Location Optimization Model of Rural Express Station with Vehicle Routes Consideration

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ABSTRACT

The construction of rural express station (RES) is an important measure to improve the express delivery outlets and save the “last mile” distribution cost. In order to scientifically plan the rural express logistics network, a location optimization model of RES is established considering vehicle routes of the distribution center and the tasks of parcel dispersion and collection. With the help of the model, the following items can be determined, such as the location of RES, the village groups serviced by different express stations, the number of delivery vehicles required by the distribution center, the route of each delivery vehicle and the optimal average daily cost of the distribution system. The model is applied to RES location problem in a certain town, and a feasible solution is obtained.

KEYWORDS

Rural Express Station, Location Optimization Model, Vehicle Routing.

INTRODUCTION

According to the China Rural Revitalization Strategic Planning 2018-2022, rural logistics infrastructure terminal network should be improved, and related enterprises should be encouraged to expand their network layout in rural areas. In order to reasonably plan the rural logistics network, some scholars have used cross median model[1], facility coverage model[2], clustering algorithm[3] or heuristic algorithm[4] to study the location optimization method of rural logistics distribution center, and a few scholars have studied the location-routing integration optimization model of the rural logistics distribution center based on the center’s delivery and pickup requirements[5]. However, few scholars have studied RES location optimization problem. In fact, since villages in towns are more dispersed and the amount of express parcel sent and received in a single village is smaller, in order to improve the success rate of one-time delivery and reduce the time asymmetry of express delivery and terminal logistics costs, it is necessary to deal with the rural express parcel business on a large scale through the establishment of RES and plan the opening positions of each station in advance. Reference[6] studied a similar problem, but due to the different focus of the study, when determining the location of the third layer facilities (RESs), this article directly takes the transportation costs from the second layer facilities (distribution centers) to the third layer facilities as distribution costs, and ignores parcel pickup requirements at third layer facilities. But in the terminal distribution
practice, because each station often has a need to send locally sourced parcels, and the parcel volume of each station is relatively small (not enough for transportation by a whole vehicle), the distribution center usually uses a circular route to deliver the express parcels to each station and complete pickup tasks at the same time. In order to make the research results more practical and guiding significance, this paper establishes a RES location optimization model with vehicle routes and parcel pickup tasks consideration.

PROBLEM DESCRIPTION

A common distribution center has several distribution vehicles, which can provide express delivery services for RESs. Each station is set up in a village, and it mainly handles express parcel sending and receiving business of several villages within a certain distance. In the daily operation, after receiving the pickup notice, the recipient will pick up the parcel at the station; when the sender needs to send an express parcel, he can go to the nearest station to go through the sending procedures. For ease of management, it is stipulated that only one vehicle can provide delivery service for each station, and each vehicle departs from the distribution center at the beginning and returns to the distribution center after completing the distribution task. Then in order to reduce the total cost of the distribution system, how to determine the location of RES and the village groups serviced by different stations?

LOCATION OPTIMIZATION MODEL

Symbol Description

The location of RES directly affects the “last mile” express delivery speed and delivery route arrangement. To facilitate modeling, define the following symbols: (1) Subscripts. \( k \) represents the delivery vehicle number, if there are \( m \) vehicles available, then \( k = 1, 2, \ldots, m \). Both \( i \) and \( j \) represent the joint number of the distribution center, village and alternative station, where the number of an alternative station is the same as that of the village where it is located; if there are \( n \) villages, and the number of the distribution center is 0, then \( i, j = 0, 1, \ldots, n \). \( I \) represents a village set that can be determined in advance according to specific circumstances. For convenience of description, the distribution center, a village or an alternative station can be referred to as a node. (2) Parameters. \( q_i \) and \( b_i \) represent the average daily receipt volume and average daily shipment volume of village \( i \), respectively; \( d_{ij} \) represents the riding distance from village \( i \) to station \( j \); \( r \) represents the allowable riding distance from each village to its service station. \( Q_k \) represents the maximum carrying capacity of vehicle \( k \); \( g_k \) represents the starting cost of vehicle \( k \); \( c_{ij} \) represents the transportation cost from node \( i \) to node \( j \) (since the terminal distribution is mostly short-distance transportation, so it is assumed that the cost is only related to the distance between two nodes and road conditions, and is less affected by the transportation volume); \( t_{ij} \) represents the transportation time of the delivery vehicle from node \( i \) to node \( j \); \( s \) represents the length of time the vehicle stays at a station (since the operation time of the vehicle at each station is not much different, it is assumed that the length of time is basically the same); \( T \) represents the allowable working time of the distribution vehicle from driving
out of the distribution center to returning to the distribution center. \( f_j \) represents the average daily construction cost of station \( j \). (3) Decision variables. \( x_{ij} \) indicates whether to let station \( j \) serve village \( i \), take 1 for yes, 0 for no. \( y_j \) indicates whether to open station \( j \), take 1 for yes, 0 for no. \( z_{ijk} \) indicates whether vehicle \( k \) is driving directly from node \( i \) to node \( j \), take 1 for yes, 0 for no. \( p_{jk} \) indicates whether vehicle \( k \)'s closed-loop path contains node \( j \), take 1 for yes, 0 for no. \( h_{ijk} \) indicates the load of vehicle \( k \) from node \( i \) to node \( j \).

**Mathematical Model Formation**

**OBJECTIVE FUNCTION**

In order to minimize the average daily total cost of the distribution system, there is

\[
\min \sum_{j=1}^{n} f_j y_j + \sum_{k=1}^{m} g_k P_{0k} + \sum_{k=1}^{m} \sum_{j=0}^{n} c_{jk} z_{ijk}
\]  

(1)

Where, the first item is the cost of opening a station, the second item is the starting cost of the vehicle, and the third item is the cost of express delivery.

**CONSTRAINTS**

For each village, only one station is needed to serve it, so

\[
\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n
\]  

(2)

For each village, it can only be served by the nearest station within a given distance \( r \). In theory, there is

\[
d_{ij} x_{ij} \leq d_{ij} y_j + r(1 - y_j), \quad i, j, j' = 1, 2, \ldots, n
\]

However, in practical applications, when \( d_{ii} = 0 \), no matter how the value of \( x_{ii} \) is taken, there is always \( d_{ii} x_{ii} = 0 \). At this situation, the condition naturally holds and the constraint is invalid. To avoid above situation, rewrite the formula as:

\[
(d_{ii} + 1) x_{ij} \leq (d_{ij} + 1) y_j + (r + 1)(1 - y_j), \quad i, j, j' = 1, 2, \ldots, n
\]  

(3)

For each village, if a station is set up in the village, the station must be the delivery node of a certain vehicle, so

\[
\sum_{k=1}^{m} p_{jk} = y_j, \quad j = 1, 2, \ldots, n
\]  

(4)

For each node (distribution center or station) on the vehicle path, the distribution vehicle must drive to the next node after exiting from this node, so

\[
z_{ijk} = 0, \quad j = 0, 1, \ldots, n; \ k = 1, 2, \ldots, m
\]  

(5)
For each node on the vehicle path, if a vehicle enters the node, the vehicle will also exit the node after completing the loading and unloading operation, so

\[
\sum_{j=0}^{n} z_{ijk} = \sum_{j=0}^{n} z_{jik}, \quad j = 0,1,\ldots,n; \quad k = 1,2,\ldots,m
\]  

(6)

For each node of the vehicle path, if a vehicle enters, the vehicle can only enter once, so

\[
\sum_{j=0}^{n} z_{ijk} = p_{jk}, \quad j = 0,1,\ldots,n; \quad k = 1,2,\ldots,m
\]  

(7)

On each road section, the vehicle load should not exceed the vehicle carrying capacity, so

\[
h_{ijk} \leq Q z_{ijk}, \quad i, j = 0,1,\ldots,n; \quad k = 1,2,\ldots,m
\]  

(8)

In the distribution center, the load of each vehicle when it enters should be the total pickup volume of all stations on the path of the vehicle, so

\[
\sum_{j=1}^{n} h_{ijk} = \sum_{j=1}^{n} \left( \sum_{i=1}^{m} h_{ijk} x_{ij} \right) p_{jk}, \quad k = 1,2,\ldots,m
\]  

(9)

In each station, according to the balance between the transportation volume and the shipment volume, there is

\[
\sum_{i=1}^{m} \left( \sum_{j=0}^{n} h_{ijk} - \sum_{i=0}^{n} h_{jik} \right) = \sum_{i=1}^{m} (q_{i} - b_{j}) x_{ij}, \quad j = 1,2,\ldots,n
\]  

(10)

\[
\sum_{i=0}^{n} \sum_{j=0}^{n} l_{ij} z_{ijk} + s \sum_{j=0}^{n} p_{jk} \leq T, \quad k = 1,2,\ldots,m
\]  

(11)

According to the actual situation, the village set where the station must be opened is determined in advance. For example, if the riding distance from a village to other villages is greater than r or if an e-commerce node has been established in a village, etc., then must open a station in this village. So

\[
y_{i} = 1, \quad i \in I
\]  

(12)

The value constraints of each decision variable are

\[
x_{ij}, y_{i} \in \{0,1\}, \quad i, j = 1,2,\ldots,n
\]  

(13)

\[
z_{ijk}, p_{jk} \in \{0,1\}, \quad i, j = 0,1,\ldots,n; \quad k = 1,2,\ldots,m
\]  

(14)

\[
h_{ijk} \geq 0, \quad i, j = 0,1,\ldots,n; \quad k = 1,2,\ldots,m
\]  

(15)

In order to improve model calculation speed, following auxiliary constraints can also be added in the model. For example, each station is bound to be responsible for the express parcel business of the village where it is located. So
\[ x_y = 1, \ i \in I \] \hspace{1cm} (16)

\[ x_y = 0, \ i \in I; j \notin I \] \hspace{1cm} (17)

For a given village \( i (i \notin I) \) that has not yet determined whether to open a station, if riding distance \( d_{ij} \) (from it to village \( j \)) is greater than \( r \), then even if a station is set up in this village, the station will not be responsible for the parcel business of village \( j \). So

\[ x_y = 0, \ i \notin I; j = 1, 2, \ldots, n; d_{ij} > r \] \hspace{1cm} (18)

It can be seen from the constraints (5) and (8), there is

\[ h_{jk} = 0, \ j = 0, 1, \ldots, n; k = 1, 2, \ldots, m \] \hspace{1cm} (19)

If the available vehicles are the same, in order to narrow the range of the feasible field of the model, the working vehicles can be assigned in order from the smallest to the largest vehicle number. In this case, if the estimated minimum number of delivery vehicles based on the shipment volume is \( m_0 \), there are

\[ p_{0k} = 1, \ k = 1, 2, \ldots, m_0 \] \hspace{1cm} (20)

\[ p_{0k} \geq p_{0(k-1)}, \ k = 1, 2, \ldots, m-1 \] \hspace{1cm} (21)

Since equation (9) contains non-linear relationship terms about decision variables, the above optimization model for site selection of RESs with equation (1) as the objective function and equations (2) to (21) as constraints is a mixed integer nonlinear programming model. For this type of model, when the scale of the problem is not large, Lingo software can be used to solve it. When the scale of the problem is large, the solution method needs to be further studied.

**MODEL APPLICATIONS**

A town has a total area of 64 km\(^2\), governs 23 villages, and has a total population of 32,000. In order to save the cost of express delivery and facilitate the delivery of express parcel, it is planned to establish a joint distribution center near the town post office. In the future operation, the express shipments sent by the courier companies to the town will be gathered to the center, and then the center will deliver them to the corresponding RES in a common distribution form. At the same time, the outposts of each station will be forwarded to the corresponding courier company through the center, and then the courier companies will transport them out of the town.

In order to facilitate management, it is planned to uniformly select DFA5030XYZFBEV1 (a kind of Dongfeng brand pure electric post car) as the delivery vehicle. According to the volume of 26×24×36 cm\(^3\) of each express parcel, each vehicle can load up to about 280 parcels. Based on the per capita express delivery volume of the village in the province where the town is located in 2018, the population of the villages in the town, and the express delivery status in recent years, the forecast of the express delivery volume in the villages
of the town in the next period is predicted, as shown in Table I. In addition, the Gaode map is used to obtain the riding distance, driving distance and driving time between the villages, as well as the driving distance and driving time between the distribution center and the villages, and calculates the transportation cost of each road segment based on the freight rate of 0.9 Yuan/km (detailed data is omitted). After analysis, assign $r = 1.5$ km, $g_k = 120$ Yuan, $s = 8$ min and $T = 120$ min. Since the riding distances from villages 11, 22, and 23 to other villages are all greater than 1.5 km, and village 13 has already established an e-commerce node, so let $I = \{11, 13, 22, 23\}$. It is planned to use village supermarkets, e-commerce outlets or convenience stores as alternative sites to set up stations, take $f_j = 120$ Yuan.

<table>
<thead>
<tr>
<th>Village number</th>
<th>Receiving volume /unit</th>
<th>Sending volume /unit</th>
<th>Village number</th>
<th>Receiving volume /unit</th>
<th>Sending volume /unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>71</td>
<td>14</td>
<td>13</td>
<td>10</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>24</td>
<td>5</td>
<td>14</td>
<td>21</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>37</td>
<td>8</td>
<td>15</td>
<td>22</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>5</td>
<td>16</td>
<td>20</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>1</td>
<td>17</td>
<td>28</td>
<td>7</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>3</td>
<td>18</td>
<td>26</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
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<td>19</td>
<td>9</td>
<td>2</td>
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<tr>
<td>8</td>
<td>13</td>
<td>3</td>
<td>20</td>
<td>30</td>
<td>6</td>
</tr>
<tr>
<td>9</td>
<td>23</td>
<td>5</td>
<td>21</td>
<td>40</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>27</td>
<td>6</td>
<td>22</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>11</td>
<td>64</td>
<td>13</td>
<td>23</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>14</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

The total receiving volume of the villages is 529, and the total sending volume is 134, so the number of vehicles for express delivery will not be less than 2 ($m_0 = 2$). Let $m = 4$, and using Lingo software to solve the location optimization model of RES. The calculation results are summarized as follows: (1) RESs should be opened in villages 1, 3, 9, 11, 13, 17, 19, 22, and 23 respectively, the stations’ service covering villages and their delivery volume is shown in Table II; (3) A total of 2 vehicles are required to deliver the express parcels, the vehicle routes and their working time are shown in Table III. (4) The system’s average daily total cost is 1364.67 Yuan.

<table>
<thead>
<tr>
<th>Station number</th>
<th>Service covering villages</th>
<th>Receiving volume /unit</th>
<th>Sending volume /unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1, 2</td>
<td>95</td>
<td>19</td>
</tr>
<tr>
<td>3</td>
<td>3, 4</td>
<td>46</td>
<td>13</td>
</tr>
<tr>
<td>9</td>
<td>5, 6, 7, 8, 9, 10</td>
<td>91</td>
<td>20</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>64</td>
<td>13</td>
</tr>
<tr>
<td>13</td>
<td>12, 13, 14</td>
<td>45</td>
<td>27</td>
</tr>
<tr>
<td>17</td>
<td>15, 16, 17</td>
<td>70</td>
<td>16</td>
</tr>
<tr>
<td>19</td>
<td>18, 19, 20, 21</td>
<td>105</td>
<td>22</td>
</tr>
<tr>
<td>22</td>
<td>22</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>23</td>
<td>23</td>
<td>6</td>
<td>2</td>
</tr>
</tbody>
</table>
TABLE III. VEHICLE ROUTES AND THEIR WORK TIME.

<table>
<thead>
<tr>
<th>Vehicle number</th>
<th>Route (sequence of node number)</th>
<th>Work time /minute</th>
<th>Dispersion volume /unit</th>
<th>Collection volume /unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0→22→17→19→1→0</td>
<td>69</td>
<td>277</td>
<td>59</td>
</tr>
<tr>
<td>2</td>
<td>0→3→9→23→11→13→0</td>
<td>116</td>
<td>252</td>
<td>75</td>
</tr>
</tbody>
</table>

Since economic fluctuations may affect the delivery volume of each village, in order to investigate the robustness of the location of RESs, on the basis of the previous forecast of the delivery volume, the location of RESs in each village’s delivery volume increases or decreases by 30% is also considered separately. The result shows: (1) When the sending and receiving volume both increase by 30%, the location and service coverage of RESs are the same as before, but at this situation, 3 vehicles are needed to deliver the express parcels, and the average daily cost of the system is 1487.46 Yuan; (2) When the sending and receiving volume both decrease by 30%, the location of each station, the villages served by each station, the number of vehicles required by the distribution center, the vehicle route and the total daily cost of the system are the same as before (since the transportation cost is only related to the distance and road conditions of the transportation road section, the average daily total cost of the system remains unchanged). It can be see that the location result of RESs in this town has strong robustness and is feasible.

CONCLUSIONS

The establishment of RES is conducive to the improvement of the express delivery outlets and helps to promote the rapid development of rural e-commerce. In order to scientifically plan the rural express logistics network and reduce the rural logistics cost, considering the vehicle routing of the distribution center and the express delivery task, a location optimization model for RES is established. The model is applied to the location problem of RES in a town, and obtains a practical location result. The results of this article can provide a theoretical basis for the scientific site selection of RES, and have practical guiding significance.

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