Figure 4, where large market value refers to market value above $10^{10}$ and mid-market value refers to market value between $10^9$ and $10^{11}$.

![Figure 4. Categories of 50 stocks.](image)
RESULT ANALYSIS

Figure 5 shows the return of the portfolio obtained by the BL-BP model and other portfolios in 16 trading days from January 2, 2019 to January 23, 2019.

![Figure 5](image)

Figure 5. Daily rate of return of different portfolio of 50 stocks.

EW refers to each stock with equal weight; MVO refers to Mean-Variance Model; BL-BP refers to BL-BP Model; STD refers to constructing portfolio by standard deviation; Classification refers to construction portfolio by dividing them according to market value and industry; Market Value refers to the proportion of each stock is the proportion of their market value.

It can be seen that the rate of return obtained by the BL-BP model for each trading day is relatively good when comparing with those of other portfolios.

In order to see the advantages of BL-BP model more intuitively, I draw the accumulated assets within 16 working days as shown in Figure 6:

![Figure 6](image)

Figure 6. Cumulative rate of return of different portfolio of 50 stocks.

It can be seen that within 16 trading days, the return of the BL-BP model is obviously better than the return of the Markowitz model and any other portfolio, and its cumulative return rate reaches 12.8%. It shows that the BL-BP model has achieved excess returns, and the Mean-Variance model has been effectively improved.
Other Trials

In addition, I also conducted an empirical analysis of the latest constituent stocks of the CSI300 Index, excluding stocks with missing data, leaving 269 stocks. And 619 samples from May 5, 2017 to November 19, 2019 were selected as the training set, and 30 trading days from November 20, 2019 to December 31, 2019 were selected as the test set.

Figure 7 shows the return of the portfolio obtained by the BL-BP model and other portfolios in 30 trading days from November 20, 2019 to December 31, 2019.

In order to see the advantages of BL-BP model more intuitively, I draw the accumulated assets within 30 working days as shown in Figure 8.

We can see that the rate of return calculated by the BL-BP model is very high, but the result calculated by the Mean-Variance model is not good.

What’s more, I also select 4 ETF from the US market considering the trading volume, market value and rate of return. And 984 samples from January 5, 2016 to November 29, 2019 were selected as the training set and 21 trading days from December 1, 2019 to December 31, 2019 were used as the test set.

Figure 9 shows the return of the portfolio obtained by the BL-BP model and 4 ETF in 21 trading days in test set.
It can be seen that the rate of return obtained by the BL-BP model for each trading day is roughly the maximum rate of return of the four ETFs. Compare the return rate of BL-BP model with Mean-Variance model and equally-weighted model as shown in Figure 10.

In order to see the advantages of BL-BP model more intuitively, I draw the accumulated assets within 21 working days as shown in Figure 11, Figure 12:
CONCLUSION AND OPTIMIZATION

This paper uses the Black-Litterman model developed on the basis of Mean-Variance model to construct portfolio and at the same time uses the neural network to predict the return of stocks as the input of views of the Black-Litterman model.

The empirical results prove that the BL-BP model optimizes the Markowitz model better and obtains more excess returns than any other portfolio. Meanwhile, I also tried other neural networks such as RBF neural network, the prediction and optimization results are far inferior to BP neural network.

However, it should be pointed out that the model still needs to be further optimized:

First, when I used BP neural network to predict the rate of return, only the second-order difference of the rate of return of the previous five days is used as an input variable, input variables such as volume can be added for improvement.

Second, we can also try some more complex neural networks, such as LSTM and DNN.

REFERENCES