Comparison of Different Chromatic Dispersion Compensation Schemes in Quasi-linear Transmission System with Intrachannel Four-wave Mixing

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Abstract. In this paper, different chromatic dispersion compensation schemes like pre-, post-, and symmetrical schemes are compared numerically in the presence of intrachannel four-wave mixing (IFWM) for return-to-zero quasi-linear transmission systems. The transmission performances have been compared in terms of the average intensity deviation of the “1” bits and the relative intensity of the ghost pulse (the “0” bits). It is found that, for a fixed amplifier spacing, optimal compensation scheme depends on transmission bit rate. As bit rate increases the performance of pre-compensation becomes better. On the contrary, post-compensation exhibits its advantage more and more as bit rate decreases.

Introduction

In high-bit-rate fiber transmission systems, intrachannel nonlinear effects are main sources of bit errors [1-4]. While intrachannel cross-phase modulation (IXPM) can be effectively suppressed by strong dispersion management in combination with a return-to-zero (RZ) modulation format, intrachannel four-wave mixing (IFWM) remains a problem which results in intensity deviation in the “1” bits and the generation of ghost pulses in the “0” bits [5-7]. It has been shown that, due to the interplay between chromatic dispersion and intrachannel nonlinear effects, system performance depends on the position of dispersion compensated fibers [8] and that intrachannel nonlinearities may be partially reduced by proper dispersion map design [9].

There are three basic dispersion compensation schemes, i.e. pre-, post-, and symmetrical compensations, and performance comparison of these schemes has been carried out [10-15]. However, most of the studies [10-13] didn't consider or discuss the IFWM effect because the single channel rate considered there was limited to 10 Gb/s where IFWM was of less importance than the other nonlinear effects such as self-phase modulation (SPM) and cross-phase modulation (XPM). The higher the channel rate is the narrower the RZ pulses should be, and the dispersive nature of the pulses gives rise to large pulse overlap and thus large IFWM effect. Randhawa et al. investigated [14] the three compensation schemes in 40 Gb/s carrier-suppressed RZ systems and found that symmetrical compensation is the best one which reduces the bit error rate (BER) to the more extent than that of pre- and post-compensations. They also showed that with increase in the input bit rate, symmetrical compensation was still the best, i.e. the optimal compensation scheme (with minimum BER) was independent of transmission bit rate. Note that the authors did not indicate whether the IFWM effect was considered in their model and there was no discussion of this effect in [14].

Recently, performance comparison of the three compensation schemes for different fiber standards has been carried out [15]. It is observed that the choice of compensation schemes depends on the characteristics of the transmission fiber: Pre-compensation is the best for ITU 655 fiber while Alcatel fiber is best for post- and symmetrical compensation. However, the simulation focused on a single bit rate of 40 Gb/s.

In this paper, pre-, post-, and symmetrical compensations are compared for 80, 40 and 20 Gb/s RZ quasi-linear transmission systems. The simulations are based on a set of coupled nonlinear
Schrödinger equations which include intrachannel nonlinearities such as IFWM, IXPM and SPM. It is found that the optimal compensation scheme depends on the transmission bit rate. Pre-compensation is the best for high bit rate such as 80 Gb/s, whereas post-compensation becomes the best for low bit rates such as 40 and 20 Gb/s. Note that the main conclusion here is quite distinct from that of [14].

**Simulation Setup**

Figure 1 (a), (b) and (c) represent, respectively, pre-, post- and symmetrical compensations. The period length is the same for all three compensation schemes. Each period comprises a 72.77 km long standard single-mode fiber (SSMF), 7.277 km long dispersion-compensating fiber (DCF), and an EDFA. We assume that the DCF can exactly compensate for chromatic dispersion of the SSMF so that the average dispersion of the transmission link is zero. The EDFA exactly compensates for energy loss caused by the SSMF and DCF.

![Simulation setup](image)

**Basic Equations**

Quasi-linear transmission can be described by the generalized nonlinear Schrödinger equation which takes the form [1]

\[
i \frac{\partial u}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u}{\partial \tau^2} + |u|^2 u = -\frac{i}{2} \Gamma u + \frac{i}{2} \mu u,
\]

where \(\xi, \tau, \) and \(u\) are, respectively, the normalized distance, time, and field envelope in soliton units. The parameters \(\Gamma\) and \(\mu\) account for, respectively, the fiber loss and the gain of the EDFA. The second term on the left side represents group-velocity dispersion (GVD) where the sign “+” or “−” is chosen, respectively, when the field is transmitted in the SSMF (anomalous GVD) and the DCF (normal GVD). The third term on the left side represents the Kerr nonlinearity.

A pseudo-random bit stream needs tedious programming especially when the IFWM effect is considered and the simulation time increases exponentially with the number of bits. However, considerable physical insight could be gained with a limited number of input bits [6,16,17] when we only focus our attention on the relative comparison. Here the input is assumed to be

\[
u(0,\tau) = u_1(0,\tau + 3q_0) + u_2(0,\tau + q_0) + u_3(0,\tau - q_0) + u_4(0,\tau - 3q_0)
\]

\[
= A_1 \text{sech}(0,\tau + 3q_0) + A_2 \text{sech}(0,\tau + q_0)
\]

\[
+ A_3 \text{sech}(0,\tau - q_0) + A_4 \text{sech}(0,\tau - 3q_0)
\]

(2)

Where \(2q_0\) represents the duration of the bit slot and \(A_j (j=1,2,3,4)\) the amplitude. We assume that all “1” bits have the same initial width and the same amplitude and that the “0” bits have a much smaller amplitude than that of the “1” bits. Substituting the input into Eq. 1, we obtain:
\[ \frac{i \partial u_i}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u_i}{\partial \tau^2} + \frac{i}{2} \Gamma u_i - i \mu u_i = - \left[ \left| u_1 \right|^2 + 2 \left| u_2 \right|^2 + 2 \left| u_3 \right|^2 \right] \right] u_i - u_2^* u_i^* - 2 u_2 u_i^* + u_3^* u_i^* - 2 u_3 u_i^* \right] \right] u_i - u_3^* u_i^* - 2 u_3 u_i^* + u_4^* u_i^* - 2 u_4 u_i^* + u_4^* u_i^*. \] 

(3)

\[ \frac{i \partial u_2}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u_2}{\partial \tau^2} + \frac{i}{2} \Gamma u_2 - i \mu u_2 = - \left[ \left| u_2 \right|^2 + 2 \left| u_3 \right|^2 + 2 \left| u_4 \right|^2 \right] \right] u_2 - u_3^* u_2^* - 2 u_3 u_2^* + u_4^* u_2^* - 2 u_4 u_2^* + u_4^* u_2^*. \] 

(4)

\[ \frac{i \partial u_3}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u_3}{\partial \tau^2} + \frac{i}{2} \Gamma u_3 - i \mu u_3 = - \left[ \left| u_3 \right|^2 + 2 \left| u_4 \right|^2 + 2 \left| u_1 \right|^2 \right] \right] u_3 - u_4^* u_3^* - 2 u_4 u_3^* + u_1^* u_3^* - 2 u_1 u_3^* \right] \right] u_3 - u_1^* u_3^* - 2 u_1 u_3^* + u_1^* u_3^* \right] u_3 - u_1^* u_3^*. \] 

(5)

\[ \frac{i \partial u_4}{\partial \xi} + \frac{1}{2} \frac{\partial^2 u_4}{\partial \tau^2} + \frac{i}{2} \Gamma u_4 - i \mu u_4 = - \left[ \left| u_4 \right|^2 + 2 \left| u_1 \right|^2 + 2 \left| u_2 \right|^2 + 2 \left| u_3 \right|^2 \right] \right] u_4 - u_1^* u_4^* - 2 u_1 u_4^* + u_2^* u_4^* - 2 u_2 u_4^* + u_2^* u_4^*. \] 

(6)

In real parameters

\[ \xi = \frac{z}{L_D} = \frac{z \beta_j}{T_0^2}, \quad \tau = \frac{t - z / v_s}{T_0}, \quad \Gamma = \alpha L_D = \frac{\alpha T_0^2}{| \beta_j |}, \quad \mu = (g_0 - \alpha) L_D. \] 

(7)

Where \( z, t, v_s \) represent, respectively, distance, time, and group velocity. \( T_0 \) is the half-width (at 1/e-intensity point) of the input “1” bits, \( \beta_j \) is the GVD coefficient, \( \alpha \) is the attenuation constant of the fiber, \( g_0 \) is the unsaturated gain parameter of the EDFA, and \( L_D = T_0^2 / | \beta_j | \) is the dispersion length. We do not include the Raman self-scattering (RSS) and self-steepening effects because, for quasi-linear strongly dispersion-managed transmission, the path-averaged bit width is very large and the peak power is very low. We also neglect high-order dispersion effects and amplifier noise and think that they have almost equal influence on the three compensation schemes. Eq. 3-6 can be used to describe bits transmission in the SSMF, DCF and EDFA. The differences are that, for transmission in the SSMF and the DCF, the gain parameter \( \mu \) is zero, while for transmission in the EDFA, the loss term (\( \Gamma \)) and even all the nonlinear terms can be neglected. In real parameters, the relation between the initial amplitude \( A_j \) of the \( j \)th bit in Eq. 2 and its real peak power is given by

\[ A_j^2 = \frac{\gamma P_j T_0^2}{| \beta_j |}. \] 

(8)

and the energy of a hyperbolic-secant pulse with peak power \( P_j \) and pulse width \( T_0 \) is

\[ E_{\text{sech}} = 2 P_j T_0, \] 

(9)

where \( \gamma \) is the nonlinearity coefficient of the fiber.

**Simulation Results and Discussion**

**The Effect of IFWM on Bit Transmission**

Quasi-linear transmission operates in the regime in which the local dispersion length \( L_D \) is much shorter than the nonlinear length \( L_{NL} \) in all fiber sections of a dispersion-managed link [1], where

\[ L_D = \frac{T_0^2}{| \beta_j |}, \quad L_{NL} = \frac{1}{\gamma P_j}, \] 

(10)

According to Eq. 8, the quasi-linear condition requires that

\[ \frac{L_D}{L_{NL}} = \frac{\gamma P_j T_0^2}{| \beta_j |} = A_j^2 \ll 1. \] 

(11)

A typical example of quasi-linear transmission is a system operating at bit rates of 40 Gb/s or more.
and employing ultra-short pulses that spread quickly over multiple bits as they propagate along the link, which is just the case discussed throughout this paper. It is shown that IFWM is the most important intrachannel nonlinear effect in quasi-linear dispersion-managed transmission system [17]. So, before we compare the three compensation schemes, it is useful to see how IFWM affects bit transmission.

It is shown that [17] the influence of IFWM on bit transmission depends on bit patterns. For 4-bit inputs, the patterns like 1110 and 0110 which contain consecutive “1” bits manifest large distortion both in shape and spectrum, whereas the distortion is very small in the case of non-consecutive “1” bits such as 1010 and 1001. Therefore, we mainly focus our attention on the transmission of bit pattern 1110. In all cases, the transmission link is fixed and is the same as described in Section 2, i.e., each transmission period comprises a 72.77 km long SSMF, 7.277 km long DCF, and an EDFA which exactly compensates for the energy loss. The GVD coefficients of the SSMF and DCF are assumed to be, respectively, \( (\beta_2)_{SSMF} = -20 \) ps\(^2\)/km and \( (\beta_2)_{DCF} = 200 \) ps\(^2\)/km near 1.55 \( \mu \)m. The DCF is assumed to have the same attenuation constant and the same nonlinearity coefficient as those of the SSMF with \( \alpha = 0.046 \) km\(^{-1}\) and \( \gamma = 1.3 \) W\(^{-1}\)km\(^{-1}\). Actually, since the DCF is much shorter than the SSMF, neglecting the difference of the parameter \( \alpha \) and \( \gamma \) between the SSMF and the DCF will have a negligible influence on the simulation results.

First, we consider transmission with all the mentioned nonlinear and dispersion effects except IFWM. Figure 2 shows the simulation results (outputs) of the three compensation schemes in both (a) shape and (b) spectrum for bit pattern 1110, where the green dotted curve represents the input, the blue solid, black dashed-dotted, and red dashed curves represent, respectively, the outputs of pre-, post-, and symmetrical-compensations. In all cases the transmission distance is fixed at 960 km that is equal to 12 transmission periods as shown by Figure 1. The input is assumed to be in the form of Eq. 2, where all the “1” bits have a same initial width of \( T_{FWHM} = 3 \) ps (\( T_0 = T_{FWHM}/1.763 \approx 1.7 \) ps) and a same initial amplitude of \( A_j = 0.2 \) (\( j = 1,2,3 \)) which, according to Equation (8), corresponds to a peak power of 213 mW, satisfying the quasi-linear condition. The “0” bit has the same initial width and shape as those of the “1” bits but with much smaller peak power of 2.13 mW (\( A_d = 0.02 \)). The initial separation (\( 2q_0 \times T_0 \)) between two adjacent bits is 12.5 ps, representing a bit rate of 80 Gb/s. The intensities of the pulse shape and spectrum are normalized with respect to the input, respectively.

We see that, in the absence of IFWM, the joint effect of GVD, SPM and IXPM has a small influence on transmission in both shape and spectrum, and the difference among the three compensation schemes are very small. However, the situation is different when IFWM is added into consideration which is shown by Figure 3. Obviously IFWM is the dominant nonlinear effect for a typical dispersion compensated quasi-linear transmission system. Because of dispersion-broadened pulse overlapping, time-domain four-wave mixing transfers energy from triples of pump pulses into
the “0” bit, generating ghost pulse, and into the “1” bits resulting in intensity deviation. Moreover, IFWM exerts different influence on different compensation schemes. In this case the pre-compensation seems to be the best because of the lowest intensity fluctuation of the “1” bits and the smallest energy of the ghost pulse. On the contrary, the post-compensation gives a worst result since the intensities of the “1” bits fluctuate dramatically and the energy of the ghost pulse is even larger than that of one of the “1” bits.

Figure 3. The same as Figure 2 except that IFWM is considered.

In the next subsection performance comparison between different compensation schemes will be made in more detail at transmission bit rates of 80, 40, and 20 Gb/s, respectively.

**Performance Comparison of the Three Compensation Schemes at Different Bit Rates**

In this subsection we compare the performance of the three schemes at different transmission bit rates of 80, 40, and 20 Gb/s, respectively. The hyperbolic-secant pulse widths $T_{FWHM}$ are, respectively, 3, 6, and 12 ps, thus each bit rate has the same duty cycle of 0.24. In all cases, the transmission link is the same as that for Figures 2 and 3, and the input bit pattern is 1110.

To make the comparison quantitative, we define two parameters, i.e., the average peak intensity deviation ($\Delta P_{\text{aver}}$) of the “1” bits and the relative peak intensity of the ghost pulse generated at the “0” bit slot. The relative peak intensity of the ghost pulse is defined as the ratio of the peak intensity of output ghost pulse to the peak intensity of the input “0” bit (note that, as mentioned earlier, the input “0” bit is assumed to have a very small amplitude). The average peak intensity deviation ($\Delta P_{\text{aver}}$) of the “1” bits is given by

$$\Delta P_{\text{aver}} = \frac{\sum_{j=1}^{3} |P_j^{(\text{out})}-1|}{3},$$

where $P_j^{(\text{out})}$ is the peak intensity of the $j$th output “1” bit. Note that, as shown by Figures 2 and 3, all the intensities are normalized with respect to the peak intensity of the input “1” bit which has a normalized value of 1.

Figure 4 compares the results of the three compensation schemes, where (a) and (b) show, respectively, the variation of $\Delta P_{\text{aver}}$ and the relative peak intensity of the ghost pulse with transmission distance. In all cases, the transmission distance is fixed at 960 km which includes 12 compensation periods, the input is the same as that for Figure 3, i.e., all the “1” bits have a same initial width of $T_{FWHM}=3$ ps and a same initial peak power of 213 mW, the separation between two adjacent bits is 12.5 ps representing a bit rate of 80 Gb/s. Note that the dots on the curves represent the calculations at the ends of each compensation periods (i.e., at the output ends of EDFAs as shown in Figure 1), and curve fitting is done based on those dots. Figure 4(a) shows that for all compensation schemes the value of $\Delta P_{\text{aver}}$ increases with transmission distance. Pre-compensation results in a smallest value of $\Delta P_{\text{aver}}$ while post-compensation causes the largest value of $\Delta P_{\text{aver}}$. Furthermore, as shown by Figure 4(b), post-compensation generates the largest ghost pulse, whereas pre- and symmetrical-compensations are much better for suppression of the ghost pulse. In
general, pre-compensation is the best and post-compensation is the worst in this case.

The results can be understood as follows: For pre-compensation, pulses always first experience quick broadening by DCF and then compressed gradually by SSMF in each transmission period. This effectively reduces the peak power of the pulse during transmission in the first half of the SSMF and thus reduces the IFWM effect. Although the pulse width is gradually recovered in the second half of the SSMF, most of the pulse energy has already been attenuated by fiber loss and the IFWM effect is also small in this stage. On the contrary, for post-compensation, pulses first experience gradual broadening in SSMF, the path-averaged peak power during the first half of the SSMF is obviously higher than that of the case of pre-compensation, leading to large IFWM effect.

It should be pointed out that, on one hand, predispersion (as in the case of pre-compensation) can decrease the peak power of the individual “1” bits and tends to suppress nonlinearities, but on the other hand, it leads to overlapping of distant bits and enhances the nonlinear interactions. However, as shown in [18], IFWM components induced between adjacent “1” bits (like patterns 1110 or 0110) are greatly larger than those between distant pulses (like 1010 or 1001). Thus, in general, predispersion is helpful for suppression of the IFWM effect in this case.

Figure 4. Comparison of pre-, post-, and symmetrical-compensations at 80 Gb/s for bit pattern 1110. Variation of (a) the average peak intensity deviation of the “1” bits ($\Delta P_{av}$) and (b) the relative peak intensity of the ghost pulse with transmission distance.

Figure 5 gives a similar comparison at a bit rate of 40 Gb/s, where all the “1” bits have a same initial width of $T_{FWHM}=6$ ps. The separation between two adjacent bits is twice as big as that for Figure 4, so the duty cycle is the same as that for Figure 4. The input peak power of the “1” bit is half of that for Figures 2-4, thus, according to Equation (9), the “1” bit contains the same energy as that for Figures 2-4.

Figure 5. The same as Figure 4 except that the transmission bit rate is 40 Gb/s. The duty cycle and the input pulse energy are identical to those for Figure 4.
Figure 5(a) shows that, relative to the case of Figure 4(a), the value $\Delta P_{\text{aver}}$ of pre-compensation is increased while $\Delta P_{\text{aver}}$ of post-compensation decreased, and the same feature applies to the ghost pulse as shown by Figure 5(b). This means that the relative performance of the three compensation schemes changes with transmission bit rate. To prove this, we further reduce the bit rate to 20 Gb/s, and the results are shown by Figure 6. Where the “1” bits have an initial width of $T_{\text{FWHM}}=12$ ps, the duty cycle and the pulse energy are identical to those for Figures 2-5. It is seen that the performance of post-compensation is further improved while pre-compensation becomes the worst one, which is exactly the opposite of the case of Figure 4. This can be explained as follows: Since pulse width increases as bit rate decreases, the dispersion length $L_D$ given by Eq. 10 increases, and the pulse broadening speed through SSMF becomes slow. So, in the case of post-compensation, fiber loss will cause a considerable degree of energy degradation before dispersion-induced pulse overlapping (which is the prerequisite for IFWM) happened, this diminishes the IFWM effect when pulses overlap during the successive transmission in the SSMF. On the contrary, for pre-compensation, pulse overlapping in the SSMF has already occurred before the energy is not attenuated by fiber loss, and therefore the IFWM effect is significant.

The conclusion here is that, for a certain duty cycle, the relative performance of the three compensation schemes depends on transmission bit rate. Pre-compensation is the best for high bit rates such as 80 Gb/s, while post-compensation is suitable for low bit rates such as 40 and 20 Gb/s. We also compared the three schemes at different bit rates but with unequal duty cycles. In this case, the transmission link and the bit pattern 1110 are the same for Figures 2-6. The hyperbolic-secant pulses representing the “1” bits have the same energy and same width of $T_{\text{FWHM}}=3$ ps for all bit rates and for all compensation schemes, so the duty cycles are different for different bit rates, i.e., 0.24, 0.18, 0.12, and 0.06, respectively, for 80, 60, 40 and 20 Gb/s. Simulations not shown here revealed that the main feature is similar to that of Figures 4-6, and the previous conclusions are also applicable for this case. The reason is basically the same as that described above. Further investigations have been done for other bit patterns such as 0110 and 1100 and found that the general feature is similar to that of bit pattern 1110. However, for patterns like 1010 and 1001 (without consecutive “1” bits), the IFWM effect on the transmission is very small and the difference among the performance of the three compensation schemes can be neglected.

Finally, it should be pointed out that all previous discussions are based on the quasi-linear condition described by Equation (11) with $A_j=0.2-0.4$ where $A_j$ is the amplitude of the “1” bit. Indeed, further simulations indicated that the above conclusions are justified as long as $A_j<0.5$. However, the situation is quite different when $A_j$ exceeds 0.5, as the soliton effect plays an important role in this case, which is beyond the scope of our discussion.
Conclusion

We compared the performances of pre-, post-, and symmetrical-dispersion compensations in the presence of IFWM in quasi-linear transmission systems. Numerical simulations show that, for a fixed amplifier spacing, the optimal compensation depends on transmission bit rate. As bit rate increases the performance of pre-compensation becomes better and better. On the contrary, post-compensation exhibits its advantage more and more as bit rate decreases. Thus, pre-compensation is the best candidate for high bit rates and post-compensation is suitable for low bit rates.

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