Decision-making Method for an Ordered Information System Based on Multi-granulation Rough Sets

Jiajia Zhang, Zhengkui Lin, Xuesheng Liu and Qing Dai

ABSTRACT

In this paper, we study an ordered problem of objects with multi-attributes based on granulation computing (GrC) for an ordered information system (OIS) and develops a novel multi-granulation rough set (MGRS) named the compromise multi-granulation rough set (CMGRS). The CMGRS is a generalized model of the classical MGRS. This new tool has a good weak transitivity and can be used to address ordered problems of objects in an OIS. Based on this novel MGRS, we construct a new approach for the ordered problem of objects for an OIS. This research introduces a method and discusses its characteristics. The results from an example investment project and the 2007 sustainability report indicate that our new approach is significant and efficient and achieves optimal results.

INTRODUCTION

In general, an information system (IS) with multi-attribute decision objects and a partial ordering of the attribute domain is called an ordered information system (OIS). Many decision problems represent ranking problems for an OIS. The ranking problem is an important branch of the decision problem. The OIS is not a serially ordered set; thus, many ranking methods are proposed [1].

Jiajia Zhang\(^1\), Zhengkui Lin\(^1\), Xuesheng Liu\(^2\) and Qing Dai\(^1\),
\(^1\)Shipping Economy and Management College, Dalian Maritime University, Dalian, China, 116026
\(^2\)College of Information Engineering, Dalian University, Dalian, China, 116622
Corresponding Author: Zhengkui Lin, emai: dalianjx@163.com
In practical problems, data may have preference information as well as inaccurate and ambiguous information. In such a scenario, the classic rough set method is not adequate. To handle information systems with partial ordering relationships, Greco [2] proposed a rough set model based on dominant relationships instead of equivalent relationships. In recent years, because of the rough set model’s many advantages, it has been widely used for the sequencing problem of the OIS. Many scholars have conducted a series of studies on ranking methods based on dominant relationships. Li et al. [3] presented a possibility ranking method with a superior degree matrix. According to the research of the ranking methods in an incomplete interval valued IS, Zhang et al. [4] proposed a ranking method with an improved possibility degree dominant relationship to solve the problem in which too many attributes may result in the ranking’s failure. This ranking method combined the interval-valued dominant relationship with the possibility degree, improved the weakness of the definition of the classic dominant relationship and defined the average comprehensive dominance degree. Moreover, the structure of the advantage matrix satisfies the complementary symmetry. Additionally, a method that accounts for the relative dominance degree was proposed by Duan et al. [5] for the multi-granularity rough set based on a dominant relationship. As IS research deepens, scholars have begun to focus on the attribute value of the object, which is fuzzy. To conduct decision analyses in a fuzzy situation, many scholars have begun to research ranking methods based on fuzzy sets. Wang et al. [6] proposed a multi-attribute decision-ranking method under the intuitionistic fuzzy environment combined with the intuitionistic fuzzy weighted average operator. The calculation formulas of the centroid positioning of the polygonal fuzzy number and the criterion of the index ranking were proposed by Wang et al. [7]. Then, according to the centre-weighted average rule and the horizontal and vertical centre formula of a trapezoidal fuzzy number, these authors [8] proposed an index ranking method.

The above discussion indicates that many scholars are studying the ranking methods of IS. However, in real life, many problems remain, and additional investigations are urgently needed to develop and perfect the ranking method theories. For each attribute, the rough set (RS) produces a cognitive model with an attribute dominant class based on known ranking relationships of OIS. We will describe a ranking object set, and then a multi-attribute multi-granulation OIS (MMGOIS) is constructed with respect to multiple attributes. We compared the information from decision objects that have multiple dominance characteristics. By using the ranking approach of RS, we rank each object. A novel ranking approach is thus obtained for the OIS and is called the ranking approach of the multi-granulation rough set (MGRS) [9]. This paper is organized as follows. Section 2 reviews the basic concepts of the MGRS of a classical ordered information system. Section 3 introduces the compromise MGRS of the OIS. Section 4 presents a new ranking method of the CMGRS of the OIS. Section 5 provides an example to demonstrate the validity and effectiveness of our new CMGRS ranking method for the OIS. Section 6 concludes the paper with a summary and directions for future work.
The optimistic and pessimistic MGRSs are defined by multiple dominant relationships in the OIS.

**Definition 2.1.** A ternary \( \Gamma = (U, AT, F) \) is an OIS in which \( U \) is a non-empty finite set of objects called the universe; \( AT \) is a non-empty finite set of attributes, and a dominant relationship is induced by a partial ordering that exists in the domain of each attribute; and \( F \) is a mapping from \( U \) to \( V_a \), and \( F = \{ f \mid f : U \rightarrow V_a, a \in AT \} \), where \( V_a \) is the finite domain of attribute \( a \).

**Definition 2.2.** Let \( \Gamma = (U, AT, F) \) be an OIS, \( A_1, A_2, \ldots, A_s \subseteq AT \), and \( AT \) be an attribute set. \( R^s_1, R^s_2, \ldots, R^s_s \) are the dominant relationships, and \( X \in P(U) \). The multi-granulation lower and upper approximation operators of \( X \) of the OIS are defined by

\[
\text{OM}^s_{\sum A_i}(X) = \left\{ \bigvee_{i=1}^{s} [x]_{R^s_i} \subseteq X \right\}
\]

(1)

\[
\overline{\text{OM}}^s_{\sum A_i}(X) = \left\{ \bigwedge_{i=1}^{s} [x]_{R^s_i} \cap X \neq \phi \right\}
\]

(2)

where \( s \leq 2^{k+1} \), \( P(U) \) is a power set of \( U \), and \( [x]_{R^s_i} \) is called the \( A_i \)-dominating class of the dominant relationship \( R^s_i \) in each \( A_i \) of \( AT \).

Similarly, the pessimistic multi-granulation lower and upper approximation operators of \( X \) of the OIS are defined as follows:

\[
\text{PM}^s_{\sum A_i}(X) = \left\{ \bigwedge_{i=1}^{s} [x]_{R^s_i} \subseteq X \right\}
\]

(3)

\[
\overline{\text{PM}}^s_{\sum A_i}(X) = \left\{ \bigvee_{i=1}^{s} [x]_{R^s_i} \cap X \neq \phi \right\}
\]

(4)

Based on the above descriptions, a target concept can be approximated by using two different approximating strategies: seeking the common reserving difference and seeking the common rejection difference in the optimistic and pessimistic MGRSs of the classical OIS. We also construct the structure of the granular space of the target concept [11].

In many practical problems, decision makers must take advantage of available decision information with many complex decision environments. Therefore, the classical MGRS must be extended. In the following, we will extend the classical MGRS to compose the CMGRS of the OIS.
COMPROMISE MULTI-GRANULATION ROUGH SET OF OIS

**Definition 3.1.** Let \( \Gamma = (U, AT, F) \) be an OIS, \( A_1, A_2, \ldots, A_r \subseteq AT \), and \( AT \) be an attribute set. \( P_r(AT) \) represents \( r \) attributes \( A_1, A_2, \ldots, A_r \) of \( AT \), where \( |P_r(AT)| = C_{r+1}^m \). For an arbitrary group of attributes \( A_1, A_2, \ldots, A_r \), that can be written as \( AT \subseteq AT \), \( R_{A_1}, R_{A_2}, \ldots, R_{A_r} \) are the dominant relationships. Moreover, for \( x \in P(U) \), the CMGRS of the lower approximation and upper approximation of \( x \) with respect to \( P(U) \) is denoted by \( \overline{CM}_{ATr}(x) \) and \( \overline{CM}_{ATr}(x) \), respectively, where

\[
\overline{CM}_{ATr}(x) = \left\{ \left[ x \right]_{\alpha_r} \subseteq X \right\} \quad (5) \quad \overline{CM}_{ATr}(x) = \left\{ \left[ x \right]_{\alpha_s} \cap X \neq \emptyset \right\} \quad (6)
\]

and where \( i \in \{2, \ldots, |P_r(AT)| \} \).

**Theorem 3.1.**

If \( r = |AT| \), then \( \overline{CM}_{ATr}(x) = PM_{\Sigma_{\alpha_r}}(x) \) and \( \overline{CM}_{ATr}(x) = PM_{\Sigma_{\alpha_s}}(x) \).

If \( r = 1 \), then \( \overline{CM}_{ATr}(x) = OM_{\Sigma_{\alpha_r}}(x) \) and \( \overline{CM}_{ATr}(x) = OM_{\Sigma_{\alpha_s}}(x) \).

**Proof:** Let \( r = |AT| \). From **Definition 3.1**, the following is easily derived:

\[ CM_{ATr}(x) = PM_{\Sigma_{\alpha_r}}(x) \quad \text{and} \quad CM_{ATr}(x) = PM_{\Sigma_{\alpha_s}}(x) \]

Let \( r = 1 \). To prove \( CM_{ATr}(x) = OM_{\Sigma_{\alpha_r}}(x) \), we need to prove \( CM_{ATr}(x) \subset OM_{\Sigma_{\alpha_r}}(x) \) and \( CM_{ATr}(x) \subset OM_{\Sigma_{\alpha_s}}(x) \).

Now, we prove that \( CM_{ATr}(x) \subset OM_{\Sigma_{\alpha_r}}(x) \).

For all \( x \in OM_{\Sigma_{\alpha_r}}(x) \), i.e., \( \forall A_i, i \in \{1, 2, \ldots, s\} \), therefore, \( \left[ x \right]_{A_i} \) is not included in \( X \) and \( \left[ x \right]_{A_i} \subseteq X \) can be obtained. Thus, \( x \in CM_{ATr}(x) \), or \( -OM_{\Sigma_{\alpha_r}}(x) \subset CM_{ATr}(x) \). Hence,

\[ CM_{ATr}(x) \subset OM_{\Sigma_{\alpha_r}}(x) \]

Then, we prove that \( CM_{ATr}(x) \subset OM_{\Sigma_{\alpha_s}}(x) \). If \( x \in CM_{ATr}(x) \) (i.e., \( x \notin CM_{ATr}(x) \)) and \( \forall A_i, i \in \{1, 2, \ldots, s\} \), then \( \left[ x \right]_{A_i} \subseteq X \) is not observed in which \( i = 1, 2, \ldots, s \).

Moreover, \( x \in OM_{\Sigma_{\alpha_r}}(x) \) can be obtained. Thus, \( -OM_{\Sigma_{\alpha_r}}(x) \subset CM_{ATr}(x) \), such that

\[ CM_{ATr}(x) \subset OM_{\Sigma_{\alpha_s}}(x) \]

In summary, \( CM_{ATr}(x) = OM_{\Sigma_{\alpha_s}}(x) \) can be obtained, which is similar to the proof of \( CM_{ATr}(x) = OM_{\Sigma_{\alpha_r}}(x) \). This completes the proof.
**Definition 3.2** Let $\tau = (U, AT, F)$ be an OIS. Then, we can define a dominant class in which all objects are at least as good as the object $x_i$ with respect to the attributes $AT_i$ in the universe. This class is written as $[x_i]_{AT_i}^p$, although it is occasionally written as $[x_i]_{AT_i}$.

$$[x_i]_{AT_i}^p = \{x_j \mid f_{A_1}(x_i) \leq f_{A_1}(x_j), \ldots, f_{A_r}(x_i) \leq f_{A_r}(x_j)\}$$

(7)

where $[x_i]_{AT_i}^p$ is the arbitrary $r$ attributes $A_1, A_2, \ldots, A_r$ of the attribute set, such that the object $x_j$ is at least as good as the object $x_i$ with respect to the attributes $A_1, A_2, \ldots, A_r$.

**Theorem 3.2.**

$[x_i]_{AT_i}^p$ satisfies the following properties:

1) If $x_j \in [x_i]_{AT_i}^p$ for all attributes in $AT_i$, then $x_j$ is at least as good as $x_i$, and then $[x_i]_{AT_i}^p \subseteq [x_j]_{AT_i}^p$.

2) If $r < \rho$, then $[x_i]_{AT_i}^p \Rightarrow [x_j]_{AT_i}^p$.

3) $J^2 = \{[x_i]_{AT_i}^p \mid x_i \in U\}$ is a coverage of $U$.

**Proof:** Items 1)-3) are easy to verify.

**ORDERED METHOD OF CMGRS OF OIS**

Based on the CMGRS, the OIS is converted to the model of the dominant relationship. The object results can be obtained by this model.

For the dominant classes $[x_i]_{AT_i}^p$ and $[x_j]_{AT_i}^p$, we utilize the inclusion degree to calculate a degree of superiority [12], where $x_i$ outranks $x_j$ with respect to the attribute set $AT_i$.

$$R_{AT_i}(x_i, x_j) = \frac{|[x_i]_{AT_i}^p \cup [x_j]_{AT_i}^p|}{|x_i|}$$

(8)

where $|x|$ represents the number of the elements of the set $x_i, x_j \in U$.

**Theorem 4.1.** $R_{AT_i}$ satisfies the following properties:

1) $0 \leq R_{AT_i}(x_i, x_j) \leq 1$;

2) If $x_j \in [x_i]_{AT_i}^p$, then $R_{AT_i}(x_i, x_j) = 1$;

3) If $x_j \in [x_i]_{AT_i}^p$, then $R_{AT_i}(x_i, x_j) \leq R_{AT_i}(x_j, x_i)$;

4) If $x_j \in [x_i]_{AT_i}^p$, then $R_{AT_i}(x_i, x_j) \geq R_{AT_i}(x_j, x_i)$.

**Proof:** 1) For $\forall i, j$, there are always $\phi \subseteq \{[x_i]_{AT_i} \cup [x_j]_{AT_i}\} \subseteq U$. Thus, $0 \leq R_{AT_i}(x_i, x_j) \leq 1$. 

182
2) If \( x_j \in [x_{ij}]_{AT} \), then \( f_{\mathcal{A}}(x_{ij}) \geq f_{\mathcal{A}}(x_{ij}) \cdots f_{\mathcal{A}}(x_{ij}) \geq f_{\mathcal{A}}(x_{ij}) \). If \( \forall x_i \in [x_{ij}]_{AT} \) such that \( f_{\mathcal{A}}(x_{ij}) \geq f_{\mathcal{A}}(x_{ij}) \cdots f_{\mathcal{A}}(x_{ij}) \geq f_{\mathcal{A}}(x_{ij}) \), then \( x_i \in [x_{ij}]_{AT} \) and \( [x_{ij}]_{AT} \subseteq [x_{ij}]_{AT} \). We can obtain \( R_{AT} (x_i, x_j) = 1 \).

3) For \( x_j \in [x_{ij}]_{AT} \), we can obtain \( [x_{ij}]_{AT} \subseteq [x_{ij}]_{AT} \) from 2). By comparing \( R_{AT} (x_i, x_j) = \frac{1}{\mid r \mid} \sum_{x_{ij}} R_{AT} (x_i, x_j) \mid \) we obtain \( R_{AT} (x_i, x_j) \leq R_{AT} (x_i, x_j) \).

4) If \( x_j \in [x_{ij}]_{AT} \), from the above proof, we have \( [x_{ij}]_{AT} \subseteq [x_{ij}]_{AT} \), and then \( -[x_{ij}]_{AT} \supseteq [x_{ij}]_{AT} \). By comparing \( R_{AT} (x_i, x_j) \) with \( R_{AT} (x_i, x_j) \), \( R_{AT} (x_i, x_j) \geq R_{AT} (x_i, x_j) \) can easily be obtained. This completes the proof.

By using the average number of each object, a synthetic degree of superiority is calculated with respect to the \( r \) attributes as follows: \( R_{AT} (x_i, x_j) = \frac{1}{\mid r \mid} \sum_{x_{ij}} R_{AT} (x_i, x_j) \).

As the value of \( R_{AT} (x_i) \) increases, object \( x_j \) improves. We can rank all objects from big to small [13].

**Theorem 4.2** Let \( \mathcal{T} = (U, AT, F) \) be an OIS. Then, \( R_{AT} (x_i) \) satisfies the following properties: 1) \( 0 \leq R_{AT} (x_i) \leq 1 \); 2) If \( x_j \in [x_{ij}]_{AT} \), then \( R_{AT} (x_i) \leq R_{AT} (x_i) \).

**Proof:** 1) Obviously, 1) can be easily proved.

2) Now, we prove 2) by comparing \( R_{AT} (x_i) \) with \( R_{AT} (x_i) \) when \( i \neq j \). From 4) of \( \mathcal{T} = (U, AT, F) \), we know that \( R_{AT} (x_i, x_j) \geq R_{AT} (x_i, x_j) \), \( x_j \in [x_{ij}]_{AT} \). Moreover, \( [x_{ij}]_{AT} \subseteq [x_{ij}]_{AT} \). By comparing \( R_{AT} (x_i, x_j) = \frac{1}{\mid r \mid} \sum_{x_{ij}} R_{AT} (x_i, x_j) \mid \) with \( R_{AT} (x_i, x_j) = \frac{1}{\mid r \mid} \sum_{x_{ij}} R_{AT} (x_i, x_j) \mid \) we can obtain \( R_{AT} (x_i, x_j) \leq R_{AT} (x_i, x_j) \). The above information indicates that \( R_{AT} (x_i) \leq R_{AT} (x_i) \). This completes the proof.

From the above ranking approach, we obtain two points. One point is the pointed order, in which all attributes are better. The other point is that this approach utilizes third-party information to rank the objects for the dissatisfied pointed order. This approach ensures the principle of pointed order (i.e., that rationality is a main principle and that whole attributes are better) and ranks the objects of the dissatisfied pointed order. This approach is effective. A ranking approach of the multi-attribute CMGRS of the OIS is verified by the following example.

**Example 4.1.** [12] Let \( U = \{x_1, x_2, x_3, x_4, x_5, x_6\} \) represent an OIS of an investment project as shown in table 1 of the supplementary material. \( AT \) is an attribute set, where \( AT = \{a_1, a_2, a_3\} \), \( a_1 \) represents the location, \( a_2 \) represents the scale of investment, and \( a_3 \) represents the density of population. Figures in brackets represent the quantity change of the attribute value. \( V_a = \{G, \} \) represents good, \( c \) represents average, and \( b \)
represents bad, \( G \geq C \geq B \); \( V_r = \{H\} \) represents high, \( M \) represents average, and \( L \) represents low; \( H \geq M \geq L \); \( V_r = \{B\} \) represents big, \( M \) represents average, and \( S \) represents small. \( B \geq M \geq S \).

In Example 4.1., the computational process is as follows: If \( r = 2 \), then \([x_k^r]\) represents a dominant class of \( x \) with respect to an arbitrary two-level granularity: \([x_k^r] = (x_1, x_2, x_3, x_4, x_5, x_6)\), \([x_2^r] = (x_1, x_2, x_3, x_4, x_5, x_6)\), \([x_3^r] = (x_1, x_2, x_3, x_4, x_5, x_6)\), \([x_4^r] = (x_1, x_2, x_3, x_4, x_5, x_6)\), \([x_5^r] = (x_1, x_2, x_3, x_4, x_5, x_6)\), \([x_6^r] = (x_1, x_2, x_3, x_4, x_5, x_6)\).

We obtain the supplementary set of the dominant classes of \( x \):
- \([x_1^r] = \emptyset \), \([x_2^r] = \{x_1, x_2, x_3, x_4, x_5, x_6\}\), \([x_3^r] = \{x_1, x_2, x_3, x_4, x_5, x_6\}\), \([x_4^r] = \{x_1, x_2, x_3, x_4, x_5, x_6\}\), \([x_5^r] = \{x_1, x_2, x_3, x_4, x_5, x_6\}\), and \([x_6^r] = \{x_1, x_2, x_3, x_4, x_5, x_6\}\).

For the calculation, the following are included:
- \( R_{AT_1}(x_1, x_2) = \frac{1}{3} \), \( R_{AT_1}(x_1, x_3) = 1 \),
- \( R_{AT_2}(x_1, x_2) = \frac{2}{3} \), \( R_{AT_2}(x_1, x_3) = \frac{2}{3} \), \( R_{AT_2}(x_1, x_4) = 1 \),
- \( R_{AT_3}(x_1, x_5) = \frac{1}{3} \), \( R_{AT_3}(x_1, x_6) = \frac{1}{3} \), \( R_{AT_3}(x_1, x_7) = 1 \),
- \( R_{AT_4}(x_1, x_8) = \frac{1}{3} \), \( R_{AT_4}(x_1, x_9) = \frac{1}{3} \), \( R_{AT_4}(x_1, x_{10}) = 1 \),
- \( R_{AT_5}(x_1, x_{11}) = \frac{1}{3} \), \( R_{AT_5}(x_1, x_{12}) = \frac{1}{3} \), \( R_{AT_5}(x_1, x_{13}) = 1 \),
- \( R_{AT_6}(x_1, x_{14}) = \frac{1}{3} \), \( R_{AT_6}(x_1, x_{15}) = \frac{1}{3} \), \( R_{AT_6}(x_1, x_{16}) = 1 \),
- \( R_{AT_7}(x_1, x_{17}) = \frac{1}{3} \), \( R_{AT_7}(x_1, x_{18}) = \frac{1}{3} \), \( R_{AT_7}(x_1, x_{19}) = 1 \),
- \( R_{AT_8}(x_1, x_{20}) = \frac{1}{3} \), \( R_{AT_8}(x_1, x_{21}) = \frac{1}{3} \), \( R_{AT_8}(x_1, x_{22}) = 1 \),
- \( R_{AT_9}(x_1, x_{23}) = \frac{1}{3} \), \( R_{AT_9}(x_1, x_{24}) = \frac{1}{3} \), \( R_{AT_9}(x_1, x_{25}) = 1 \),
- \( R_{AT_{10}}(x_1, x_{26}) = \frac{1}{3} \), \( R_{AT_{10}}(x_1, x_{27}) = \frac{1}{3} \), \( R_{AT_{10}}(x_1, x_{28}) = 1 \),
- \( R_{AT_{11}}(x_1, x_{29}) = \frac{1}{3} \), \( R_{AT_{11}}(x_1, x_{30}) = \frac{1}{3} \), \( R_{AT_{11}}(x_1, x_{31}) = 1 \).

The ranking result is \( x_2 > x_5 > x_4 = x_6 > x_1 = x_3 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
<th>( a_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_1 )</td>
<td>(1)B</td>
<td>(2)M</td>
<td>(1)S</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>(3)G</td>
<td>(2)M</td>
<td>(2)M</td>
</tr>
<tr>
<td>( x_3 )</td>
<td>(1)B</td>
<td>(1)L</td>
<td>(2)M</td>
</tr>
<tr>
<td>( x_4 )</td>
<td>(2)C</td>
<td>(1)L</td>
<td>(3)B</td>
</tr>
<tr>
<td>( x_5 )</td>
<td>(2)C</td>
<td>(3)H</td>
<td>(2)M</td>
</tr>
<tr>
<td>( x_6 )</td>
<td>(2)C</td>
<td>(2)M</td>
<td>(1)S</td>
</tr>
</tbody>
</table>

**EXAMPLE**

Based on the data from the 2007 sustainability report (the data can obtain from zi892177892@163.com), we conduct a ranking study of the capacity of regional sustainable development. The data are included in the supplement. From the index system, five indicators are used to determine the capacity of sustainable development. We assess the sustainability of 31 regions in our country. \( U = \{\text{Beijing, Tianjin, Hebei, Shanxi, Neimenggu, Liaoning, Jilin, Heilongjiang, Shanghai, Jiangsu, Zhejiang, Anhui, Fujian, Jiangxi, Shandong, Henan, Hubei, Hunan, Guangdong, Guangxi, Hainan, Chongqing, Sichuan, Guizhou, Yunnan, Xizang, Shanxi, Gansu, Qinghai, Ningxia, and Xinjiang} \} \) is an object set, and
\( \mathbf{AT} = \{ \text{survival attribute index, developmental attribute index, environmental attribute index, social support attribute index, and intelligence support system index} \} = \{ a_1, a_2, a_3, a_4, a_5 \} \) is an attribute set. \( F \) is a mapping from \( U \) to \( V_{a_j} \). For each \( x_i \) and \( V_{a_j} \), we have \( F_{a_j}(x_i) = v_{a_j} \). Thus, a sustainable IS can be obtained. First, we preprocess the data and use equal widths to discretize the data to facilitate data comparisons. Then, we use the method proposed in this paper for the calculations. We take \( r = 3 \) in this example. MATLAB is used for the computations, and the following result is obtained:

Using the average comprehensive dominance degree (ACDD) in [4], the result is as follows:

A comparison with the ACDD method [4] shows that our method can better separate different target areas; thus, our method has a higher degree of differentiation. Our method is both effective and simple. The results show that Beijing has the strongest sustainable development and Shanghai presents the second strongest sustainable development, whereas Qinghai has the weakest sustainable development. Shanghai and Beijing are China's first-tier cities and represent economic centres; thus, these cities will have the strongest sustainable development, which is consistent with known data. However, in the ACDD, Shanghai has the strongest sustainable development and Zhejiang has the second strongest sustainable development. Thus, the sustainable development of Beijing is lower than that of Zhejiang, which is not consistent with known data. The weakest sustainable development is observed in the same city as that obtained using our method. Therefore, the experiment shows that our method is effective and practical.

CONCLUSIONS

Based on optimistic and pessimistic MGRS, this paper constructs the CMGRS in the OIS. In this theory, for OIS decision-making problems (i.e., multi-attribute ranking problems of partial ordering), a ranking method is proposed based on the CMGRS of the OIS. This method utilizes the degree of superiority to rank the objects. The features of this approach maintain the principle of the pointed order and ranks the objects of the dissatisfied pointed order, and this approach is effective and reasonable. This ranking approach maintains the weak transitivity of a partially ordered set and converts a serially ordered set to a partially ordered set [14]. In our further work, we will focus on the granularity parameter of the MGRS and its method of selection. In addition to the ranking problems, a wide range of studies are focused on decision problems in an OIS.
ACKNOWLEDGEMENTS

This work is supported by the National Science Foundation of China (Nos. 61170255).

REFERENCES