Option Arbitrage and Quantification Realization Based on Time Value

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Abstract. Option is a very important financial derivative instrument, with hedging, speculation, price discovery and other functions. Based on the option parity formula, this paper finds out that the time value of the option has arbitrage opportunity by studying the spot price of the 50ETF and 50ETF fund. According to the relationship between the time value of the option and the trend of the subject matter, some arbitrage strategies are developed. The result of the last back test shows that these strategies have a good return.

Introduction

Approved by China securities regulatory commission (CSRC), Shanghai stock exchange listed and traded the 50ETF option contract on February 9, 2015, which means that China's financial system has been further improved and has truly entered the trading era of options. China's option market is still an emerging market with immature development, and its activity is far less than that of foreign options market. Therefore, Chinese options have more arbitrage opportunities and it is easier to take advantage of the arbitrage opportunities to gain profits.

Option value is composed of connotation value and time value. Connotation value refers to the immediate profit obtained from the exercise of the option. The time value of the option is the part of the option value minus the connotation value. The traditional definition of arbitrage refers to the activity of profiting by buying and selling the same subject matter at the same favorable price in different markets at the same time. Options portfolio trading strategies include buy call option to sell the subject matter, buy put option to sell the subject matter.

Literature Review

Yuan (2014) introduced the European option parity formula, as well as the application of the classic black-schole model and binary tree model in European and American options. And then synthesized long and short of the subject matter with call and put options, buy long and sell short, found risk-free arbitrage opportunities. Han (2015) conducted empirical studies on bullish call options, bullish put options, bearish call options and bearish put options through soybean meal option data, and found that there were arbitrage opportunities. Based on the option parity formula, Qian and Zhu (2016) deduced the option arbitrage spread formula. Taking the 50ETF option as an example, some combination cases that made the option arbitrage spread greater than 0 were found through experimental pairing. Miao (2016) introduced the actual strategies of futures companies on simulated options, and pointed out that there was not much innovation in relevant arbitrage strategies, and a certain profit could be obtained by using traditional arbitrage strategies.

From domestic and foreign studies, the arbitrage of options is more based on the pricing formula and original definition of options. This paper tries to study the option arbitrage based on time value, because the change of time value is largely based on the residual expiration time, so the arbitrage risk is more controllable and the risk is smaller. The arbitrage in this paper is based on the difference of the time value of the option, which uses the put option and the buy subject matter to synthesize the call option, and a certain strike price call option on the same expiration date to carry out the arbitrage to buy the cheap price, while the standard to judge the value of time is the value of time. This process through the Portland quantitative trading, completely objective.
Research Method

The theoretical basis of arbitrage is the parity formula: \( C + X e^{-rt} = P + S \), where \( C \) stands for the call option price, \( X \) stands for the strike price, \( r \) stands for the discount rate, \( t \) stands for the expiration time of the option, \( P \) stands for the put option price and \( S \) stands for the subject matter price. In the actual transaction process time is shorter, \( X e^{-rt} \) can be ignored. Therefore, the parity formula can be changed to \( C = P + S \).

Table 1. Variable comment table.

<table>
<thead>
<tr>
<th>Time</th>
<th>The price of the subject matter</th>
<th>Firm price</th>
<th>The value of time</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>( S_{t1} )</td>
<td>( X )</td>
<td>( P_{t1} )</td>
</tr>
<tr>
<td>T2</td>
<td>( S_{t2} )</td>
<td>( X )</td>
<td>( P_{t2} )</td>
</tr>
</tbody>
</table>

The profit is calculated by buying the call option and selling the put option and the subject matter. Case 1: the call option and the put option are all real value options

\[
\Delta H = \Delta C + \Delta P + \Delta C = (S_{t2} - S_{t1}) - (S_{t1} - x_c) + (C_{t2}^T - C_{t1}^T) + (x_p - S_{t1}) - (x_p - S_{t2}) + (P_{t1}^T - P_{t2}^T) + (S_{t1} - S_{t2})
\]

Where \( A \) stands for the price difference between the late period and the early period, and \( B \) stands for the difference between the time value of the late bullish put option and the value of the earlier period.

Case 2: the call option is the dummy option and the put is the real option

\[
\Delta H = \Delta C + \Delta P + \Delta C = (C_{t2}^T - C_{t1}^T) + (S_{t2} - S_{t1}) + (P_{t1}^T - P_{t2}^T) + (S_{t1} - S_{t2})
\]

\[
\Delta H = \Delta C + \Delta P + \Delta C = (C_{t2}^T - C_{t1}^T) + (P_{t1}^T - P_{t2}^T) - (C_{t1}^T - P_{t1}^T)
\]

\[
\Delta H = \Delta C + \Delta P + \Delta C = A + B
\]

The case that the call option is the real value option and the put is the virtual value option is the same as the above result.

Case 3: The call and put options are both phantom options

\[
\Delta H = \Delta C + \Delta P + \Delta C = (C_{t2}^T - C_{t1}^T) + (P_{t1}^T - P_{t2}^T) + (S_{t1} - S_{t2})
\]

\[
\Delta H = \Delta C + \Delta P + \Delta C = (S_{t1} - S_{t2}) + [(C_{t2}^T - P_{t2}^T) - (C_{t1}^T - P_{t1}^T)]
\]

\[
\Delta H = \Delta C + \Delta P + \Delta C = -A + B
\]

According to the comparison of the above situations, the final arbitrage result is irrelevant to the price of the subject matter only when one of the bullish and bearish options is the real value and the other is the imaginary value, so the arbitrage adopts this combination.

Empirical Analysis

Data Selection

The data of option contracts whose expiration time is December 2015, March 2016, June 2016 and March 2017 are selected. Based on the amount of option strike pricing issued in that month, the daily line data of the same call and put options with different logarithms are selected. In June 2016, four pairs of call put options with the same price at 2.15, 2.20, 2.6 and 2.65 were selected. In March 2017, four pairs of call put options with the same price at 2.05, 2.10, 2.15 and 2.25 were selected. Three equal strike and put options were selected in March 2016 and December 2015.
Data Analysis

Since there are many options contracts, only three typical options pairs are selected in this paper. The strike price is 1.95 call put option in December 2015, 2.15 call put option in June 2016 and 2.15 call put option in March 2017. The difference in time value is the time value of the call option minus the time value of the put.

Figures for 2016 and 2017 were omitted due to space limitation. It can be seen from these graphs that the time value difference and the price of 50ETF are in a positive ratio.

![Figure 1. December 2015 expiration option prices, time value difference and ETF price trend.](image1.png)

Stationarity Test. Set the Shanghai 50ETF price as Y, the call option price as X1, and the put option price as X2. For example, the December 2015 combination.

![Figure 2. Price stability test.](image2.png)

Under the confidence level of 0.05, the price of 50ETF, call options and put options are all zero order integrals, indicating that all three time series are stable. As for the June 2016 combination and the March 2017 combination, the situation is the same.

Cointegration Test. The price of call option, put option and 50ETF are all stable. The precondition of arbitrage is that there is a long-term stable equilibrium relationship between the prices of the three, so it needs to carry out co-integration test. The three regressions are the return of the put option to the call option, the return of the ETF to the call option and the return of the put option to the ETF. For example, the December 2015 combination.

![Figure 3. Co-integration test.](image3.png)

It can be seen from the test results that at the confidence level of 0.05, the residuals are all zero order integral. It shows that there is a long-term stable equilibrium relationship among the three variables. As for the June 2016 combination and the March 2017 combination, the situation is the same, there is a long-term stable equilibrium relationship among the three variables.

Quantitative Implementation of Policies. After determining that there is a stable long-term equilibrium relationship between all three variables, arbitrage can be conducted. The time value
arbitrage strategy can be profitable on the premise of judging the correct price trend. Sell the subject matter and put options and buy the call option when the subject matter is short. The signal is required to be stable, not to be able to stop loss and gain in a timely manner, as long as it is guaranteed to filter out small consolidation areas and judge the general direction.

A total of 112,000 yuan was invested from August 2015 to March 2017, with a maximum of 18,568 yuan being used. And net income of 16440 yuan, yield 14.68%.

Comparison of Results between Different Periods and Pricing Options. On the daily line cycle, the profit and loss of holding position changes greatly, so try arbitrage on a 30-minute cycle, and also try arbitrage between different strike prices and different expiration months, and compare it with the arbitrage method above.

As for the same month: the effect of 30 minutes was not good, and the benefit was -7990. As for the different month: the 30-minute period of the option with the same strike pricing for different months in March and June 2016 was selected for testing.

Summary

This paper carries out arbitrage from the perspective of time value and studies the option parity formula from another perspective. Using a real value option and an imaginary value option makes the final return only contain the time value. When the false value of the right becomes the real value in the period when the price of the subject matter changes, the corresponding real value option will also become the virtual value option because the strike price is the same. To some extent, the final
return only contains the time value. When compared to 30 minutes, 30 minutes is not very profitable. And the effect of the same strike pricing option on different expiration months is not very different from that of the same strike pricing option on the same month. Due to the short position in the 30-minute time scale, the time value does not change enough time.

Although the research results are satisfactory, there are still many shortcomings and problems. For example, the judgment of the trend of the subject matter is purely random, which is closely related to the trading signal, so it is not a risk-free arbitrage in the strict sense.

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