Analysis of Nonlinear Seepage Characteristics of High Water Cut Preferential Channel
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Abstract. After long-term water injection, some preferential seepage channels would form which are several times or even hundreds of times larger than the original channels in the reservoir. The inefficient and ineffective circulating water injection caused by the preferential channel restricts the efficient development of the oilfield. In this paper, based on the seepage law of the preferential channel, the seepage model for the preferential channel is established. The pressure gradient, seepage velocity and average formation pressure under the nonlinear seepage flow are calculated. The nonlinear seepage characteristics in the preferential channel are analyzed. After comparing with the Darcy linear flow, it is considered that the existence of the preferential channel has a great influence on the production, and the influence of the nonlinear flow in the preferential channel should be considered in the calculation of the development index.

Introduction

Waterflooding development is an effective way to exploit oil reservoirs. Most oilfields in China are exploited by different forms of water injection to improve oil recovery. However, with the deepening of reservoir exploitation, most oil fields are currently in high water cut or extra high water cut. Due to the long-term scouring action of the injected water, the force around the rock particles changes, local peeling or migration occurs, and the reservoir properties and parameters change accordingly, especially the local permeability changes greatly, and then preferential channels with large permeability would form in the reservoir. Through dynamic monitoring and analysis, Dou Zhilin found that in a specific part of high-water cut reservoirs, mutually connected seepage channels are formed, and the seepage capacity of these seepage channels is several tens or even hundreds of times higher than that of the original channels [1].

The existence of preferential seepage channels leads to inefficient water injection in oil fields, and even inefficient recirculation water injection, which has become an important factor restricting the efficient and rational development of oil fields and enhancing oil recovery. Therefore, it has important practical significance to study the structure and seepage characteristics of preferential channels [2-4]. Liu Rui carried out an indoor physical simulation experiment of the turbulent channel for the reservoir, studied the Darcy linear flow and the nonlinear seepage law in the porous medium, and obtained the corresponding relationship of the nonlinear coefficient under high-speed flow [5]. Based on the existing studies, this paper studies the seepage characteristics of the high water cut preferential channel based on the Forchheimer binomial nonlinear seepage equation.

Seepage Law in the Preferential Channel

In the process of waterflooding development, due to the different physical properties of the injected water and the reservoir fluid, coupled with the immersion and erosion of the reservoir, the physical and chemical changes of the reservoir occur, there would result in a large change in the reservoir parameters. In addition, due to the difference in physical properties of the reservoir, the fluid flows to a certain local area. The long-term flow causes the fluid to locally generate preferential seepage, and forms a preferential channel. The turbulent flow is prone to occur in the preferential channel. Its main performance is in [2,5]: after the formation of the preferential seepage channel, the local
permeability increases, the fluid is easy to move, and the percolation velocity increases. In addition to the viscous resistance, the flow pressure must be considered more inertial resistance due to the high velocity \[6-7\].

The current expressions describing the nonlinear seepage of fluids in reservoirs are mainly \[4,8\]:

Forchheimer Binomial equation as:

\[
\frac{dp}{dr} = \frac{\mu}{k} v + \beta \rho v^2
\]  
(1)

Exponential equation as:

\[
\frac{dp}{dr} = c v^n
\]  
(2)

where: \(\mu\)—fluid viscosity, mpa.s; \(k\)—permeability, mD; \(v\)—seepage velocity, cm/s; \(\rho\)—fluid density, g/cm\(^3\); \(\beta\)—Non-Darcian coefficient, \(f\); \(c\)—coefficients related to porous media and fluids, \(f\); \(n\)—percolation index, \(1 < n \leq 2\).

The first term on the right side of the Forchheimer binomial equation represents the influence of viscous force, and the second term reflects the influence of inertial force. This formula can be derived from the Navier-Stokes equation of fluid mechanics and has a good theoretical basis.

**Establishment and Solution of Nonlinear Seepage Model**

Assume that there is sufficient liquid supply on the edge of a circular formation with homogeneous thickness. The supply pressure is \(p_e\), its center is a well, the radius of the formation is \(r_e\), oil well radius \(r_w\), the bottom hole pressure of the oil well is \(p_w\), the bottom hole production of the oil well is \(Q\), the preferential channel present in the reservoir has a thickness of \(h\), permeability is \(k\), fluid viscosity is \(\mu\), density is \(\rho\).

The pressure gradient at any point in the formation when the seepage velocity is substituted into the Forchheimer formula (1) for nonlinear seepage is:

\[
\frac{dp}{dr} = \frac{\mu}{k} \frac{Q}{2\pi rh} + \beta \rho \left(\frac{Q}{2\pi rh}\right)^2
\]  
(3)

Combine equation (3) with the boundary conditions to obtain the pressure at any point in the formation during nonlinear seepage:

\[
p(r) = p_e - \frac{Q\mu}{2\pi kh} \ln \frac{r_e}{r} - \frac{Q^2 \beta \rho}{4\pi^2 h^2} \left(\frac{1}{r_e} - \frac{1}{r}\right)
\]  
(4)

When the well is fixed at the flow pressure \(p_w\), we make such assumptions as:

\[
a = \frac{\beta \rho}{4\pi^2 h^2} \left(\frac{1}{r_e} - \frac{1}{r_w}\right), b = \frac{\mu}{2\pi kh} \ln \frac{r_e}{r_w}, c = p_w - p_e.
\]

Then equation (4) is transformed into a quadratic equation for \(Q\), and considering that the well production \(Q\) is greater than zero, the production expression is:

\[
Q = \frac{-b + \sqrt{b^2 - 4ac}}{2a}
\]  
(5)

Substituting \(a, b, c\) into equation (5), the velocity at any place in the formation during nonlinear seepage based on the Forchheimer equation is written by:
\[ v = \frac{-\mu}{2\pi kh} \ln \frac{r}{r_w} + \left( \frac{\mu}{2\pi kh} \ln \frac{r}{r_w} \right)^2 + \frac{\beta \rho}{\pi h} \left( \frac{1}{r_w} - \frac{1}{r_e} \right) \left( p_e - p_w \right) \]  

Substituting the above formula into equation (1), the pressure gradient at any place in the formation can be given as:

\[ \frac{dp}{dr} = \frac{\mu}{k} \frac{1}{r} A + \frac{\beta \rho}{r^2} A^2 \]  

where \( A = \frac{-\mu}{2\pi kh} \ln \frac{r}{r_w} + \left( \frac{\mu}{2\pi kh} \ln \frac{r}{r_w} \right)^2 + \frac{\beta \rho}{\pi h} \left( \frac{1}{r_w} - \frac{1}{r_e} \right) \left( p_e - p_w \right) \)

The pressure at any place in the formation when the nonlinear seepage is integrated with the upper boundary condition in combination with the outer boundary condition is:

\[ p(r) = p_e - \frac{\mu}{k} A \ln \frac{r}{r_w} - A^2 \beta \rho \left( \frac{1}{r} - \frac{1}{r_e} \right) \]  

where \( v \) is seepage velocity, cm/s; \( \beta \) is nonlinear flow coefficient [7-8] cm-1; \( p \) is pressure, 0.1MPa; \( r \) is distance from a point in the formation to the well, cm.

**Nonlinear Seepage Characteristics Analysis**

In order to quantitatively analyze the nonlinear flow law and its impact on actual production, the value of the formation properties of the preferential channel is shown in Table 1: reservoir original pressure \( p_w \) equals 20MPa, reservoir thickness \( h \) is 20cm, porosity \( \Phi \) equals 0.2, outer boundary radius \( r_e \) equals 200m, well radius \( r_w \) is 10cm, permeability \( k \) equals 5D, fluid viscosity \( \mu = 1 \text{mPa.s} \), density \( \rho = 0.85 \text{g/cm}^3 \).

<table>
<thead>
<tr>
<th>Parameters and units</th>
<th>Numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well production ((Q:\text{cm}^3/s))</td>
<td>1000</td>
</tr>
<tr>
<td>Well radius ((r_w:\text{cm}))</td>
<td>10</td>
</tr>
<tr>
<td>Formation radius ((r_e:\text{cm}))</td>
<td>20000</td>
</tr>
<tr>
<td>Initial formation pressure ((p_e:10^4\text{Mpa}))</td>
<td>200</td>
</tr>
<tr>
<td>Formation thickness ((h:\text{cm}))</td>
<td>50</td>
</tr>
<tr>
<td>Formation permeability ((k:D))</td>
<td>5</td>
</tr>
<tr>
<td>Formation porosity ((\Phi:%))</td>
<td>0.2</td>
</tr>
<tr>
<td>Fluid viscosity ((\mu:\text{mPa.s}))</td>
<td>1</td>
</tr>
<tr>
<td>Fluid density ((\rho:\text{g/cm}^3))</td>
<td>0.85</td>
</tr>
</tbody>
</table>

**Constant Inner Boundary Pressure and External Boundary Pressure**

When the bottom hole pressure of the oil well \( p_w \) equals 18MPa, the formation pressure gradient, the seepage velocity and the differential pressure distribution calculated by Darcy’s law and Forchheimer’s formula are shown in Table 2 and Figure 1-3.
Table 2. Pressure gradient, velocity and pressure difference distribution at different locations.

<table>
<thead>
<tr>
<th>distance (m)</th>
<th>Darcy flow</th>
<th>Nonlinear flow</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pressure gradient (MPa/m)</td>
<td>velocity (cm/s)</td>
</tr>
<tr>
<td>0.1</td>
<td>2.6313</td>
<td>0.6599</td>
</tr>
<tr>
<td>0.2</td>
<td>1.2304</td>
<td>0.3086</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5754</td>
<td>0.1443</td>
</tr>
<tr>
<td>1.0</td>
<td>0.2691</td>
<td>0.0675</td>
</tr>
<tr>
<td>2.1</td>
<td>0.1258</td>
<td>0.0316</td>
</tr>
<tr>
<td>4.5</td>
<td>0.0588</td>
<td>0.0148</td>
</tr>
<tr>
<td>9.6</td>
<td>0.0275</td>
<td>0.0069</td>
</tr>
<tr>
<td>20.5</td>
<td>0.0129</td>
<td>0.0032</td>
</tr>
<tr>
<td>43.7</td>
<td>0.0060</td>
<td>0.0015</td>
</tr>
<tr>
<td>93.5</td>
<td>0.0028</td>
<td>0.0007</td>
</tr>
<tr>
<td>200.0</td>
<td>0.0013</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

The Forchheimer equation describes the influence of the inertial force caused by the large seepage velocity in the preferential channel near the wellbore, and the inertial force plays an important role at this time [7]. The same pattern can be found from Figure 2-3: When the oil well is produced at a constant pressure, when the production pressure difference is equal, the seepage velocity at any point in the formation obtained based on the nonlinear flow described by Forchheimer is smaller than that of the Darcy linear flow, and any point in the formation in the case of nonlinear flow. The pressure difference from the bottom of the well is larger than that of the Darcy flow. This is because, as described by the Forchheimer equation, the energy consumption of the nonlinear flow is in addition to the viscous resistance (the resistance considered by the Darcy linear flow), and the inertial resistance due to the large velocity is also consumed in the formation. The fluid should be produced as much as possible, and there should be a larger pressure difference for nonlinear flow.

However, although the velocity of the nonlinear flow in the formation is less than that of the Darcy flow (Fig. 2), the pressure gradient under the nonlinear flow is larger than that of Darcy near the bottom of the well (Fig. 1). This is because in the vicinity of the well, the velocity in the reservoir is large, and the inertial resistance of the fluid movement is large, and plays a major role. At the same time, as the distance increases, the velocity gradually decreases, and the inertial resistance caused by the speed gradually weakens. After the distance reaches a certain value, that is, the seepage velocity reaches a certain range, the inertial resistance hardly works. The nonlinearity described by Forchheimer approaches the linear flow (Fig 3).

![Figure 1. Comparison of pressure gradient distribution of Darcy flow and Forchheimer nonlinear flow.](image)
At the same time, the bottom hole production based on nonlinear flow and Darcy flow is 31.5 m$^3$/d and 71.4 m$^3$/d respectively. The production of nonlinear flow is less than half of the linear flow, which further illustrates the inertial resistance in the case of large velocity in the preferential channel. Most of the energy in the formation is consumed, that is, Forchheimer is more responsive to formation seepage.

**Constant Inner Boundary Production and External Boundary Pressure**

When the well production is set to $Q=86.4$ m$^3$/d, the formation pressure gradient, pressure and differential pressure distribution calculated by Darcy's law and Forchheimer's formula are shown in Table 3 and Figures 4-6.

The Forchheimer equation describes the influence of the inertial force caused by the large seepage velocity in the preferential channel near the wellbore, and the inertial force dominates at this time [7]. The same rule can be found from Figure 5-6: when the well is produced in constant production, the pressure at any point in the formation based on the nonlinear flow described by Forchheimer is smaller than that in the Darcy linear flow. The pressure difference between any point and the bottom of the well is larger than that of the Darcy flow. This is because, as described by the Forchheimer equation, the energy consumption of the nonlinear flow is in addition to the viscous resistance (the resistance considered by the Darcy linear flow), and the inertial resistance due to the large velocity is also consumed in the formation. The fluid should be produced as much as possible, and there should be a larger pressure difference in the nonlinear flow.
Table 3. Pressure gradient, pressure and pressure difference distribution at different locations.

<table>
<thead>
<tr>
<th>distance (m)</th>
<th>Darcy flow</th>
<th></th>
<th>Nonlinear flow</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pressure gradient (MPa/m)</td>
<td>velocity (cm/s)</td>
<td>Pressure difference (MPa)</td>
<td>Pressure gradient (MPa/m)</td>
<td>velocity (cm/s)</td>
<td>Pressure difference (MPa)</td>
</tr>
<tr>
<td>0.1</td>
<td>3.1847</td>
<td>19.52</td>
<td>0.00</td>
<td>14.1073</td>
<td>18.17</td>
<td>0.00</td>
</tr>
<tr>
<td>0.2</td>
<td>1.4892</td>
<td>19.56</td>
<td>0.05</td>
<td>3.2434</td>
<td>18.94</td>
<td>0.77</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6964</td>
<td>19.61</td>
<td>0.10</td>
<td>0.7834</td>
<td>19.32</td>
<td>1.15</td>
</tr>
<tr>
<td>1.0</td>
<td>0.3257</td>
<td>19.66</td>
<td>0.15</td>
<td>0.2060</td>
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<tr>
<td>2.1</td>
<td>0.1523</td>
<td>19.71</td>
<td>0.19</td>
<td>0.0613</td>
<td>19.65</td>
<td>1.48</td>
</tr>
<tr>
<td>4.5</td>
<td>0.0712</td>
<td>19.76</td>
<td>0.24</td>
<td>0.0210</td>
<td>19.73</td>
<td>1.56</td>
</tr>
<tr>
<td>9.6</td>
<td>0.0333</td>
<td>19.81</td>
<td>0.29</td>
<td>0.0081</td>
<td>19.79</td>
<td>1.62</td>
</tr>
<tr>
<td>20.5</td>
<td>0.0156</td>
<td>19.85</td>
<td>0.34</td>
<td>0.0034</td>
<td>19.85</td>
<td>1.68</td>
</tr>
<tr>
<td>43.7</td>
<td>0.0073</td>
<td>19.90</td>
<td>0.39</td>
<td>0.0015</td>
<td>19.90</td>
<td>1.73</td>
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<td>93.5</td>
<td>0.0034</td>
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<td>0.44</td>
<td>0.0007</td>
<td>19.95</td>
<td>1.78</td>
</tr>
<tr>
<td>200.0</td>
<td>0.0016</td>
<td>20.00</td>
<td>0.48</td>
<td>0.0003</td>
<td>20.00</td>
<td>1.83</td>
</tr>
</tbody>
</table>

However, although the pressure in the formation is less than the Darcy flow at this time (Fig. 5), the pressure gradient under the nonlinear flow is larger than that of Darcy near the bottom of the well (Fig. 4). This is because in the vicinity of the well, the seepage velocity in the reservoir is large, and the inertial resistance of the fluid movement is large, and plays a major role.

Figure 4. Comparison of pressure gradient distribution of Darcy flow and Forchheimer nonlinear flow.

Figure 5. Comparison of pressure distribution between Darcy flow and Forchheimer nonlinear flow.
At the same time, as the distance increases, the velocity gradually decreases, and the inertial resistance caused by the speed gradually weakens. After the distance reaches a certain value, which means that the seepage velocity reaches a certain range, the inertial resistance hardly works. The nonlinearity described by Forchheimer approaches the linear flow (Fig. 6).

For quantitative calculations, the bottom hole pressure based on nonlinear flow and Darcy flow is 18.17MPa and 19.52MPa respectively, and the pressure of nonlinear flow is 1.35MPa less than the pressure of Darcy linear flow, which further shows that inertial resistance consumes most of the energy in the formation.

**Average Formation Pressure**

The average formation pressure is obtained by the area weighted average method. From the supply edge to the bottom of the well, the average formation pressure in the entire drainage zone is [4]:

\[
\bar{P} = \int_{r_e}^{r_w} 2\pi r p dr / \left( \pi \left( r_e^2 - r_w^2 \right) \right)
\]  

(9)

The nonlinear flow and the Darcy flow pressure formula are respectively brought into the upper formula to obtain the Nonlinear flow as written as Eq.(10) and darcy linear flow as given as Eq.(11), respectively.

\[
\bar{P} = P_e + \frac{A^2 \beta \rho}{r_e} - \frac{2A^2 \beta \rho}{r_e + r_w} - \frac{A \mu}{k \left( r_e^2 - r_w^2 \right)} \left( \frac{r_e^2}{2} - \frac{r_w^2}{2} - r_w^2 \ln \left( \frac{r_e}{r_w} \right) \right)
\]

(10)

\[
\bar{P} = P_e - \frac{P_e - P_w}{\ln \left( \frac{r_e}{r_w} \right)} + r_w^2 \left( P_e - P_w \right) \frac{1}{2 \left( r_e^2 - r_w^2 \right)}
\]

(11)

With the same parameters as above, the average formation pressure under the conditions of Darcy linear flow and nonlinear flow using the above two formulas is 19.74Mpa and 19.94Mpa, respectively, and the error is 78%. The average formation pressure for high-speed nonlinear flow is higher than that for Darcy linear flow. This is because the pressure is mainly consumed in a small area near the bottom of the well (Fig.1 and 4), and the value obtained by the non-linear flow when the average formation pressure is averaged by the area weighting method is very close to the boundary pressure.

**Conclusions**

In this paper based on the law of seepage of preferential channel, the steady seepage model considering the nonlinear flow of the Forchheimer equation is established. The analytical
relationship between the formation pressure and the production under Forchheimer nonlinear seepage is obtained by mathematical methods such as integral. The formation pressure, pressure gradient and seepage velocity calculated are presented, and compared with that of Darcy linear flow. The main conclusions are as follows:

(1) At the same differential pressure, the production based on Forchheimer nonlinear flow is lower than that based on Darcy linear flow. The velocity of Darcy flow at any point in the formation is larger than that of the nonlinear flow. This is because the nonlinear flow model considers the inertia effect caused by the large velocity near the bottom of the well. The formation pressure consumption is composed of the inertial resistance and the viscous force. The greater the velocity is, the greater the influence of the inertial resistance is.

(2) In the near-well zone, the inertial force plays a leading role. Although the nonlinear flow has a smaller velocity than the Darcy flow, the pressure gradient is larger than the Darcy flow. When the distance from the well becomes a certain value, the velocity becomes small, and the quadratic velocity of the nonlinear flow equation is negligible, that is, the inertial force is negligible compared with the viscous force.

(3) Due to the large velocity near the bottom of the well, the inertial resistance cannot be ignored, and the formation pressure and pressure gradient calculated by the Darcy linear flow do not take into account the influence of the inertial force, which brings a large error. This is because the formation pressure consumption is mainly near the bottom of the well; the nonlinear flow equation considers the influence of inertial force and can more accurately explain the seepage law in the formation. Therefore, the Darcy linear flow in the preferential channel reservoir cannot be used to describe the flow law of the formation near the bottom of the well.

References


