Electromechanical Coupling Analysis of Shaped Reflector Antenna Based on Standard Discrete Parabola Set

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Abstract. The mechanical-electromagnetic-field-coupling-model (MEFCM) is widely used in analysing the effect of structural deformation on radiation pattern of reflector antennas. Due to lack a closed-form optical path difference expression for shaped reflector antennas, the MEFCM cannot be directly applied. For this shortcoming, in order to apply the coupling model into shaped dual-reflector antennas, we proposed a new shaped surface description approach based on standard discrete parabola set. By employing this approach, the MEFCM can be applied to analyse the radiation pattern of shaped dual-reflector antenna with different types of structural deformation. Application on a 25 m shaped Cassegrain antenna shows the effectiveness of the proposed approach for the pattern analysis of shaped dual-reflector antennas.

Introduction

Large reflector antenna’s structural deformation is inevitably affected in operation conditions due to its exterior loads, such as gravity, temperature and wind, which results in its gain loss and pointing error caused by beam distortions. The mechanical-electromagnetic-field-coupling-model (MEFCM) [1,2] is one widely applied model to analyze the influence of structural deformation on the electromagnetic (EM) performance of reflector antenna. It can be used to rapidly analyze the influence of different types of errors on far filed beam pattern, such as surface random error and structural deformation error [3,4]. In MEFCM, one key step is to obtain the aperture field phase error (PE) or optical path different (OPD), especially for dual-reflector antennas [5].

Many researchers have studied the influence of reflector antenna structural errors on OPD and EM performance. Duan[1] studied influence of surface random errors and systematic errors on EM performance and established the optimization model of the integrated mechanical electromagnetic performance. Baars[6] studied influence of different types of structural deformation on OPD for primary-focus antennas, such as axial and lateral feed defocus errors. Ruze[7] studied the influence of different types of structural deformation on the EM performance for dual-reflector antennas and derived the relationship between different types of structural and OPD for dual-reflector, such as feed displacement, subreflector translation and rotation offsets, etc.

The foregoing relationships between antenna structural deformation and OPD can be used in MEFCM to study the influence of structural errors on far field beam pattern. However, these relationships are only suitable for standard reflector, which can be accurately described by a closed-form expression, and therefore, cannot be applied for shaped reflector. In order to enable MEFCM to be applied in the rapid analysis shaped reflector’s structural deformation, in this paper, a surface description method of shaped reflector based on standard discrete parabola set is proposed and

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applied to relationship between different types of structural deformation and OPDs. Thus, shaped dual-reflector surface can be accurately described and aperture field PE distribution caused by structural deformation can be rapidly calculated, from which the distorted far filed beam pattern can be rapidly obtained.

In this paper, we propose a new method that mainly deals with electro-mechanical coupling analysis of shaped dual-reflector antennas. In order to obtain high aperture efficiency and low sidelobe levels, reflector’s shaping design is adopted to obtain the desired aperture field distribution. The shaping design should satisfy three conditions[8]: conservation of power, equality of path-length and law of reflection. Shaped reflectors can spread spherical waves into a desired pattern based on geometric optics. After shaping design, as shown in Fig.1, generatrices of primary and secondary reflector are not standard parabola and hyperbola, respectively, and cannot be accurately described by a closed form equation. The shaped surfaces do not satisfy the geometric relationship of classical Cassegrain system and then, the classical dual-reflector OPD relationships cannot be employed. In order to apply Ruze’s OPD relationships, in this paper, the shaped dual-reflector surface is accurately described based on standard discrete parabola set.

**Description of Shaped Reflector Surface**

The proposed description method of shaped dual-reflector surface is shown in Fig.2. The shaped primary reflector's generatrix is expressed as a discrete point set. Similarly, the shaped subreflector's generatrix is also expressed as a discrete point set. Supposed that each point of primary reflector is located in a standard parabola, for the ith point, the corresponding parabolic equation can be expressed as:

\[ x_i^2 = 4f_i(z_i - h_i) \]  

where \( h_i \) and \( f_i \) signify the offset of z-direction and focal length of the ith parabola, respectively. Here we use \( L = \{P_1, \ldots, P_n\} \) to express the point set for primary reflector and \( Q = \{S_1, \ldots, S_n\} \) for secondary reflector, and their points \( P_i(x_i, z_i) \) and \( S_i(x_i, z_i) \) are one-to-one correspondence. The slope
of the shaped primary surface at \( i \)th point can be obtained by discrete point difference or mean weighted by areas of adjacent triangles. As the nature of the parabola, \( k_{si} \) is signified as follows:

\[
k_{si} = x_i / (2f_i) = \tan(\phi_i / 2)
\]

Table 1. Relative parameters of standard discrete parabola set.

<table>
<thead>
<tr>
<th></th>
<th>( f_i ) = ( x_i / (2 \tan(\phi_i / 2) )</th>
<th>( h_i = z_i - x_i^2 / 4f_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnification</td>
<td>( M_i = \tan(\phi_i / 2) / \tan(\theta_i / 2) )</td>
<td>Eccentricity ( e_i = M_i + 1 / M_i - 1 )</td>
</tr>
<tr>
<td>Secondary-focal length (m)</td>
<td>( 2c_i = \frac{x_{a_i}}{\tan(\phi_i)} + \frac{x_{u_i}}{\tan(\theta_i)} )</td>
<td>Semi-major axis distance (m) ( a_i = \frac{c_i}{e_i} )</td>
</tr>
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</table>

As shown in Table 1, the relative parameters of shaped dual-reflector can be obtained based on the property of the classical dual-reflector system.

**Optical-Path-Difference Caused by Structural Deformation**

The Optical-Path-Difference (OFPD) \( \delta \) of aperture plane caused by antenna structural deformation is the sum of the OFPDs due to the deformation of the primary reflector \( \delta_p \), the offset of subreflector \( \delta_s \), and the displacements of the feed \( \delta_f \). The OFPD \( \delta_p \) of the deformation of the primary reflector is defined as follows:

\[
\delta_p(r_i, \phi_i) = \mathbf{c}_{pi} \cdot \mathbf{u}_{pi}
\]

where the displacement vector of primary surface is \( \mathbf{u}_{pi} = (\Delta x_i, \Delta y_i, \Delta z_i) \). The components of the coefficient vector in Eq. (3) are \( c_{p1} = -2n_{u_i}n_{a_i} \), \( c_{p2} = -2n_{y_i}n_{a_i} \), \( c_{p3} = -2n_{z_i}^2 \), and \( n_{a_i}, n_{y_i}, n_{z_i} \) signify the components of a unit vector normal. The OFPD \( \delta_s \) of the displacements of the feed is defined as follows:

\[
\delta_s(r_i, \phi_i) = \mathbf{c}_{fi} \cdot \mathbf{u}_{fi}
\]

where the displacement vector of the feed is \( \mathbf{u}_{fi} = (\Delta x_f, \Delta y_f, \Delta z_f) \). The components of the coefficient vector in Eq. (4) are \( c_{f1} = -\sin \theta \cos \phi \), \( c_{f2} = -\sin \theta \sin \phi \), \( c_{f3} = 1 - \cos \theta \). The OFPD \( \delta_i \) of the offset of the subreflector is defined as follows:

\[
\delta_i(r_i, \phi_i) = \mathbf{c}_{si} \cdot \mathbf{p}
\]

where the offset vector of the subreflector is \( \mathbf{p} = (\Delta x_s, \Delta y_s, \Delta z_s, \Delta y_f, \Delta z_f)^T \); The components of the coefficient vector in Eq. (5) are \( c_{si1} = (\sin \theta_i - \sin \phi_i) \cos \phi_i \), \( c_{si2} = -\sin \theta_i \cos \phi_i \), \( c_{si3} = -(\cos \phi_i + \cos \theta_i) \), \( c_{si4} = -(c_i - a_i)(\sin \theta_i + M_i \sin \phi_i) \sin \phi_i \), \( c_{si5} = -(c_i - a_i)(\sin \theta_i + M_i \sin \phi_i) \cos \phi_i \).

**MEFCM for Shaped Reflector**

The geometric diagram of dual-reflector antenna is shown in Fig.3, and the MEFCM for shaped dual-reflector can be expressed as follows:

\[
T(\theta', \phi') = \sum_{i=1}^{m} F(r_i, \phi_i) \exp[jkr_i \sin \theta' \cos(\phi' - \phi_i)] \cdot \exp(jk\delta_i) \Delta s_i
\]
where \( F(r, \theta) \) is the aperture field distribution of shaped reflector; \( k \) is the free space wave constant, \( k = 2\pi / \lambda \); \( \delta \) is the OPD of point \( i \) in aperture plane. The Gauss Integration method can be adopted to solve the MEFCM.

![Figure 3. Geometric diagram of dual-reflector antenna.](image)

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![Figure 4. A 25 m shaped Cassegrain antenna characteristic: (a). Cassegrain generatrix; (b). curve of relative parameters.](image)

Figure 4. A 25 m shaped Cassegrain antenna characteristic: (a). Cassegrain generatrix; (b). curve of relative parameters.

![Figure 5. OPD distributions and beam patterns: (a) and (b) are OPD distributions of subreflector 0.1\( \lambda \) x-offset and 0.1\( \lambda \) z-offset; (c) and (d) are beam patterns of subreflector 0.1\( \lambda \) x-offset and 0.1\( \lambda \) z-offset.](image)

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Analysis and Discussion

In order to verify effectiveness, the MEFCM for shaped reflector is adopted in the analysis of the structural deformation of a 25 m Cassegrain antenna operating with a wave length $\lambda$ of 0.06m, and the antenna characteristic is shown in Fig.4. The basic normal parameters of the model include: primary reflector diameter-25m, subreflector diameter-3m, half-angles subtended of primary reflector and sub-reflector-79.61° and 14.43°, paraboloid focal length-7.8m, hyperboloid focal length-0.53m, eccentricity-1.358, magnification-6.583. As shown in Fig.5, the relative parameters (eccentricity, paraboloid focal length, magnification, offset) are different over aperture, and vary with radius.

As shown in Fig.5, the MEFCMs of two types of subreflector offsets, 0.1$\lambda$ x-offset and 0.1$\lambda$ z-offset, are analysed respectively. The aperture OPD distribution caused by x-offset (lateral defocus) shows asymmetry and the beam pattern shows pointing deviation. The aperture OPD distribution caused by z-offset (axial defocus) shows symmetry and the beam pattern shows large distortion.

Conclusion

The proposed approach is able to accurately describe the shaped dual-reflector surface as standard discrete parabola set and the feature of classical Cassegrain system can be used in OPD relationship, and it realizes the rapid solution of MEFCM for shaped dual-reflector antenna. The analysis of MEFCM for a 25 m shaped Cassegrain antenna demonstrated the feasibility and effectiveness of the approach.

Acknowledgments

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