Normalized LMS Filtering of Self-Mixing Interference Signal with Varying Frequency

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Abstract. In actual measurement, the vibration frequency of the measuring object may change. In order to improve the accuracy of parameter estimation and measurement, this paper introduced the composition and filter principle of self-mixing interference system, and used the normalized LMS algorithm to simulate the interference experiment signal with frequency variation. The simulation results show that when the frequency of the signal changes, the weight coefficient of the filter is always in a small order of magnitude ($10^{-3}$) dynamic change, which can quickly converge to a suitable value. The filtering effect is no different from the actual single frequency.

Introduction

With the development of modern information and digital technology, the requirements for noise interference elimination in many fields are becoming increasingly strong, and real-time adaptive processing of signals is required. In 1970, Widrow and Hoff proposed the least mean square (LMS) algorithm. The algorithm was an adaptive signal processing algorithm based on the Wienerhoff equation. It can implement filtering, smoothing and prediction [1]. It is widely used because of its small computational complexity and easy implementation. However, the LMS algorithm has the contradiction between convergence speed and steady state error. Many scholars have carried out in-depth research and improvement on these problems. In order to improve the convergence speed of the LMS algorithm, Bitmead and Bershard et al. proposed a normalized LMS (NLMS) algorithm [2] [3]. NLMS has increased the iterative step size of the algorithm and has a faster convergence speed, so it has been widely used. NLMS has increased the iterative step size of the algorithm and has a faster convergence speed, so it has been widely used.

The laser interference signal has a simple structure and is easy to collimate, and is widely used in laser self-parameters and various high-precision quantities. In the optical feedback self-mixing interference system, since the interference of the external environment affects the signal, the collected experimental data is accompanied by interference and noise. In order to improve the accuracy of the measurement, the original data needs to be effectively processed to reduce various noises and extract useful interference signals.

This paper studies the principle of self-mixing interference systems and theoretical models and adaptive filters. At the same time, the filtering effect of the normalized least mean square filtering algorithm on the measured interference signal with the change of processing frequency is also studied. The variation of the weight coefficient of filtering at different moments is analyzed. The algorithm allows the filtered weight coefficients to quickly converge to an appropriate value after the signal frequency changes.

System Model and Filtering Principle

Composition and Theoretical Model of Self-Mixing Interference System

Optical feedback self-mixing interference (OFSMI) [4] means that after the laser output light is reflected or scattered by an external object, a part of the returned light and the cavity light are coherently mixed in the laser cavity, and the amplitude and frequency of the output light change.
The self-mixing interference signal not only carries the information of the characteristics and motion laws of the external target, but also carries the information of the laser's own parameters.

The optical feedback self-mixing interference (OFSMI) system consists of a semiconductor laser, a lens and an externally reflecting object. The physical model is shown in Figure 1.

![Figure 1. Three-mirror F-P cavity equivalent diagram of self-mixing interference system.](image)

The mathematical model of the optical feedback self-mixing interference system is expressed as:

\[ \Phi_F(\tau) = \Phi_0(\tau) - C \cdot \sin[\Phi_F(\tau) + \arctan \alpha] \]  

(1)

\[ G(\Phi_F(\tau)) = \cos(\Phi_F(\tau)) \]  

(2)

\[ P(\Phi_F(\tau)) = P_0[1 + mG(\Phi_F(\tau))] \]  

(3)

where: equation (1) is the phase equation of the laser self-mixing interference system; equation (2) represents an interference function of the relationship between the normalized optical power and the external cavity phase when optical feedback is included; equation (3) is the power equation of the laser self-mixing interference system; \( \tau = 2L / c \), \( L \) is the length of the outer cavity, \( c \) is the speed of light in vacuum; \( \alpha \) is the line width broadening factor; \( m \) is the feedback level factor.

When the external reflector \( M \) is displaced, the output power \( P(\Phi_F(\tau)) \) of the laser with optical feedback changes. The \( P(\Phi_F(\tau)) \) detected by the photodetector carries the size and direction information of the displacement of the external reflector. The displacement trajectory of the external reflector can be reconstructed by processing the \( P(\Phi_F(\tau)) \) by selecting a suitable displacement measurement algorithm.

**Principle of Adaptive Filter**

The basic principle of adaptive filtering is to automatically adjust the current filtering parameters by using the results of the filtering parameters at the previous moment to adapt to the statistical characteristics of the signal and noise unknown or changing with time, so as to achieve optimal filtering. Therefore, the adaptive filter has self-adjusting and tracking capabilities. The general structure of the adaptive filter is shown in Figure 2.

![Figure 2. Adaptive filter structure.](image)

In figure 2, \( x(n) \) represents the input signal at time \( n \). \( y(n) \) represents the output signal after passing the parameter-adjustable digital filter at time \( n \), \( d(n) \) represents the desired signal at time \( n \), the difference between the desired signal \( d(n) \) and the output signal \( y(n) \) yields an error signal \( e(n) \).
e(n) and x(n) adjust the parameters of the filter through an adaptive algorithm. By adjusting the error signal e(n) to be minimized, the output signal y(n+1) at the next moment is brought closer to the desired signal.

**Normalized LMS Algorithm**

The normalized LMS algorithm speeds up the convergence of the algorithm and improves the stability of the algorithm by changing the step factor. The compensation factor needs to be normalized by the power value of the input signal to obtain a variable step factor μ related to the input signal power. This factor changes as the input power changes, resulting in lower steady-state errors at higher convergence speeds[5].

The normalization process is as follows:

Normalized step factor calculation:

$$μ = \frac{\mu}{\sigma_x^2}$$  \hspace{1cm} (4)

$$\sigma_x^2 = X^T(n)X(n)$$  \hspace{1cm} (5)

where: $\sigma_x^2$ is the input power value of the signal.

Weight coefficient update process:

$$W(n + 1) = W(n) + 2\frac{\mu}{\gamma + X^T(n)X(n)}X(n)e(n)$$  \hspace{1cm} (6)

The step factor at this time changes with the change of the input power, so that the normalized LMS algorithm has both the convergence speed and the steady state error. Here, $\gamma$ is a small amount set to avoid the occurrence of a false score. The normalized LMS algorithm solves the contradiction between convergence speed and steady state error.

**Normalized LMS Algorithm for Processing Self-Mixing Interference Signals with Varying Frequency**

In actual measurements, the frequency of the measured object motion may change. Next, filtering is performed on the signal whose frequency has changed. In the experiment, since there is no signal that changes the frequency, in this paper, the signals of two different frequencies are connected back and forth to achieve the purpose of signal frequency conversion. This frequency change is a jump.

![Figure 3. a(a) 20hz plus 40hz experimentally acquired interference signal,(b) NLMS algorithm filtered interference signal,(c) error signal; b shows the dynamic change of the tap weight coefficient of the 20hz plus 40hz experimental acquisition signal.](image)

Figure 3.a shows the filtering effect of the 20hz plus 40hz signal, with the filter order $M=128$ and the step factor $\mu=0.0001$. It can be seen that the effect of filtering is no different from the actual single frequency. When the frequency jumps, the algorithm converges quickly. Because, in the
initial stage, after the algorithm runs converged, the tap weight coefficient fluctuates within a small range (as shown in Figure 3.b, the tap weight is on the order of $10^{-3}$), that is, randomly moving around the minimum point of the error performance surface. This confirms that the filter weight coefficient obtained after the convergence of the least mean square algorithm does not end in the Wiener solution, but rather fluctuates around a small range of the Wiener solution. When the frequency is changed, the tap weight coefficient is quickly updated to an adapted value.

Figure 3.b shows the dynamic change of the tap weight coefficient of the 20hz plus 40hz experimental acquisition signal. The sampling points of the signal have a total of 200,000 points, 100,000 points before and after. When $w_1$ is $i=50000$, this time is equivalent to the weight signal of the tap when the sampling point is 50000, and the algorithm has already converged. When $w_2$ is $i=100000$, this is equivalent to the weight signal of the tap at the end of the 20hz signal. It can be seen that $w_1$ and $w_2$ are almost the same, the tap weight coefficient changes within a small range because both $w_1$ and $w_2$ are in a state of convergence. When $w_3$ is $i=150000$, it is equivalent to the tap weight coefficient at the 150,000th sampling point, according to the filtering effect, the algorithm has also converged. $W_4$ is the final tap weight coefficient, which is the tap weight coefficient after the end of the algorithm. The following conclusions can be drawn: 1) Due to the operation of the algorithm, the tap weight coefficient is in a dynamic adjustment process. 2) When the frequency of the signal changes, the tap weight coefficient can be quickly adjusted to an appropriate value because the tap weight coefficient is of a small magnitude ($10^{-3}$). 3) Adaptive filtering can process signals whose signal frequency changes, which also verifies the adaptiveness of adaptive filtering.

Figure 4 is the filtering effect of 20hz plus 500hz and the weight coefficient of each moment.

![Figure 4](image)

From Fig. 3 and Fig. 4, the filtering effect of the NLMS algorithm on the signal can be seen that the NLMS algorithm has a better filtering effect on the high frequency impact noise signal.

**Conclusions**

In this paper, the performance of the normalized minimum mean square adaptive filter in processing the signal with varying frequency is tested. The simulation results show that the normalized minimum mean square adaptive filtering weight coefficient can be quickly adjusted to the appropriate value in the order of magnitude ($10^{-3}$) when the signal frequency changes. And the filtering effect is no different from the actual single frequency.
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References


