Robust Analysis of Freight Comprehensive Transportation Networks
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Abstract. The robustness of Freight Comprehensive Transportation Networks (FCTNs) refers to the ability that FCTNs can maintain their function or performance tolerating perturbations from uncertainties. To evaluate the robustness of FCTNs, this paper proposes several quantitative indicators including accessibility, delivery, carrying capacity and transferring capacity. Numerical results based on the case of coal comprehensive transportation networks in Shanxi, China show the effectiveness of the proposed method.

Introduction
Freight comprehensive transportation networks (FCTNs) are complex networks, such as railways, highways, waterways and other means of transportation. Since many uncertainties arise from demands and supplies of goods, the network structure, and other external causes such as earthquakes, floods, and fires et al. [1, 2], the equilibrium of the freight flows might be severely affected. Thus, to evaluate the ability of FCTNs of maintaining their function or performance under these uncertainties (i.e., robustness), has become more and more important.

Recent research on robust analysis of networks has focused on many areas, such as supply chain networks, reverse logistics networks, urban traffic networks, and airline networks. Thadakamaila and Raghavan et al. [3] present a methodology for building survivable large-scale supply network topologies that can be extended to other large-scale multiagent systems. Yan, Liu and Zhuang [4] propose a node importance evaluation method to analyze cascading failure characteristics in a supply chain network. For road networks facing future travel demand uncertainty, Yina, Madanat and Lu [5] develop robust improvement schemes that make the system performance insensitive to demand uncertainty or allow the system to perform better than the worst-case demand scenario. Soh and Lim et al. [6] analyze a complex weighted network of travel routes on the Singapore rail and bus transportation systems. They show that the dynamical assortativity of the bus networks differ from its topological counterpart. Taylor [7] discusses a method for assessing critical locations in urban road networks and evaluates their criticality using the impact of each section on the road networks. D’Lima and Medda [8] propose a quantitative measure of resilience using a mean-reverting stochastic model and apply this model to the case of the London Underground. For the robust analysis of airline networks, Lordan and Sallan et al. [9] analyze the topology and robustness of the network route of airlines following low cost carriers (LCCs) and full service carriers (FSCs) business models, and show that LCC networks are more robust than FSC networks. Verma, Araújo and Herrmann [10] empirically reveal that the world airline network is a redundant and resilient network for long-distance air travels, but otherwise breaks down completely due to removal of short and apparently insignificant connections. These short-ranged connections with a moderate number of passengers and alternate flights are the connections that keep remote parts of the world accessible. Wuellner, Roy and D’Souza [11] analyze the individual structures of the seven largest passenger
carriers in the USA and find that networks with dense inter-connectivity are extremely resilient to both targeted removal of airports and random removal of flight paths.

Although robust analysis of transportation networks has received more and more attention in the literature, most of the above papers are at the theoretical level with simplifications, for example, only modeling networks with single means of transportation. In this paper, motivated by real applications, we focus on FCTNs considering the impact of transshipment on the robustness. We propose quantitative indicators to evaluate the robustness of FCTNs and then show their effectiveness numerically based on the case of coal comprehensive transportation networks in Shanxi, China.

Evaluating Indicators

In this section, we define some quantitative indicators to evaluate the robustness of FCTNs, including accessibility, delivery, carrying capacity and transferring capacity respectively as follows.

**Accessibility.** Under the influence of high traffic and uncertainties, some nodes/edges of FCTNs will be blocked or broken down and the goods will have to bypass these nodes/edges and go by a rather indirect route. In these situations, the indicator of accessibility is used to describe the convenience that goods travel from their origin node to the destination node. For FCTNs, there are two kinds of accessibility, that is, mileage accessibility and traveling time accessibility. Mileage accessibility, denoted by $D$, reflects the degree of difficulty that goods overcome the obstacles of distance, and is defined as follows:

$$D = \frac{\sum_i \sum_j \bar{d}_{ij}}{\sum_i \sum_j d_{ij}},$$

where $d_{ij}$ is the shortest traveling distance between nodes $i$ and $j$ when FCTNs are disturbed and $\bar{d}_{ij}$ is the shortest traveling distance between nodes $i$ and $j$ when FCTNs are undisturbed.

Traveling time includes transportation time spent on different kinds of transportation modes and transferring time among these modes. It is related to both the routes and the traveling speeds. Traveling time accessibility, denoted by $T$, reflects the degree of difficulty in travel time between node pairs in networks, and is defined as follows:

$$T = \frac{\sum_i \sum_j \bar{t}_{ij}}{\sum_i \sum_j t_{ij}},$$

where $t_{ij}$ is the shortest traveling time between nodes $i$ and $j$ when FCTNs are disturbed, and $\bar{t}_{ij}$ is the shortest traveling time between nodes $i$ and $j$ when FCTNs are undisturbed.

**Delivery.** The influence of uncertainties on FCTNs may lead to the variation of goods in transit. The greater the influence, the greater the reduction in the amount of freight between nodes. Thus the indicator of delivery is used to describe the ability which FCTNs can maintain their amount of freight facing uncertainties. Let $E$ be the set of all neighboring node pairs $(i,j)$ in FCTNs, and denote $v_{ij}$ and $\bar{v}_{ij}$ the amount of freight for node pair $(i,j)$ with and without disturbance respectively. Then the reduction in the amount of freight for the node pair $(i,j)$ is

$$\xi_{ij} = \begin{cases} \bar{v}_{ij} - v_{ij} & \text{if } \bar{v}_{ij} \geq v_{ij}, \\ 0 & \text{else}. \end{cases}$$

We use the mean value $\mu$ and standard deviation $\sigma$ of $\xi_{ij}$ as indicators for delivery, that is

$$\mu = \frac{1}{|E|} \sum_{(i,j) \in E} \xi_{ij},$$

and
\[ \sigma = \sqrt{\frac{1}{|E|} \sum_{(i,j) \in E} (\xi_{ij} - \mu)^2} \]

where \(|E|\) is the cardinal number of the set \(E\). The mean value \(\mu\) suggests the average level that the amount of freight decreases by disturbance. Thus, the smaller \(\mu\) indicates the better robustness of FCTNs. The standard deviation \(\sigma\) indicates the degree that the amount of freight deviates from the average level. A high \(\sigma\) implies that there are sections with large reductions, and these sections are critical.

**Carrying Capacity.** The indicator of carrying capacity measures how the maximum amount of freight that can travel through a given FCTN in a given amount of time with disturbance deviate from the maximum amount without disturbance. It is defined by the ratio of transport capacity of FCTNs with disturbance to transport capacity without disturbance, that is

\[ C = \frac{\sum_{(i,j) \in E} c_{ij} d_{ij}}{\sum_{(i,j) \in E} \bar{c}_{ij} \bar{d}_{ij}} \]

where \(c_{ij}\) and \(\bar{c}_{ij}\) are the maximum amount of freight for node pair \((i,j)\) with and without disturbance respectively.

**Transferring Capacity.** FCTNs consist of the railways, highways, waterways, aviation, and other modes. Freights will be transferred in different modes of transport at transferring nodes. Thus, the processing capacity of transferring nodes are very critical for the robustness of FCTNs. The indicator of transferring capacity is to describe the ability that transferring nodes of FCTNs can maintain their processing capacities under the interference of uncertainties, and is defined as follows:

\[ S = \frac{\sum_{i \in M} s_i}{\sum_{i \in \overline{M}} \bar{s}_i} \]

where \(s_i\) and \(\bar{s}_i\) are the processing capacity of the transferring node \(i\) with and without disturbance respectively. In addition, \(M\) and \(\overline{M}\) are the set of transferring nodes of FCTNs with and without disturbance respectively.

**Numerical Results**

In this section, numerical examples are reported to illustrate the effectiveness of the proposed quantitative indicators to evaluate the robustness of FCTNs. We conduct numerical experiments by MATLAB based on the data from the coal comprehensive transportation network (CCTN) in Shanxi, China, which is shown in Fig. 1.
With the rapid development of economy, the coal demand in China is increasing, especially in the five coastal provinces, Shanghai, Jiangsu, Zhejiang, Fujian, Guangdong due to the fast growth of the consumption of coal for power generation. This not only further aggravates the sluggish situation of coal transportation in Shanxi, but also makes this transportation network more and more complicated. Based on the statistical yearbooks for coal production in Shanxi province and related literatures, we get data on railways, waterways and highways for coal transportation, including origins, destinations, transportation capacity, transportation amount, transferring capacity, and distance and speed for each sections.

There are many uncertainties in the coal comprehensive transportation network, such as demands and supplies of goods, the network structure, and earthquakes, floods, fires et al. The influence of these uncertainties on the network can be divided into two kinds of attacks, that is, random attack and selective attack. Random attack refers to the removal of edges or nodes from the network with certain probability while selective attack is the destruction on edges or nodes according to some strategy (for example, the degree of nodes or the capacity of edges).

We first investigate the accessibility of CCTN under random attack. Removing edges in CCTN randomly and irreversibly, the trends of mileage accessibility and traveling time accessibility are shown in Fig. 2.

Fig. 2 (a) shows that when the ratio of the number of edges being randomly removed from the network is less than 15%, the indicator of mileage accessibility changes little. That is, it is not sensitive to the random attack of edges, since the structure of the network has not been seriously damaged and transport between nodes can be accomplished by circuitous routes. When the ratio of the number of removed edges exceeds 15%, the indicator of mileage accessibility decreases rapidly. In this case, the structure of the network has been destructed very seriously.

Fig. 2 (b) shows that when the ratio of the number of removed edges is less than 12%, the indicator of traveling time accessibility does not change very much. This is because that the structure of the network has not been seriously destructed, but it takes much more time to travel in circuitous routes. When the ratio of the number of removed edges exceeds 12%, the indicator of traveling time accessibility decreases rapidly.
The coal producing and consuming locations constitute a set of supply and demand node pairs in the network. By destructing edges in the network randomly, and redistributing the flow according to the set of supply and demand node pairs, we get the trends of the mean value $\mu$ and standard deviation $\sigma$ of the reduction, which are shown in Fig. 3. From the numerical results, we can see that when the deleted edges increase, the mean value and standard deviation tend to increase as well.

Next, we further compare the indicator of carrying capacity between random attack and selective attack. Compared with random attack, selective attack is more harmful to the network, because it can attack the network in accordance with the capacity of edges or the degree of nodes. Using the strategies of removing edges randomly and by the capacity of edges especially, the trends of carrying capacity are shown in Fig. 4 (a). We can see that, under the strategy of removing edges randomly, the indicator of carrying capacity decreases much more slower than that under the strategy of removing edges by the capacity of edges. That is, the indicator of carrying capacity is more sensitive to selective attack than to random attack.

Since the transshipment of coal among different modes of transport is conducted at transferring nodes and the distributing flows are much more larger than other nodes in general, transferring nodes play an important role in the network. We compare the indicator of transferring capacity under selective attack on nodes between by degree and by transshipment capacity. Using the strategies of removing nodes by degree and by capacity especially, the trends of transferring capacity are shown in
Fig. 4 (b). The figure illustrates that, under the strategy of removing nodes by degree, the indicator of transferring capacity decreases much more slower than under the strategy of removing nodes by transshipment capacity. That is, the indicator of transferring capacity is more sensitive to selective attack on nodes by degree than by transshipment capacity.

Conclusion

In this paper, we investigate the robust evaluation of FCTNs under random attack and selective attack, considering the impact of transshipment. Quantitative indicators, including accessibility, delivery, carrying capacity and transferring capacity, are developed. Numerical examples are given based on the case of coal comprehensive transportation networks in Shanxi, China. Numerical results show that: (i) FCTNs are not sensitive to the random attack of a few edges, since the structure of the network has not been destructed seriously. (ii) FCTNs are more sensitive to selective attack than to random attack, that is, selective attack is more harmful to the network than random attack.

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References


