Maneuvering Estimation for Guidance Law in Polar Coordinate System

Fu-gui LI¹, Sheng-wei JIA¹,², Yang ZHANG¹, Jian-fu LI¹ and Hong ZHAO¹

¹China Academy of Launch Vehicle Technology, Beijing 100076, China
²Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

*Corresponding author

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Abstract. Target maneuvering had important effect to the miss distance. An extended Kalman filter was designed to estimate target maneuvering. The polar coordinate system was chosen to construct the state-space equations. The measurement equations was linear in the polar coordinate system. The measurement information was measured by radar seekers. The initialization parameters for the extended Kalman filter was also proposed. Then methods were simulated under typical intercept conditions. The results show that the extended Kalman filter could estimate target maneuvering very well.

Introduction

The guidance law was the key part of the guidance system, which had important effect to the guidance performance[1-3]. Some optimal guidance laws were mathematically derived to reduce the miss distance due to target maneuvering. The optimal guidance law had perfect performance when the guidance information was correct. However, the optimal guidance laws needed a lot of guidance information which couldn't measured by sensors mounted on the missiles, such as target manoeuvring acceleration, time-to-go and guidance system dynamics. In order to use optimal guidance laws, the guidance information should be estimated above all first.

At present, most methods to estimate maneuvering targets are based on one-dimensional linear model[4-5]. Some of them assumed the situation too ideally[6-7], and lacked reality value. The measured information with seeker should be made the best of. Also the estimation state equation should be close to the reality intercept model. Then the guidance information estimated by the filter was meaningful.

In this paper, the extended Kalman filter was constructed in polar coordinate system to estimate guidance signals with the information measured by phased array radar seekers was introduced at first. Then, the initialization parameters for the extended Kalman filter was also proposed. Also, under the typical condition, the simulation was done to verify the performance of methods.

Extended Kalman Filter in Polar Coordinate System

State Equation of Extended Kalman Filter

Taking the missile as their origin, the inertia coordinate system and LOS (the line of sight) coordinate system could be established as in Fig.1, where $q_p$ and $R$ are the LOS angle and the distance between the missile and the target, respectively. Both $q_p$ and $R$ could be measured by phased array radar seekers or derived from other measurements.
Let the displacement, velocity, and acceleration of the missile and the target in the inertia coordinate system be $x_m, y_m, v_{xm}, v_{ym}, a_{xm}, a_{ym}$ and $x_t, y_t, v_{xt}, v_{yt}, a_{xt}, a_{yt}$ respectively. Let the relative displacement, relative velocity, and relative acceleration of the missile to target in the inertia coordinate system be $x, y, v_x, v_y, a_x, a_y$ respectively.

\[
\begin{align*}
    x &= x_t - x_m \\
    y &= y_t - y_m \\
    v_x &= v_{xt} - v_{xm} \\
    v_y &= v_{yt} - v_{ym} \\
    a_x &= a_{xt} - a_{xm} \\
    a_y &= a_{yt} - a_{ym}
\end{align*}
\]  

(1)

With kinematics, Eq.2 could be obtained.

\[
\begin{align*}
    v_x &= \dot{x} \\
    v_y &= \dot{y} \\
    a_x &= \ddot{x} \\
    a_y &= \ddot{y}
\end{align*}
\]  

(2)

From Fig.1, it was easily to got Eq.3.

\[
\begin{align*}
    q_p &= \arctan \left( \frac{y}{x} \right) \\
    R &= \sqrt{x^2 + y^2} \\
    x &= R \cos \left( q_p \right) \\
    y &= R \sin \left( q_p \right)
\end{align*}
\]  

(3)

With the state variable vector $X = [q_p, \dot{q}_p, R, \dot{R}, a_u, \dot{a}_u]$, where $\dot{q}_p$ was angular of LOS, $\dot{R}$ was relative velocity along the LOS between the missile and the target, $a_u$ and $\dot{a}_u$ was the projection of target acceleration in LOS coordinate system.

Took the derivative of $q_p$

\[
\frac{dq_p}{dt} = \dot{q}_p
\]  

(4)

Took the derivative of Eq.3, then

\[
\begin{align*}
    \dot{x} &= \dot{R} \cos (q_p) - R \dot{q}_p \sin (q_p) \\
    \dot{y} &= \dot{R} \sin (q_p) + R \dot{q}_p \cos (q_p)
\end{align*}
\]  

(5)

Took the derivative of Eq.5, then
\[
\begin{align*}
\dot{x} &= \ddot{R} \cos (q_p) - 2 \dot{R} \dot{q}_p \sin (q_p) - R \ddot{q}_p^2 \cos (q_p) - R \dot{q}_p \sin (q_p) \\
\dot{y} &= \ddot{R} \sin (q_p) + 2 \dot{R} \dot{q}_p \cos (q_p) - R \ddot{q}_p^2 \sin (q_p) + R \dot{q}_p \cos (q_p)
\end{align*}
\]  
(6)

Eq. 7 was the transfer equation from inertia coordinate system to LOS coordinate system.

\[
C_i = \begin{bmatrix}
\cos q_p & \sin q_p & 0 \\
-\sin q_p & \cos q_p & 0 \\
0 & 0 & 1
\end{bmatrix}
\]  
(7)

Then the projection of target and missile acceleration in LOS coordinate system could get easily as Eq.8.

\[
\begin{bmatrix}
\dot{a}_{tx}' \\
\dot{a}_{ty}' \\
0
\end{bmatrix} = C_i \begin{bmatrix}
\ddot{x}_t \\
\ddot{y}_t \\
0
\end{bmatrix}
\]  
(8)

The projection of missile acceleration in LOS coordinate system.

\[
\begin{bmatrix}
\dot{a}_{mx}' \\
\dot{a}_{my}' \\
0
\end{bmatrix} = C_i \begin{bmatrix}
\ddot{x}_m \\
\ddot{y}_m \\
0
\end{bmatrix}
\]  
(9)

The projection of relative acceleration in LOS coordinate system.

\[
\begin{bmatrix}
\ddot{x}' \\
\ddot{y}' \\
0
\end{bmatrix} = C_i \begin{bmatrix}
\ddot{x} \\
\ddot{y} \\
0
\end{bmatrix}
\]  
(10)

Took some mathematical computation of the first line of Eq.10, then

\[
\frac{d \dot{q}_p}{dt} = -2 \left( \frac{\ddot{R}}{R} \right) \dot{q}_p + \frac{\dot{a}_{ty}'}{R} - \frac{a_{mx}'}{R}
\]  
(11)

Took some mathematical calculation of the second line of Eq.10, then

\[
\frac{d \ddot{R}}{dt} = R \ddot{q}_p^2 - \dot{a}_{tx}' - \dot{a}_{mx}'
\]  
(12)

Took the derivative of \( R \), then

\[
\frac{d}{dt} R = \dot{R}
\]  
(13)

Assuming the acceleration of the target in the inertia coordinate system was constant, then

\[
\begin{cases}
\frac{d}{dt} a_{tx} = w_x \\
\frac{d}{dt} a_{ty} = w_y
\end{cases}
\]  
(14)

\( w_x \) and \( w_y \) are Gaussian white noises.

With Euler's Theorem and some calculation, the derivative of \( a_{tx}' \) and \( a_{ty}' \) could be obtained.
The state equation of extended Kalman filter could be derived with Eq.10, Eq.11, Eq.12, Eq.13 and Eq.15.

\[
\frac{dq_p}{dt} = \dot{q}_p \\
\frac{da_p^i}{dt} = -2\left(\frac{\dot{R}}{R}\right)\dot{q}_p + \frac{a_{qy}}{R} - \frac{a_{my}}{R} \\
\frac{dR}{dt} = \dot{R} \\
\frac{d\dot{R}}{dt} = R\dot{a}_p^i + a_{tx} - a_{mx} \\
\frac{da_x^i}{dt} = a_{tx}^i \dot{q}_p + \cos q_p w_x + \sin q_p w_y \\
\frac{da_y^i}{dt} = -a_{tx}^i \dot{q}_p - \sin q_p w_x + \cos q_p w_y
\]

we could have the following state-space equation.

\[
\frac{d}{dt} X = AX + w
\]

where

\[
A(X) = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & -2\left(\frac{\dot{R}}{R}\right) & -a_{ty}^i - a_{my}^i - 2\dot{R}\ddot{q}_p & -2\ddot{q}_p & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 2\dot{R}\ddot{q}_p & 0 & -1 & 0 & \ddot{q}_p \\
0 & a_{ty}^i & 0 & 0 & 0 & -\dot{q}_p \\
0 & -a_{tx}^i & 0 & 0 & -\dot{q}_p & 0
\end{bmatrix}
\]

\[
w = \begin{bmatrix}
0 \\
0 \\
\cos q_p w_x + \sin q_p w_y \\
-\sin q_p w_x + \cos q_p w_y
\end{bmatrix}
\]

**Measurement Equation**

Considering the missile equipped with phased array radar seekers, which could measure the LOS angle \(q_p\) and the distance \(R\) between the missile and the target. The measurement equations could be described as follows.

\[
\begin{cases}
q_p^g = q_p + v_q \\
R^g = R + v_R
\end{cases}
\]

where \(v_q\) and \(v_R\) are measurement Gaussian white noises with their variances \(\sigma_q^2\) and \(\sigma_R^2\), respectively. To employ the Kalman filter, the measurement Eq.18 were described as follows

\[
Z = CX + v
\]

where
Discrete Kalman Filter

Let \( T_s \) be the discrete time step. Based on Eq.17 and Eq.19, the state variables could be estimated by the following discrete extended Kalman filter.

\[
\hat{X}_k = \Phi_k \hat{X}_{k-1} + K_k ( Z_k - C_k \hat{X}_{k-1} )
\]

where \( Q_k ( \hat{X} ) = \int_0^T \Phi(t) G S_n G^T \Phi^T(t) dt \) was the discrete system noise matrix; \( Z_k \) was the discrete measurement matrix; \( \Phi_k = \Phi_j (T_s) \) was the discrete state transition matrix which could be calculated as follows.

\[
\Phi(t) = I + A t + \frac{A^2 t^2}{2}
\]

Furthermore, \( K_k \) was the gain matrix of the extended Kalman filter, which could be calculated as follows.

\[
\begin{align*}
M_k &= \Phi_k ( \hat{X} ) P_{k-1} \Phi_k^T ( \hat{X} ) + Q_k ( \hat{X} ) \\
K_k &= M_k C^T ( \hat{X} ) \left[ C ( \hat{X} ) M_k C^T ( \hat{X} ) + R_k \right]^{-1} \\
P_k &= \left( I - K_k C ( \hat{X} ) \right) M_k
\end{align*}
\]

where \( M_k \) was the covariance matrix before the update; \( R_k \) was the covariance matrix of the measurement noises, \( P_k \) was the covariance matrix.

The convergence of extended Kalman filter was sensitive to the initialization. In practice, the estimate state of the Kalman filter was typically set to be zero. The mesurement state could be set as before measured information. Then

\[
\hat{X}(0) = [ \hat{q}_{p0}, 0, \hat{R}_0, \hat{R}_0, 0, 0 ]^T
\]

where \( \hat{q}_{p0}, \hat{R}_0 \) and \( \hat{R}_0 \) were the initial estimation of the LOS angle, relative distance and relative velocity, respectively. The maximum measured error of \( q_p \) and \( R \) were \( \Delta q \) and \( \Delta R \). The maximum estimated error of \( \hat{q}_p \) and \( \hat{R} \) was \( \Delta \hat{q} \) and \( \Delta \hat{R} \). The covariance matrix was set as follows.

\[
P(0) =
\begin{bmatrix}
\Delta q^2 & 0 & 0 & 0 & 0 & 0 \\
0 & \Delta \hat{q}^2 & 0 & 0 & 0 & 0 \\
0 & 0 & \Delta R^2 & 0 & 0 & 0 \\
0 & 0 & 0 & \Delta \hat{R}^2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

With the equations (20)~(24), the extended Kalman filter could be start to estimate the state values. And time-to-go could estimate as follows.

\[
\hat{t}_{go} = \hat{R}/\Delta \hat{R}
\]
Simulation

To illustrate the performance of our proposed methods, the head-to-head intercept simulation was done. The initial conditions for the simulation were listed in Table 1. The standard deviations of measurement noises were listed in Table 2.

Table 1. The initial conditions for the simulation.

<table>
<thead>
<tr>
<th>$q_{p0}$ (°)</th>
<th>$\dot{q}_{p0}$ (°/s)</th>
<th>$R_0$ (km)</th>
<th>$\dot{R}_0$ (m/s)</th>
<th>$a_{x0}$ (m/s²)</th>
<th>$a_{y0}$ (m/s²)</th>
<th>$a_{t0}$ (m/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5000</td>
<td>45</td>
<td>5</td>
<td>-100</td>
<td>10</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 2. The standard deviations of measurement noises.

<table>
<thead>
<tr>
<th>$\sigma_q$ (°)</th>
<th>$\sigma_R$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>7</td>
</tr>
</tbody>
</table>

Both the discretization step and the simulation step are set to be 1ms. The Kalman filter was used in polar coordinate system and in rectangular coordinate system. Which could verify the performance of the filter better.

Fig.2 and Fig.3 were the estimated acceleration $a^I_x$ and $a^I_y$ respectively. Fig.4 and Fig.5 were the estimated LOS angle $q_p$ and LOS angular velocity $\dot{q}_p$. Fig.6 and Fig.7 were the estimated e distance $R$ and the approaching velocity $\dot{R}$. After a short transient time, the estimated values converge to the true values very well. And the estimated result in polar coordinate system and rectangular coordinate system were basically identical. For the seeker could measure $q_p$ and $R$, which were the information along and perpendicular to the LOS. The estimated information was strongly correlated with the measured information. The extended Kalman filter had sufficient robustness.
Summary
Based on the measurements of phased array radar seekers, the extended Kalman filter has been designed in the polar coordinate system to estimate the maneuvering target acceleration, the time-to-go, and the LOS angular velocity. The simulation results show that the extended Kalman filter can accurately estimate the maneuvering information. After a short transient time, the estimates can converge to their true values. The estimated information was strongly correlated with the measured information. The extended Kalman filter had sufficient robustness.

References