Study on Fuzzy Reliability for Wear of Key Components in Concrete Pumping System

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Abstract. In the wear process of the glass-like plates, taking the double uncertainty of the blur and randomness of the event into account, this paper proposes a fuzzy reliability analysis method for the wear of the glass-like plates. This paper discretely describes the four states of extremely reliable, very reliable, relatively reliable and unreliable. By using fuzzy conditional probability, this paper constructs a multidimensional membership function and obtains a series of reliability conditions under the floating threshold $\lambda$, which helps to predict when to replace the glass-like plates accurately and improves the reliability of concrete pump trucks.

Introduction

Abrasion is the process of continuous loss of surface materials on a component during friction. From intactness to fault, the product or component has actually undergone a dynamic transfer process. Concrete pump truck is a machine that continuously transports concrete along the pipeline by pressure, which is composed of the pump body and the pipeline$^{[9]}$. As an important part of the concrete pumping system, the glass-like plate interfaces with the concrete conveying cylinder. Driven by the swing cylinder, its moving part S-pipe swings from one side of the concrete cylinder port to the other side, and thereby the cylinder ports on both sides draw concrete from the hopper in turn. The glass-like plate and the cutting ring form a mutually frictional combination. With the reciprocating swing of the S pipe, the cutting ring performs relative motion against the glass-like plate, forming a surface-to-surface friction wear in the initial state. However, due to the fact that both the glass-like plate and cutting ring are completely covered by the concrete in the hopper during the work process, the cutting ring and the aperture edge of the glass-like plate become blunt under the repeated flow and impact of the high-pressure concrete, with a small arc-shaped wedge formed, leading the tiny solid particles of concrete into the contact surface and resulting in three-body abrasive wear conditions$^{[10]}$. Therefore, abrasive wear is one of the major failure modes of the glass-like plate. Through accurate calculation of the glass-like plate reliability, the parts of the plate can be replaced appropriately, which is practically significant to the improvement of production efficiency.

The wear process is subject to the dual uncertainty of event fuzziness and randomness, thus in diagnosing the fault of the wear process, the traditional reliability of the \{0,1\} (binary logic) set is unable to reflect the dynamic process of the abrasion. The reliability of allowable wear extent obtained by combining the normal distribution function (as the fuzzy failure membership function of wear) with the probability distribution (as commonly used in the classical reliability design) fails to accurately reflect the change of reliability in the wear process as well. Instead, capturing the specific difference states in the wear process, coupled with the fuzzy conditional probability, can reflect the dynamic process of the wear from the initial state to the maximum state in a more direct way based on the transformation of the difference states.
Analysis on Fuzziness in Wear Process of Glass-like Plates

Fuzziness of the Wear Process

The essence of fuzziness lies in the inherently objective development and change of things, rather than artificially subjective desires. Nevertheless, in order to obtain more practical and reasonable results, the human subjectivity is expected to reflect this objectivity. Fuzziness is derived from the intermediate transition process between things \{0,1\}, so if a specific state is captured in this transition process, then for this captured state, the original fuzziness is transformed into certainty. Therefore, fuzzy sets can give full consideration to the intermediate transitional properties of things and make full use of the intermediate transitional information of things to obtain a series of analysis results under floating thresholds so as to deepen our understanding of the difference states between things \{0,1\}.

We defined four states, including extremely reliable, very reliable, reliable and unreliable, to discretely describe the wear process of the glass-like plate. It is obviously no longer fuzzy for a captured reliability states. Instead, it is transformed into a common problem of reliability solving, so as to obtain the reliability at each discrete state. To this end, a series of different reliability states are captured during the intermediate transition process of glass-like plate wear to measure the degrees of reliability under different reliability states, so as to further describe the essence of the wear process and enable designers to deal with them in a flexible way.

Basic Laws of Wear

It is typically assumed that the wear extent $U$ obeys a normal distribution. According to the working requirements of the machine, the maximum allowable post-wearing gap is $U_{max}$, after operation the part for a period of $t = T$, the fit clearance $U$ increases gradually due to the accumulation of wear damage and the part fails once $U \geq U_{max}$. The wear law of parts can be expressed by the function of wear extent and load, relative speed, surface condition of material, lubrication state and working time $t$. The function relationship can be expressed as follows:

$$U = k \cdot f(p, \nu, t)$$

where: $k$ - the coefficient of wear determined by the friction auxiliary material and lubrication state; $p$ - unit pressure of the friction surface; $\nu$ - relative sliding speed of the friction surface.

Under normal circumstances, the rate of wear is:

$$\gamma = kp^n \nu^p$$

Under the first-order approximation, the mean and variance of wear rate are:

$$\overline{\gamma} = kp^n \cdot \nu^n$$

$$S_\gamma^2 = \gamma^2 \left[ \left( \frac{ms_n}{\overline{p}} \right)^2 + \left( \frac{ms_n}{\overline{D}} \right)^2 \right]$$

The relationship between wear extent and time can be expressed as:

$$U = \gamma t = k p \nu t$$

where: $\gamma$ - rate of wear.

The probability of the wear gap $U$ being less than the limit $U_{max}$ is the reliability at the given life expectancy $T$, that is:

$$R = F(U < U_{max}) = \int_{-\infty}^{U_{max}} f(U) dU = \int_{-\infty}^{U_{max}} \frac{1}{\sqrt{2\pi}} \exp \left[ -\frac{(U - \overline{U})^2}{2s_U^2} \right] dU = \int_{-\infty}^{U_{max} - \overline{U}} \frac{1}{\sqrt{2\pi}} \frac{z^2}{2s_U^2} dz$$

The right side indicates the normal distribution, so the reliability:
Mathematical Model

**Fuzzy Conditional Probability Basis** [2]

Assuming that \( C \) is the clear event that "the product fulfills the specified functions under specified working conditions within specified time" in the definition of classical reliability and \( \zeta_1, \zeta_2, \ldots, \zeta_n \) are the fuzzy events represented by the fuzzy function subsets, then it is obvious that \( C \) to varying degrees falls into \( \zeta_1, \zeta_2, \ldots, \zeta_n \), \( \zeta_i \) to indicate the fuzzy function subset that we are discussing.

According to the definition of fuzzy conditional probability:

\[
P(C | A_i) = P(\zeta_i | C)P(C)
\]

where: \( P(C) \) - the classical reliability, generally indicated by \( R \); 
\( P(C | A_i) \) - "in the case of \( C \), the probability for \( C \) to fall into \( \zeta_i \)";
\( \Delta \) - triangle norm operator (algebraic product).
\( P(C | A_i) \) - fuzzy reliability, generally expressed as \( R \).

So equation (3) is also expressed as:

\[
R = P(\zeta_i | C)R
\]

where: \( P(\zeta_i | C) \) - in the case of \( C \), the probability of \( \zeta_i \) is the fuzzy conditional probability; it can be replaced by the membership function \( \mu_{\zeta_i}(C) \); the availability of \( C \) is indicated by its probability \( R \), so \( \mu_{\zeta_i}(C) \) can also be expressed as \( \mu_{\zeta_i}(R) \), then equation (4) can be rewritten as:

\[
R = \mu_{\zeta_i}(R)R
\]

**Establishment of \( m \)-phase Fuzzy Statistical Test Model**

Assuming that \( A_1, A_2, \ldots, A_m \) is a fuzzy subset on the domain of discourse \( U: P_m = \{A_1, A_2, \ldots, A_m\} \)

Extract a set of samples with a capacity of \( n \) from the domain of discourse \( U \), where the sample numbers for \( A_1, A_2, \ldots, A_m \) are \( n_1, n_2, \ldots, n_m \) respectively. Select \( p \) indicators (or parameter) that can characterize the fuzzy set \( A_j (j = 1, 2, \ldots, m) \) for each sample, and denote the characteristic index of the \( i \)th sample as \( p \) dimension vector.

\[
u_i = (x_{i1}, x_{i2}, \ldots, x_{ip})
\]

where: \( i = 1, 2, \ldots, n \), \( n = \sum_{j=1}^{m} n_j \)

Therefore, the domain is expressed as:

\( U = \{\nu\} \quad \nu = (x_{11}, x_{12}, \ldots, x_{ip}) \quad x_{ij} \) is the \( i \)th characteristic index value

As the mapping from domain \( U \) to set \( P_m \)

\[
f(\nu) = \begin{cases} 
1, \text{ when } \nu \text{ is classified as } A_1 \\
1/2, \text{ when } \nu \text{ is classified as } A_2 \\
\ldots \ldots \ldots \\
1/m, \text{ when } \nu \text{ is classified as } A_m
\end{cases}
\]

Assuming:
Example of Fuzzy Reliability Calculation for Wear of Glass-like Plate

Calculation of General Reliability

Example: taking the parameters of HBMC-50/16-132S low-pressure concrete pump of Sany Heavy Industry as an example, the theoretical concrete delivery pressure is 10±2.4MPa, with a theoretical concrete delivery capacity of 45m³/h. The glass-like plate is known as a wear part that adopts a
cemented carbide structure. At low pressure, the major failure mode is abrasive wear, with a maximum allowable wear extent of 5mm, initial wear extent $\overline{U}_0 = 0$, standard deviation $S_u = 1\, \mu m$ and relative sliding speed of glass-like plate $\nu = 0.2 \pm 0.06 m/s$. According to the abrasive wear test of test specimen under average usage specification, the wear extent is 0.52mm within 100 hours, with a load spectrum approaching to normal distribution. Please try to calculate the fuzzy reliability of the specimen in 800 hours.

Solution: the service life of cemented carbide-inlaid glass-like plat is about pumping 43,000m$^3$ of concrete, with a theoretical pumping capacity of 45m$^3$/h. Assuming that the machine continuously pumps concrete, the ultimate service life is $T = (43,000m^3)/(45m^3/h) = 956(h)$.

According to the wear test, the wear extent within 100 hours is 0.52mm, so the average rate of wear is $\overline{\gamma} = 0.0052 (mm/h)$

As the load spectrum obeys normal distribution:

- The mean pressure $\bar{p}$ is $\bar{p} = 10 MPa$, with a standard deviation of $S_p = 2.4/3 = 0.8(MPa)$
- The mean relative sliding speed $\bar{\nu} = 0.2 m/s$, standard deviation $S_{\nu} = 0.06/3 = 0.02 (m/s)$

For abrasive wear, $m = 1, n = 1$. According to equation (1), the standard deviation of the wear rate is:

$$s_{\gamma} = \sqrt{\left[\left(\frac{mS_p}{\bar{p}}\right)^2 + \left(\frac{nS_{\nu}}{\bar{\nu}}\right)^2\right]} = 0.00066 (mm/h)$$

Then the average of wear extent $U$ in 800 hours is:

$$\overline{U} = \overline{U}_0 + \overline{\gamma}t = 0 + 0.0052 \times 800 = 4.16 (mm)$$

Standard deviation of wear extent $U$:

$$s_U = \sqrt{s_{U_0}^2 + s_{\gamma}^2T^2} = 0.528 (mm)$$

According to equation (2),

$$R = \phi\left(\frac{\overline{U}_{\text{max}} - \overline{U}}{S_U}\right) = \phi\left(\frac{5 - 4.88}{0.528}\right) = 0.9441$$

So the general reliability of the component within the given life expectancy is $R = 94.41\%$.

**Calculation of Degree of Membership $\mu_\delta(R)$ for Each Fuzzy Subset of Reliability**

1. Determine the fuzzy set on the domain $U$; $P_m = \{\text{extremely reliable } A_1, \text{very reliable } A_2, \text{reliable } A_3, \text{unreliable } A_4\}$;

2. In order to facilitate the calculation and explanation of the problem, 2 samples are selected for each fuzzy set, and the wear extent and working time are selected as the characteristic indexes to characterize each fuzzy set.

   That is: $x_1$ - wear extent ($U$); $x_2$ - working time ($T$).

   The characteristic index of the $i^{th}$ fuzzy subset can be expressed by two-dimensional vector as:

   $$u_i = (x_{i1}, x_{i2})$$

   Based on expert selection, the mean values of the characteristic indexes for the fuzzy subsets are defined and listed as follows:
Table 1. Expert Ratings on Characteristic Indexes of Samples.

<table>
<thead>
<tr>
<th>Category</th>
<th>Sample No.</th>
<th>Wear Extent U (mean) (mm)</th>
<th>Working Time t (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extremely reliable</td>
<td>1</td>
<td>2.0</td>
<td>650</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2.5</td>
<td>700</td>
</tr>
<tr>
<td>Very reliable</td>
<td>3</td>
<td>2.8</td>
<td>780</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>3.0</td>
<td>830</td>
</tr>
<tr>
<td>Reliable</td>
<td>5</td>
<td>3.5</td>
<td>850</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>3.8</td>
<td>870</td>
</tr>
<tr>
<td>Very unreliable</td>
<td>7</td>
<td>4.6</td>
<td>905</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>5.2</td>
<td>960</td>
</tr>
</tbody>
</table>

(3) Calculation of $\hat{\beta}$

As mapping:

$$u_i = \begin{cases} 
1 & \text{when } u_i \text{ is classified as } A_1 \\
1/2 & \text{when } u_i \text{ is classified as } A_2 \\
1/3 & \text{when } u_i \text{ is classified as } A_3 \\
1/4 & \text{when } u_i \text{ is classified as } A_4 
\end{cases}$$

According to equation (6):

$$Y_{3x1} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2.5 & 3 \\ 1 & 2.8 & 3 \\ 1 & 3.0 & 4 \\ 1 & 3.5 & 4 \\ 1 & 4.2 & 4 \\ 1 & 5 & 4 \\ 1 & 5.6 & 4 
\end{bmatrix}^T$$

$$X_{8x3} = \begin{bmatrix} 1 \\ 2 \\ 650 \\ 1 \\ 2.5 \\ 700 \\ 1 \\ 2.8 \\ 780 \\ 1 \\ 3.0 \\ 830 \\ 1 \\ 3.5 \\ 842 \\ 1 \\ 4.2 \\ 850 \\ 1 \\ 5 \\ 956 \\ 1 \\ 5.6 \\ 975 
\end{bmatrix}$$

According to equation (7), the least squares estimation of $U$ is calculated as:

$$\hat{\beta} = (X'X)^{-1}X'Y = (\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2)^T$$

Wherein,

$$\hat{\beta}_{3x1} = (3.3404, 0.1382, -0.0040)^T$$

(4) Construction of multivariate membership function

According to Equation (8), a multivariate membership function is constructed on the basis of Logical function:

According to the actual situation, $\alpha = -6$.

That is:

$$\mu_j(u) = \frac{1}{1 + \exp[\alpha(\beta_0 + \sum_{i=1}^{2} \hat{\beta}_i X_i)]} = \{1 + \exp(-6)(3.3404 + 0.1382 - 0.0040)]^{-1}$$

The 2 characteristic indexes of each sample are substitute for the above equation, and the calculation results are listed in Table 2.
Table 2. Membership of Samples to Reliability.

<table>
<thead>
<tr>
<th>Category</th>
<th>Sample No.</th>
<th>Membership $\mu_s(u)$</th>
<th>Category</th>
<th>Sample No.</th>
<th>Membership $\mu_s(u)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extremely reliable</td>
<td>1</td>
<td>0.998</td>
<td>Reliable</td>
<td>5</td>
<td>0.927</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.995</td>
<td></td>
<td>6</td>
<td>0.910</td>
</tr>
<tr>
<td>Very reliable</td>
<td>3</td>
<td>0.975</td>
<td>Very unreliable</td>
<td>7</td>
<td>0.894</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.932</td>
<td></td>
<td>8</td>
<td>0.782</td>
</tr>
</tbody>
</table>

Select 3 thresholds $\lambda_1=0.975$, $\lambda_2=0.927$, $\lambda_3=0.894$ from Table 2.

Namely, cut the fuzzy subset of the reliability $A$ into 3 common sets.

That is: Extremely reliable $A_1^* = \{ \mu_s(u) \geq 0.975 \}$

Very reliable $A_2^* = \{ 0.927 \leq \mu_s(u) \leq 0.975 \}$

Reliable $A_3^* = \{ 0.894 \leq \mu_s(u) \leq 0.927 \}$

Very unreliable $A_4^* = \{ \mu_s(u) \leq 0.894 \}$

The mean membership of each reliability fuzzy subset of the sample is taken as the membership function $\mu_s(C)$:

1. $\mu_s(C) = 0.9965$
2. $\mu_s(C) = 0.9535$
3. $\mu_s(C) = 0.9185$
4. $\mu_s(C) = 0.8380$

Calculation of Fuzzy Reliability

The fuzzy reliability at all stages throughout the 800-hour wear process can be calculated by substituting the membership $\mu_s(u)$ of each fuzzy subset into the equation.

Then according to equation (5), the glass-like plate is at the extremely reliable stage, with a reliability of:

$R = \mu_s(C) \times 0.9441 = 94.08\%$

At very reliable phase, the reliability is: $R = \mu_s(C) \times 0.9441 = 90.02\%$

At reliable stage, the reliability is: $R = \mu_s(C) \times 0.9441 = 86.72\%$

At very unreliable stage, the reliability is: $R = \mu_s(C) \times 0.9441 = 79.12\%$

If the average wear extent is known to be 3.2mm and the working time is 840h, the membership of this state can be obtained by the equation, and thus the reliability states can be determined according to the selected threshold.

According to the equation: $\mu_s(u) = 0.9266$

Based on the selected $\lambda$ threshold, we know: $0.894 \leq \mu_s(u) \leq 0.927$

Working under this state is at a reliable stage with a general reliability of 87.29%, so the fuzzy reliability for this stage is $R = \mu_s(u) \times R = 0.9266 \times 0.8729 = 80.88\%$.

Conclusion

The traditional reliability is established on the basis of the $\{0,1\}$ binary logic, which fails to describe the wear process due to the inaccurate calculation results. So far, the half-normal distribution membership function has been widely used in the calculations of wear-related fuzzy reliability, which to a certain extent considers the fuzziness of the wear process and provides the reliability for allowable wear extent. This paper, taking into full consideration the transition process of wear, determines the difference states in the transition process, to obtain a series of reliability states at the floating threshold value $\lambda$ through transforming the glass-like plate from a reliability states to an adjacent reliability state. On this basis, the fuzziness of the wear of the glass-like plate is further analyzed to get the analysis results that are unavailable for the existing fuzzy reliability, which is instructive for the timely replacement of the glass-like plate and the improvement of the working efficiency of the concrete pump truck.
References
