Application of Generalized RBF Network Based on K-means Clustering in Solving Complex Mappings

Xun-lai HE¹, Jun-hui YIN²*, Wei-zhao ZHANG² and Zhen-qian YANG³

¹Nantong Institute of Technology, Jiangsu China
²Shijiazhuang Campus of Army Engineering University, Hebei China
³Xi’an Satellite Measurement and Control Center, Shanxi China

*Corresponding author

Keywords: K-means, Neural network, Initial disturbance, Uniform design, Complex mapping.

Abstract. Initial perturbation is one of the important sources of the distribution of projectile dropping points. However, due to the complex structure of the virtual prototyping system of coupling system, there is no clear functional relationship between the parameters of the initial disturbance index and its influencing parameters. It is difficult to establish scientific mapping relationship. In this paper, for the complex mapping problem of multi-objective optimization of initial perturbation, based on K-means clustering generalized RBF network, the nonlinear mapping relationship between initial disturbance index parameters and its important influence parameters is established. Uniform design is used to establish a virtual shooting test scheme, and a generalized radial basis neural network is used to solve complex mappings between initial disturbance index parameters and important influencing parameters, thereby providing an important basis for initial disturbance optimization.

Introduction

Theoretical and experimental studies have shown that the initial disturbance is the result of the combined effects of various complex factors such as bombs, guns, gunpowder, and the environment during the movement of the radon. From the structural parameters of the missile-gun coupling system, multiple parameters that have a great influence on the initial disturbance are selected as the parameter vector. Because the structure of the virtual prototype of the missile-gun coupling system is very complicated, it is difficult to establish a comparison between the initial disturbance index parameter and its influencing parameters.

In this paper, based on the K-means clustering-based Radial Basic Function (RBF) neural network, the initial perturbation parameters and their important influences are established for the complex nonlinear mapping problem in multi-objective optimization of initial perturbation. The RBF neural network is proposed by Moody and Darken based on the local response characteristics of biological neurons. It is suitable for solving nonlinear mapping problems [1]. It has been successfully applied to data classification, system modeling and control, pattern recognition, and nonlinearity. Function approximation, fault diagnosis and other fields [2]. The generalized RBF network uses the radial basis function as the "base" of the hidden unit to form the hidden layer space. The low-dimensional space model is transformed into the high-dimensional space by the input vector transformation so that the linear inseparability problem in the low-dimensional space is in the high-dimensional space. First, establish a virtual shooting test scheme by uniform design, and then extract the initial disturbance index parameters and important influence parameter sample database to provide a data source for generalized RBF neural network training, and use the generalized radial basis neural network to solve complex mapping between initial disturbance index parameters and important influences parameters.

Generalized RBF Network based on K-means Clustering

The radial basis function is usually defined as a monotonic function of the Euclidean distance from
any point in space to a certain center. The activation function takes the distance between the input vector and the weight vector \( \| \text{dist} \| \) as the independent variable, and the expression is

\[
R(\| \text{dist} \|) = e^{-\frac{\text{dist}^2}{2\sigma^2}}
\]  

(1)

The basic forms of radial basis functions mainly include Gauss functions, Reflected Sigmoidal functions, and Inverse multi-quadratics functions. Since the Gauss function has many advantages such as good resolution and smoothness, radial symmetry, and the existence of an arbitrary order derivative [3], it is used as a radial basis function and its expression is

\[
\varphi(r) = \exp\left(-\frac{r^2}{2\sigma^2}\right)
\]  

(2)

The activation function of the radial basis network can be expressed as

\[
R(x_p - c_i) = \exp\left(-\frac{1}{2\sigma^2}\| x_p - c_i \|^2 \right)
\]  

(3)

In formula (3), \( \| x_p - c_i \| \) is Euclidean norm, \( c_i \) is Gaussian function center, \( \sigma \) is Gaussian function variance.

The network output is obtained from a radial-based network structure

\[
y_j = \sum_{i=1}^{h} w_{ij} \exp\left(-\frac{1}{2\sigma^2}\| x_p - c_i \|^2 \right) \quad j = 1,2,\ldots,n
\]  

(4)

In formula (4), \( x_p = (x_{p1}, x_{p2}, \ldots, x_{pn})^T \), that is the NO. \( P \) input sample, \( p = 1,2,\ldots,P \), \( P \) is the total number of samples, \( c_i \) is hidden node center, \( i = 1,2,\ldots,h \), indicates the number of hidden layer nodes, \( w_{ij} \) is the connection weight from the hidden layer to the output layer, \( y_j \) is The actual output of NO. \( j \) output node of the network corresponding to the input sample.

Assuming that \( d \) is the expected output value, the basis function variance is expressed as

\[
\sigma = \frac{1}{P} \sum_{j=1}^{n} \| d_j - y_j c_i \|^2
\]  

(5)

The K-means clustering algorithm proposed by Duda and Hart uses the self-organizing clustering method to determine the appropriate data center for the hidden base node's radial basis function [4]. The advantage is that it can be implicitly determined according to the distance between clustering centers. The expansion constant of the node is high in computational efficiency. Since the clustered data center is not the sample data \( X_p \), so \( c(k) \) represents the center at NO.\( k \) time of the iteration. The process of using the K-means clustering algorithm to determine the data center is as follows,

**Step 1.** Initialization and assignment. Select \( M \) mutual vectors as the initial cluster center: \( c_1(0), c_2(0), \ldots, c_M(0) \), a method of assigning a small random number to each cluster center vector may be used when selecting.

**Step 2.** Euclidean distance calculation. Solving the Euclidean distance between each sample point of the input space and the cluster center \( \| X_p - c_j(k) \| \), \( p = 1,2,\ldots,P \), \( j = 1,2,\ldots,M \).

**Step 3.** Similar matches. Let \( j^* \) represent the subscript of the winning hidden node, and determine the classification \( j^*(X_p) \) of each input sample \( X_p \) according to its minimum Euclidean distance from the cluster center, when there is the following equation,
\[ j^*(X^p) = \min_j \|X^p - c_j(k)\| \quad p = 1, 2, \ldots, P \]  

\( X^p \) is classified as NO. \( j^* \) category so that all samples are divided into \( M \) subsets: \( U_1(k), U_2(k), \ldots, U_M(k) \). Each subset constitutes a clustering domain that is typically represented by a clustering center.

**Step4.** Update all types of cluster centers and average samples in each cluster domain. Let \( U_j(k) \) represents NO. \( J \) cluster domain, \( N_j \) is number of samples in the NO. \( J \) clustering domain, so

\[ c_j(k+1) = \frac{1}{N_j} \sum_{X \in U_j(k)} X \]  

**Step5.** Add 1 to \( k \) value and go to Step2. Repeat the above process until changes of \( c_k \) less than the required value.

After each cluster center is determined, the corresponding spreading constant of the radial basis function can be determined according to the distance between the centers. Definition

\[ d_j = \min_i \|c_j - c_i\| \]  

The extension constant is \( \delta_j = \lambda d_j \), where \( \lambda \) is the overlap factor.

**Mapping Relationship between Initial Disturbance Index and Influencing Parameters**

From the structural parameters of the missile-gun coupling system, 9 parameters with great influence on the initial disturbance are selected, namely the parameter vector. \( \{ L_{ny}, L_{my}, K_{gd}, L_{mc}, K_{dd}, C_{dd}, L_d, l_b, m \} \), there is no definite functional relationship between the projectile perturbation indicator parameter and the vector, so it is difficult to establish a more scientific mapping relationship.

**Method of Constructing Complex Nonlinear Mappings**

Firstly, a virtual shooting test scheme is established by uniform design, and then a complex mapping between initial disturbance index parameters and important influence parameters is solved based on K-means clustering generalized RBF network. As shown in Fig.1, the specific implementation method can be expressed as,

**Step1.** Determine the basic parameters such as the gun firing angle, charge conditions, weather conditions, projectile landing spot sample capacity (the virtual shooting times \( n \), determined in the uniform design), and assume that the projectile posture is not interfered by the mouth flow field.

**Step2.** Uniformly design the important influence parameters of the initial disturbance of the projectile, establish a virtual test scheme for the missile-gun coupling system, and obtain \( n \) sets of input samples required for the training of the RBF neural network.

**Step3.** According to the test plan, run the missile-coupled virtual prototype under different conditions, extract the longitudinal component \( \delta_{01} \) and lateral component \( \delta_{02} \) of the initial attack angle of the projectile, the vertical component \( \phi_{01} \) and horizontal component \( \phi_{02} \) of the swing angular velocity, and other four indicators, and obtain the group output sample.

**Step4.** Use the generalized RBF neural network to train the input and output samples obtained in Step2 and Step3 to solve the nonlinear mapping relationship between the 4 indicator of the initial perturbation of the projectile and the 9 important influence parameters.

**Uniformly Designed Virtual Shooting Test Sample Library Extraction**

In order to compress the number of virtual shooting tests and obtain a good mapping relationship, the
global variation law of the initial perturbation of the projectile is reflected with fewer samples and their combinations. The virtual shooting test program is established based on the uniform design method to extract the initial perturbation index parameters of the projectile. The important influence parameters sample database provides data sources for generalized RBF network training.

Uniform design theory is widely used in many fields such as aviation industry, automobile industry[5], medical and health[6], defense industry[7], and is more suitable than orthogonal design for multi-factorial and multi-level tests.

According to the range of values of various important influence parameters of the initial disturbance of the projectile in Table 1, each parameter takes 17 levels, using 9 columns in the uniform design table $U_{17}(17^{16})$ to establish a 9 factor 17 level test scheme to ensure the minimum number of tests and the test arrangement is the most uniform.

<table>
<thead>
<tr>
<th>Influences parameter</th>
<th>$l_b$ (mm)</th>
<th>$K_{dd}$ (N/mm)</th>
<th>$C_{dd}$ (N·s/mm)</th>
<th>$L_{my}$ (mm)</th>
<th>$L_{nc}$ (mm)</th>
<th>$m$ (kg)</th>
<th>$L_F$ (mm)</th>
<th>$K_{gd}$ (N/mm)</th>
<th>$L_{My}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial value</td>
<td>222</td>
<td>79313.7</td>
<td>0.044</td>
<td>0.000</td>
<td>0.000</td>
<td>43.56</td>
<td>0.000</td>
<td>15750</td>
<td>0.000</td>
</tr>
<tr>
<td>Upper limit</td>
<td>180</td>
<td>50000</td>
<td>0.000</td>
<td>-0.500</td>
<td>-0.500</td>
<td>42.25</td>
<td>-5.000</td>
<td>10000</td>
<td>-5.000</td>
</tr>
<tr>
<td>Lower limit</td>
<td>260</td>
<td>100000</td>
<td>0.100</td>
<td>0.500</td>
<td>0.500</td>
<td>44.87</td>
<td>5.000</td>
<td>20000</td>
<td>5.000</td>
</tr>
</tbody>
</table>

In order to facilitate the analysis of the optimization effects of the initial disturbance's important influence parameters, the actual structural parameters of the missile-gun coupling system can be based on the parameters given in Table 1. Initial value set of tests was conducted. Therefore, the virtual shooting test considering the initial disturbance was performed 18 times in total, and the calculated initial disturbance indicator parameters of the projectile were taken as the neural network output sample database. Radial basic function (RBF) neural network is used to obtain the nonlinear mapping relationship between initial disturbance index parameters and important influencing parameters.

Solving Complex Nonlinear Mapping Relationship

Because the physical quantities in the input and output samples all have different dimensions and physical meanings, and the range of values varies greatly, for example, the distance between projecting center and elastic belt center $l_b$ varies from 180 to 260. The equivalent spring stiffness $K_{dd}$ in the model ranged from 50,000 to 100,000. In order to improve the training accuracy, the sample is scaled and standardized, and the data in the network is limited to [0, 1] or [-1, 1] intervals. The calculation method is

$$x_{mid} = \frac{x_{max} + x_{min}}{2}, \quad \bar{x}_i = 2 \frac{x_i - x_{mid}}{x_{max} - x_{mid}}$$  \hspace{1cm} (9)

In equation (9), $x_i$ represents the pre-transformation data, $x_{max}$ and $x_{min}$ are the maximum and minimum values in the data variation range, $x_{mid}$ represents the center value of the data variation interval.

According to above algorithm, the data center of each radial basis function is calculated. After several trials, 12 cluster centers are identified as hidden nodes, and 9 important parameters of the initial disturbance of the projectile to be optimized are taken as the input nodes of the RBF network, and 4 index parameters $\delta_{01}$, $\delta_{02}$, $\phi_{01}$, $\phi_{02}$, are used as output nodes.

The initial perturbation index parameter value is obtained as the output sample database through the simulation of the artillery coupling system dynamics. Based on this, the input and output sample
database of input node 9, hidden node 12 and output node 4 of the generalized RBF network are constructed for training. The connection weight value $w_{ij}$ from the layer to the output layer and the threshold parameter $B_x$ of each output node, end the training, when the output error satisfies the preset target error.

Figure 1 shows the network training process. Its standard error rapidly declines within the first few generations of evolution. After the evolution of 10 generations, the error elimination is more than 90%. After 100 generations, the error variation decrease to 1.95E-003, so the network converges.

After the training was completed, the test samples were used for testing. The results showed that the fitting effect was better. Finally, the threshold parameter [1.0715, 0.5899, 0.6073, 0.3943] is obtained. From this, nonlinear mapping relationship can be determined between the initial perturbation index parameters and important influence parameters of the projectile.

**Figure 1.** The convergence process of generalized RBF network training error.

**Summary**

In this paper, the virtual shooting test scheme is established by the uniform design method, and the initial perturbation index parameters and important influence parameter sample libraries of the projectiles are extracted. The generalized RBF neural network based on K-means clustering algorithm is used to solve complex mapping between the initial perturbation index parameters and important influence parameters.

**Acknowledgements**

Thanks for support of the following fund projects,

[1] ‘Thirteen Five’ Jiangsu Province, a subject key provincial construction discipline project (2016-0802 Mechanical Engineering)

[2] Jiangsu university brand professional construction project funded projects (PPZY2015C251 mechanical design and manufacture and automation)

**References**


