Stopper Parameters Design for Micro-elastic Beam Under High Shock Environment

Tao Jiang and Ning Ma

ABSTRACT

Aimed at to the proper design of the stopper parameters about micro-elastic beam of MEMS gyroscope under high-g shock, Micro-elastic beam and stopper collision system are built to analyze that how elastic coefficient and limit distance of stopper to affect the shock response of a micro-elastic beam, then determining the optimal range of elastic coefficient and limit distance and their correctness will be proved through FEA software. Results show that: On the premise of meeting the limit requirements, to minimize the value of limit distance is advantageous to improve the impact resistance of the micro-elastic beam, and proper elastic coefficient can restrain second force; stopper parameters design in this paper have some reference value for MEMS gyroscope design under high-g shock.

INTRODUCTION

MEMS gyroscope can rely on movable parts to detect changes in angular velocity. Under shock environment, the displacement of these movable parts suddenly increases, causing the elastic beam to excessively bend and break [1]. MEMS gyroscope needs to withstand high-g shock in most applications, so it is essential to improve the shock resistance of MEMS gyroscope.

Stoppers are often used to improve the impact resistance of MEMS devices. Reference [2] used stoppers for the first time in the design of the accelerometer; Reference [3] used T type limit stopper for shock protection, but this will generate the secondary impact that may cause problems such as debris and failure. Reference [4] proposed a new type of anti-impact design. The limit structure uses a multi-layer elastic baffle which can protect the micro-devices better than the traditional rigid one.

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In reference [5], the impact response of the MEMS accelerometer with elastic stopper was studied, and the double linkage stopper was designed to better resist high frequency impact.

However, the previous researches about the stopper focused on the stopper’s structural design, and most of them just limited the whole micro device. Researches on the limit parameters design and the limitation at the micro-elastic beam are less. This paper adds a stopper to the micro-elastic beam of the MEMS gyroscope, and clearly analyzes the influence of stopper parameters on the impact response of the micro-elastic beam, then designs reasonable parameters’ range of stopper increase the anti-impact ability of MEMS gyroscope.

**DYNAMIC EQUATION OF MICROCANTILEVER-STOPPER COLLISION SYSTEM**

Under high-g shock environment, the micro-elastic beam is the most vulnerable parts of MEMS gyroscope. As shown in Figure 1, one end of the cantilever beam is fixed on the substrate of the same material, and the other end is suspended by a mass. The huge force generated by the mass under impact acts on the end of the cantilever beam, and the deflection displacement of the cantilever beam increases until the stress of the fixed end exceeds its fracture stress, then the cantilever beam fractures and fails, and the MEMS gyroscope eventually cannot work normally.

Therefore, in order to improve the impact resistance of the MEMS gyroscope, a basic solution is to limit the maximum displacement of the micro-elastic beam by using a stopper. Figure 2 shows the mechanical model of the microcantilever-stopper collision system. The mass M of the microcantilever is concentrated at the end, the length is L, the Young's modulus is E, and the inertia constant is I. M is subjected to a half sinusoidal excitation of size P0sin(ωt), which produces a deflection displacement x. When the deflection displacement is greater than the limit distance d, the stopper begins to function as a limit, and its stiffness coefficient is k.
According to reference [6-7], the differential equation of the microcantilever-stopper collision system can be established:

\[ Mx'' + cx' + f(x) + kH(x)(x - d) = P_0 \sin(\omega t) \]  

(1)

Where: \( c \) is the damping coefficient inside the vibration system and it is negligible. \( H(x) \) is the Heaviside step function, and \( f(x) \) is the nonlinear restoring force of the microcantilever [6-7]. They are respectively given by

\[
H(x) = \begin{cases} 
0 & x < d \\
1 & x \geq d 
\end{cases} \\
f(x) = \frac{12EI}{L^3}x + \frac{432EI}{35L^5}x^3
\]

(2)

After simplifying equation (1), the differential equation of the non-smooth and nonlinear microcantilever-stopper collision system can be obtained

\[ y'' + b(y + \frac{36}{35}y^3) + eH(y)(y - d) = z \sin(\omega t) \]

(3)

Where:

\[
y = \frac{x}{L}, \quad b = \frac{12EI}{ML^2}, \quad e = \frac{k}{M}, \quad z = \frac{P_0}{M}
\]

(4)

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Figure 2. Microcantilever-stopper collision system.
It can be seen from equation (2) that there are three types of parameters affecting the shock response of the microcantilever. (1) Microcantilever self-parameters: mass \( M \), length \( L \), bending stiffness \( EI \). (2) External shock parameters: amplitude \( p_0 \), time \( t \). (3) Stopper parameters: stiffness \( k \), limit distance \( d \). Among them, according to the performance requirements of MEMS gyroscope, the parameters of the micro-elastic beam have been determined. According to the basic technical indicators of MEMS gyroscope in terms of impact resistance, MEMS gyroscope needs to withstand a single-peak transient ultra-high acceleration shock with an impact time of less than 0.2ms and a peak of at least 15000g or even more than 20000g, so the external impact parameters are basically determined. Therefore, special attention has to be paid to changing stiffness coefficient \( k \) and the limit distance \( d \) of the stopper, to study the influence of the two on the shock response of the microcantilever, and to design reasonable stopper parameters.

### INFLUENCE OF STOPPER PARAMETERS ON SHOCK RESPONSE

Taking a single crystal silicon microcantilever of a vibrating MEMS gyroscope as an example, main parameters are shown in TABLE I. External half sinusoidal impact acceleration amplitude is 20000g and impact time is 0.2ms.

According to parameters above, the impact dynamic response of microcantilever without a stopper is simulated in LS-DYNA. The simulation results show that when the root stress of microcantilever exceeds the fracture stress of 440MPa, the free end displacement exceeds 18 μm, so the fracture displacement of cantilever beam is 18 μm.

### TABLE I. PHYSICAL PARAMETERS OF SINGLE CRYSTAL SILICON CANTILEVER.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Numerical value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length(μm)</td>
<td>160</td>
</tr>
<tr>
<td>Width(μm)</td>
<td>20</td>
</tr>
<tr>
<td>Height(μm)</td>
<td>4</td>
</tr>
<tr>
<td>Density(kg/m³)</td>
<td>2330</td>
</tr>
<tr>
<td>Young's modulus (GPa)</td>
<td>160</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.28</td>
</tr>
<tr>
<td>Fracture stress (MPa)</td>
<td>440</td>
</tr>
</tbody>
</table>

**Influence of Stopper Stiffness on Shock Response**

Stopper with different stiffness is considered, and property parameters of stopper take the following values: \( k_1=40N/m \), \( K_2=140N/m \), \( k_3=185N/m \), \( k_4=190N/m \), initial limit distance \( d=6\mu m \). In MATLAB, the fourth-order Runge-Kutta method is used to solve the dynamic equation of microcantilever-stopper collision system presented above. As shown in Figure 3, the maximum deflection displacement when stopper is
not added is 45μm, far exceeding the fracture displacement of 18μm, and cantilever beam fractures and fails. After stopper is added, when stiffness is k1=40N/m, the maximum response displacement is 27.6μm, which still exceeds the fracture displacement. This indicates that if the stiffness of stopper is designed too low, it will not protect cantilever beam. When limit stiffness is k2=140N/m and k3=185N/m, the maximum displacement is limited to 16.3μm and 14.3μm respectively. This shows that the larger the stiffness is, the more obvious the limit effect is, and the more cantilever beam is protected from shock. However, when limit block stiffness is k4=190N/m, the displacement curve begins to be in disorder, and the maximum displacement of cantilever beam increases, instead of decreases, meanwhile the collision system begins to destabilize.

Figure 4(a) and 4(b) respectively are curves of microcantilever velocity response when k3=185N/m and k4=190N/m. Obviously, it can be seen that when k3=185N/m, the speed of cantilever beam becomes gradually smooth after contacting stopper, but when k4=190N/m, the speed of cantilever beam drastically changes, far exceeding the speed before contacting with stopper. Such a dramatic speed change generates huge secondary impact force between beam and stopper. According to reference [6], this secondary impact force can be given by:

$$F_{impact} = m \frac{\partial v_{relative}(t)}{\partial t}$$

(5)

If F impact is large enough, the displacement response of the microcantilever will also drastically changes, and the microcantilever-stopper collision system collapses. When K2=140N/m, k3=185N/m and k4=190N/m, the secondary impact force calculated by formula (5) respectively is 3.48e-5N, 5.52e-5N and 3.12e-4N. So, the stiffness design of stopper must limit deformation of the micro cantilever beam, and is also be considered to reduce the secondary impact force between micro cantilever beam and stopper.

Figure 3. Displacement response curve.
Influence of Limit Distance on Shock Response

To study the influence of limit distance on microcantilever beam response, the first thing is to ensure that limit distance is greater than the maximum displacement of microcantilever beam when MEMS gyroscope works normally. It is known that when MEMS gyroscope works normally, the driving force is more than 10g, and the maximum displacement response is much less than 1μm, and the fracture displacement is 18μm. So, limit distance can be taken as follows: $d_1=10μm$, $d_2=6μm$, $d_3=1μm$, and limit stiffness is set as $k_3=185N/m$. As shown in Fig. 5(a), the displacement response of cantilever beam under different limit distances. And $d_1$, $d_2$, $d_3$ respectively limits the maximum displacement to 17.4μm, 14.3μm, 10.4μm, which indicates low limit distance is good to relieve the maximum amplitude. Figure 5(b) shows the velocity response of cantilever beam with different limit distances. It is obvious that the value of limit distance has little effect on
velocity response. Therefore, limit distance should be set to a lower value under the premise of normal operation of MEMS gyroscope.

STOPPER PARAMETERS DESIGN

The analysis shows that topper design must simultaneously meet the following three principles: (1) the maximum displacement response of cantilever beam should be smaller than the fracture displacement, which requires stiffness to be designed to be bigger. (2) In order to reduce the secondary impact force, the compliance of stopper should be increased, which requires stiffness to be designed to be smaller, but it must be firstly guaranteed to meet principle (1). (3) Limit distance should be as small as possible under the premise of MEMS gyroscope’s normal function. With these three principles, the optimization of stopper parameters can maximize the ability of protecting cantilever beam.

Stiffness $k$ is designed according to principle (1) and (2). The lower limiting value should satisfy that the maximum displacement is less than the fracture displacement. The upper limiting value should realize the maximum effect of the cantilever beam displacement reduction. Limit distance $d$ is designed according to principle (3). It is much easier for micro-accelerometer made by surface process to achieve the distance less than 2 μm, while the MEMS device for lateral motion by bulk silicon process can hardly realize limit distance less than 2 μm [8]. So, $d=2\mu m$.

Fig. 6 shows the relationship curves between stopper stiffness and the maximum deflection displacement and the maximum secondary impact force of microcantilever, when MEMS gyroscope is subjected to 20000g (0.2ms). The abscissa is the change of stiffness, and the ordinate is the change of maximum displacement and secondary impact force. According to the design principle (1) (2), stiffness range can be determined as (90, 188) N/m.

Therefore, taking stopper parameters $d=2\mu m$, $k=180N/m$, FEA model of the microcantilever-stopper collision system is established in LS-DYNA, to simulate the stress response of the microcantilever after adding the stopper, and compared with the response of microcantilever without a stopper. As shown in Figure 7(a) and 7(b), when stopper is not added, the maximum stress at the root of the cantilever beam is 1.08GPa, far exceeding the fracture stress 440MPa, and cantilever beam fractures and fails. After adding the designed stopper, the maximum stress at the root of cantilever beam is 122MPa, which is less than the fracture stress 440MPa, so cantilever beam is effectively protected. This shows that the reasonable design of stopper parameters plays an important role in improving the impact resistance of MEMS gyroscope.
CONCLUSIONS

In order to meet the requirement of 20000g anti-impact ability of MEMS gyroscope, this paper studies the influence of stopper parameters on shock response of micro-elastic beam and draws a conclusion that stopper parameters design must consider many factors. The selection of stiffness should not only limit the maximum displacement of elastic beam as much as possible, but also reduce the secondary impact force generated between micro-elastic beam and stopper; and the smaller the limit distance, the more micro-elastic beam can be protected. The design method of stopper parameters in this paper is general, so it has some reference value for improving the impact resistance of MEMS gyroscope under shock environment.
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REFERENCES