**Process Capability Evaluation and Analysis for Product Quality Management**

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**Abstract.** Process capability indices (PCIs) have been widely used to evaluate and determine whether the manufacturing process meets the preset target of the customer in manufacturing industries. A larger PCI value means a higher quality yield. This study applies two PCIs, the accuracy index \(\delta\) and the precision index \(\gamma\), to analyze, improve and manage the process performance of a product. To address statistical sampling error, the \((1 - \alpha) \times 100\%\) upper confidence intervals (UCLs) of \(\delta\) and \(\gamma\) are derived. An example is given to illustrate the use of the proposed method, and is proven to give good results for practical applications.

**Introduction**

Process capability indices (PCIs) are considered as a practical measurement tool and used in many industries in order to measure the capability of a process. PCIs are used to determine whether a manufacturing process is capable of producing items with dimensions within a specified tolerance range stipulated by customers. The PCI is based on the process parameters \((\mu, \sigma)\) and the process specification limits to quantify the performance of the process. Over the past decade, a plethora of such PCIs works has already been proposed by several researchers [1-7].

The first PCI \(C_p\) was developed by Juran [8], but it only considered process variability. For this reason, Kane [9] presented a PCI \(C_{pk}\) with respect to the process mean \(\mu\). Chan et al. [10] developed a PCI \(C_{pm}\) which incorporates variation with respect to the target value \(T\). Vännman [11] proposed two PCIs, the accuracy index \(\delta\) and the precision index \(\gamma\), for measuring the degrees of process centring and process deviation, respectively. These are defined as follows:

\[
\delta = \frac{\mu - T}{d} \quad \text{and} \quad \gamma = \frac{\sigma}{d},
\]

where \(d = (USL - LSL) / 2\) is the half specification width related to the process tolerance, and \(T = (USL + LSL) / 2\) is the target value, \(\sigma\) is the process standard deviation, and USL and LSL are the upper and lower engineering specification limits, respectively, of the process.

**UCLs for Accuracy and Precision Indices**

Suppose that a quality characteristic \(X\) is normally distributed with \(\mu\) and \(\sigma\) and let \(X_{j1}, X_{j2}, \ldots, X_{jm}, j = 1, 2, \ldots, m\), be \(m\) random samples, each of size \(n\) with a normal distribution on the quality characteristic. Then, \(\mu\) and \(\sigma\) can be estimated using:

\[
\bar{X} = \frac{1}{n} \sum_{j=1}^{m} X_{jr} \quad \text{and} \quad S^2 = \frac{1}{n-1} \sum_{j=1}^{m} (X_{jr} - \bar{X})^2
\]
The overall sample mean \( \hat{\mu} \) and the pooled sample variance \( \hat{\sigma}^2 \), which are unbiased estimators, can be used as follows:

\[
\hat{\mu} = \bar{X} = \frac{\sum_{j=1}^{m} X_j}{m} \quad \text{and} \quad \hat{\sigma}^2 = S_p^2 = \frac{\sum_{j=1}^{m} (n_j-1)S_j^2}{\theta - m} = \frac{\sum_{j=1}^{m} S_j^2}{m}.
\]

(3)

In general, the formula for the \( Z \)-statistic is defined as:

\[
Z = \frac{\bar{X} - \mu}{\sigma / \sqrt{\theta}} \sim N(0, 1),
\]

(4)

where the \( Z \)-statistic follows the standard normal distribution \( N(0, 1) \) and \( \theta = \sum_{j=1}^{m} n_j = m \times n \) is the total sample size.

Let

\[
Z = \frac{\hat{\delta} - \delta}{\gamma / \sqrt{\theta}} \quad \text{and} \quad K = \frac{m(n-1)\hat{\gamma}^2}{\gamma^2},
\]

(5)

where \( K \) obeys the \( \chi^2 \) distribution with \( m(n-1) \) degrees of freedom and \( \hat{\gamma} = \frac{\hat{\sigma}}{d} \) denotes the natural estimator of \( \gamma \).

As mentioned above, the natural estimator of \( \delta \) based on the samples is as follows:

\[
\hat{\delta} = \frac{\hat{\mu} - T}{d} = \frac{\bar{X} - T}{d}.
\]

(6)

The sample distribution of \( \hat{\delta} \) can be derived as \( N(\delta, \frac{\gamma}{\theta}) \) on the basis of the assumption of normality.

Let

\[
P(Z \geq -Z_{\alpha'}) = 1 - \alpha' .
\]

(7)

where \( \alpha' = 1 - \sqrt{1 - \alpha} \) and \( \alpha \) is a given level of significance.

Then

\[
P[K \geq \chi^2_{\alpha'}(m(n-1))] = 1 - \alpha'.
\]

(8)

To derive the \( (1 - \alpha) \times 100\% \) upper confidence intervals (UCLs) of \( \delta \) and \( \gamma \), this study defines events \( A \) and \( B \) as follows:

\[
A = \left\{ \frac{\hat{\delta} - \delta}{\gamma / \sqrt{\theta}} \geq -Z_{\alpha'} \right\} \quad \text{and} \quad B = \left\{ \frac{m(n-1)\hat{\gamma}^2}{\gamma^2} \geq \chi^2_{\alpha'}(m(n-1)) \right\}.
\]

(9)

As can be seen, \( P(A) = P(B) = 1 - \alpha' \), which means that \( P(A^c) = P(B^c) = \alpha' \). Using Boole’s inequality and the DeMorgan theorem, we can derive that \( P(A \cap B) \geq 1 - P(A^c) - P(B^c) \), and therefore:
\[
P\left( \frac{\hat{\delta} - \delta}{\gamma \sqrt{\theta}} \leq -Z_{\alpha} \sqrt{\frac{m(n-1)\hat{\gamma}^2}{\chi^2(m(n-1))}} \right) = 1 - \alpha
\]
\[
\Rightarrow P\left( \delta \leq \hat{\delta} + Z_{\alpha} \frac{\gamma}{\sqrt{\theta}} \right) = 1 - \alpha.
\]

Therefore, the \(1 - \alpha\)×100% upper confidence interval of \(\delta\) is \(\hat{\delta} + Z_{\alpha} \frac{\gamma}{\sqrt{\theta}}\) and the \(1 - \alpha\)×100% upper confidence interval of \(\gamma\) is \(\sqrt{\frac{m(n-1)}{\chi^2(m(n-1))}} \hat{\gamma}\), as follows:

\[
(\delta_U, \gamma_U) = \left( \hat{\delta} + Z_{\alpha} \frac{\gamma}{\sqrt{\theta}}, \sqrt{\frac{m(n-1)}{\chi^2(m(n-1))}} \hat{\gamma} \right).
\]

**Numerical Example**

As an example, consider the process that produced circular infrared (IR) filter given in Hsu et al. [6] to investigate the performance of the proposed approach. The circular IR filter has two key process characteristics: lens thickness and lens diameter, and the specifications of its process characteristics are found in Table 1.

**Table 1. Specifications and capability indices for the circular IR filter.**

<table>
<thead>
<tr>
<th>Process characteristic</th>
<th>USL</th>
<th>LSL</th>
<th>T</th>
<th>d</th>
<th>(\hat{\mu})</th>
<th>(\hat{\sigma})</th>
<th>(\hat{\delta})</th>
<th>(\hat{\gamma})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness</td>
<td>0.63</td>
<td>0.61</td>
<td>0.62</td>
<td>0.01</td>
<td>0.625</td>
<td>0.001</td>
<td>0.50</td>
<td>0.10</td>
</tr>
<tr>
<td>Diameter</td>
<td>8.30</td>
<td>8.20</td>
<td>8.25</td>
<td>0.05</td>
<td>8.263</td>
<td>0.011</td>
<td>0.26</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Sample data of the circular IR filter were obtained from 25 random samples, each of size 12 (that is, \(\theta = 25 \times 12 = 300\)) and specified \(\alpha = 0.025\). The related data, USL, LSL, T, d, \(\hat{\mu}\), \(\hat{\sigma}\), \(\hat{\delta}\) and \(\hat{\gamma}\), for the two process characteristics of the circular IR filter are summarized in Table 1. As a result, the UCLs for the two quality characteristics of the circular IR filter were derived, as per the summary in Table 2. As shown in Table 2, the values of \(\delta_U\) and \(\gamma_U\) for thickness are 0.07 and 0.11, respectively; and the values of \(\delta_U\) and \(\gamma_U\) for diameter are 0.04 and 0.24, respectively.

**Table 2. Results for the circular IR filter.**

<table>
<thead>
<tr>
<th>Process characteristic</th>
<th>(\delta_U)</th>
<th>(\gamma_U)</th>
</tr>
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<tbody>
<tr>
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</tr>
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</tr>
</tbody>
</table>

**Conclusions**

Process capability indices (PCIs) comprise three basic parameters that have been used widely throughout the world as one of the important statistical process control tools to measuring product potential and performance. In practice, many quality engineers and manufacturers have emphasized research into PCIs to propose more powerful approaches of evaluating process levels for product quality management. In this study, two improving process performance PCIs called accuracy index \(\delta\) and the precision index \(\gamma\) are adopted for measuring the degrees of process centring and process deviation, respectively. The \((1 - \alpha)\times100\%\) upper confidence intervals (UCLs) of \(\delta\) and \(\gamma\) are
developed for effective evaluating the capability and performance of processes and improving buyer and seller relationships when the consumers cast doubt on the quality of the products.

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References