Scheduling with Proportional Linear Deteriorating and a Maintenance Activity

Peng SHEN, Xue HUANG and Ji-bo WANG*

School of Science, Shenyang Aerospace University, Shenyang 110136, China

*Corresponding author

Keywords: Scheduling, Deteriorating effect, A maintenance activity.

Abstract. In this paper, we consider single machine scheduling with proportional linear deteriorating in which the processing time of a job depends on its starting time in a sequence. We assume that a maintenance activity is performed in a schedule. After a maintenance activity, machine will revert to its initial condition. The objective is to find simultaneously the optimal sequence for jobs and the position of a maintenance activity to minimize the makespan. A polynomial time algorithm is provided to solve the problem.

Introduction

In the classical scheduling problems, the processing time of jobs is assumed to be independent of its starting time in a scheduling sequence, i.e. it is a constant. However, the actual processing time of jobs usually change due to the deteriorating effect (see Brown and Yechiali[1], Alidaee and Womer [2], Cheng et al. [3], Gawiejnowicz [4])). In scheduling with the deteriorating effect, the later a given job is scheduled in the sequence, the longer its processing time.

The deteriorating (aging) effect on scheduling problem was first introduced by Brown and Yechiali[1]. Since then, scheduling problem with considerations of the deteriorating (aging) effects has been one of the most popular topics among researchers in recent years. For details on this stream of research, time-dependent (deteriorating effects) scheduling problems are studied in comprehensive surveys [2, 3, 4]. Recently, Wang and Wang [5] considered single machine scheduling problem with group technology, ready times and time dependent processing times. For the makespan minimization scheduling, they proved that the problem can be solved in polynomial time. Wang and Wang [6] considered single machine scheduling problem with precedence constraints and simple linear deterioration.

In many real-life situations, it is important that the maintenance activity improve the production efficiency of the machines or the quality of the products. However, the maintenance activity disturbs machine availability for production because machine is not available for processing jobs during the maintenance activity. Scheduling under such an environment is known as scheduling with the maintenance activity, which has received the attention of many researchers, including Sanlaville and Schmidt [7], Schmidt [8], Ma et al. [9] and Lee[10], among others.

Additionally, as so to model a more realistic production system, the scheduling problem with simultaneous considerations of deteriorating effect and the maintenance activity have been popular topics to researchers. Wu and Lee [11] considered a single-machine scheduling with deteriorating effect under a resumable availability constraint in order to find an optimal schedule to minimize the makespan. Kuo and Yang [12] explored single-machine scheduling problems with a cyclic process of deteriorating effects and some maintenance activities. The problem with job-independent and position-dependent deteriorating effects was investigated to minimize the makespan and a polynomial time algorithm was provided to solve it. Jr and Geiger [13] studied a single-machine scheduling with time-dependent processing times and a rate-modifying activity. The objective was to find the optimal policy for assigning a single rate-modifying activity in a sequence to minimize the makespan. Rustogi and Strusevich [14] considered single machine scheduling with generalized positional deterioration effects and machine maintenance activities.
In this study, we extend the model of Jr and Geiger [13] to the case of proportional linear deteriorating effect and a maintenance activity. It is shown that the problem of minimizing makespan remains polynomially solvable.

**Problem Description**

There are \( n \) jobs to be processed on a single-machine setting. All jobs are available at time zero and job preemption is not allowed. As in Jr and Geiger [13], we consider a proportional linear deterioration processing times, i.e., the processing time under position-specific proportional deterioration is given by

\[
p[j] = \begin{cases} 
1, & j = 1 \\
\alpha[j](a + bs[j]), & j = 2, \ldots, n 
\end{cases}
\]  

where \( \alpha[j] \geq 1 \) is the deteriorating ratio and \( [j] \) denotes the \( j \)th position in a sequence, \( s[j] \) is the start time of job \( [j] \) in a given schedule, and \( a > 0, b \geq 1 \) is the linear deteriorating factor. As in Jr and Geiger [13], we assume that \( \alpha[1] = 0 \), but \( p[j] = \alpha[1](a + bs[1]) = 1 \).

We assume that a maintenance activity (RMA) is performed in a schedule and the maintenance time is \( t \). During the maintenance activity the machine is turned off and no production is performed. After a maintenance activity, machine will be restored to its initial stage and the deteriorating effect will start anew. Let \( k \) denote the sequence position assign to an RMA, i.e.,

\[
p[j] = \begin{cases} 
\alpha[j](a + bs[j]), & j = 1,2, \ldots, k - 1 \\
p[j-k], & j = k + 1, \ldots, n, n + 1 
\end{cases}
\]  

The objective is to find the optimal sequence for jobs and the position of the maintenance activity such that the makespan is minimized. Adopting the three-field notation of Graham et al [15], we denote the problem studied in this paper as \( 1|p[j] = \alpha[j](a + bs[j]), rma|C_{max} \).

Denote the completion time of job \( [j] \) as \( C[j] \), according to Eq.(1), we have

\[ C[j] = (1 + \frac{a}{b}) \Pi_{i=1}^{j-1} (1 + b\alpha[i]) - \frac{a}{b}. \]  

For jobs scheduled before an RMA, we have

\[ p[j] = \begin{cases} 
1, & j = 1 \\
\alpha[j](a + bs[j]), & j = 2, \ldots, k - 1 
\end{cases} \]  

For jobs scheduled after an RMA, let \( r \) denote the \( r \)th job scheduled after an RMA, we have

\[ p[k+r] = \begin{cases} 
1, & r = 1 \\
\alpha[r]b(1 + \frac{a}{b}) \Pi_{i=1}^{r-1} (1 + b\alpha[i]), & r = 2,3, \ldots, n - k + 1 
\end{cases} \]  

Obviously, the job \( n \) occupies sequence position \( n + 1 \) if an RMA is scheduled in position \( k(k = 1,2, \ldots, n) \). Let \( C_{max}(k) \) be the makespan associated with scheduling an RMA in sequence position \( n \), we have

\[ C_{k+r}(k) = t + (1 + \frac{a}{b}) \Pi_{i=1}^{k-1} (1 + b\alpha[i]) + (1 + \frac{a}{b}) \Pi_{i=1}^{r-1} (1 + b\alpha[i]) - 2 \frac{a}{b} \]  

\[ C_{max}(k) = C_{n+1} + (1 + \frac{a}{b}) \Pi_{i=1}^{k-1} (1 + b\alpha[i]) + (1 + \frac{a}{b}) \Pi_{i=1}^{n-k+1} (1 + b\alpha[i]) - 2 \frac{a}{b} \]  

Obviously the makespan is sequence-independent for the problem \( 1|p[j] = \alpha[j](a + bs[j]), rma|C_{max} \). Hence, it is only necessary to determine the sequence position \( k(k = 1,2, \ldots, n) \) in which the RMA should be inserted. This can be accomplished by determining the value \( k^* \) that minimizes Eq. (7). Obviously, it is never optimal to schedule the RMA in the first sequence position \( k = 1 \) because not scheduling an RMA (i.e., \( k > n \)) always dominates \( k = 1 \).
Proposition 1.

a) \( C_{\max}(\frac{n}{2} + 1 - q) = C_{\max}(\frac{n}{2} + 1 + q) \) for even \( n \) and \( q = 1, 2, ..., \frac{n}{2} - 1 \),

b) \( C_{\max}(\frac{n+1}{2} - q) = C_{\max}(\frac{n+1}{2} + 1 + q) \) for odd \( n \) and \( q = 1, 2, ..., \frac{n-1}{2} - 1 \).

Proof. a): If \( n \) is an even integer, then \( k = \frac{n}{2} + 1 - q \) and \( k = \frac{n}{2} + 1 + q \) are also integers and both represent feasible sequence positions. If \( k = \frac{n}{2} + 1 - q \), it follows that \( n - k + 1 = \frac{n}{2} + q \), and \( k - 1 = \frac{n}{2} - q \). Similarly, if \( k = \frac{n}{2} + 1 + q \), it follows that \( n - k + 1 = \frac{n}{2} + q \), and \( k - 1 = \frac{n}{2} - q \). From Eq. (7), we have

\[
C_{\max}(\frac{n}{2} + 1 - q) = t + (1 + \frac{a}{b}) \prod_{i=1}^{n+q} (1 + b \alpha(i)) + (1 + \frac{a}{b}) \prod_{i=1}^{n-q} (1 + b \alpha(i)) - 2 \frac{a}{b} = C_{\max}(\frac{n}{2} + 1 + q)
\]

Similarly, b) can be proved.

Lemma 1. Let \( C_{\max}(j)(k) \) be the completion time of the job in sequence position \( j \) if the RMA is scheduled in sequence position \( k \), where \( k > n \) denotes that no RMA is scheduled. Then

a) \( C_{\max}(j)(k > n) = 1 + \sum_{a=1}^{j} \prod_{i=1}^{n+1} \alpha(i) b \left( 1 + \frac{a}{b} \right) \left( 1 + b \alpha(i) \right), j = 1, 2, ..., n \),

b) \( C_{\max}(\frac{n+1}{2} - q) = C_{\max}(\frac{n+1}{2} + 1 + q) \) for odd \( n \) and \( q = 1, 2, ..., \frac{n-1}{2} - 1 \).

Proof. a): Expanding the equation given by Lemma 1 part a, we have

\[
C_{\max}(j)(k > n) = 1 + \alpha[j-1] b \left( \frac{a}{b} + 1 \right) \left( 1 + b \alpha[j-1] \right) - \frac{a}{b} + \alpha[1] b \left( \frac{a}{b} + 1 \right) + \alpha[2] b \left( \frac{a}{b} + 1 \right) \left( 1 + b \alpha[1] \right) + \cdots + \alpha[j] b \left( \frac{a}{b} + 1 \right) \left( 1 + b \alpha[j-1] \right) - \frac{a}{b}
\]

The last line is the same as Eq. (2). Similarly, the proof of b) can be obtained. This concludes the proof of Lemma 1.

Corollary 1. \( C_{\max}(n+1)(k) = C_{\max}(k) = t + 1 + b \left( 1 + \frac{a}{b} \right) \sum_{m=1}^{n-k+1} \alpha_m \prod_{i=1}^{m-1} \left( 1 + b \alpha(i) \right) + (1 + \frac{a}{b}) \prod_{i=1}^{k-1} \left( 1 + b \alpha(i) \right) - \frac{a}{b} \)

Proof. As mentioned previously, job \( n \) will always be assigned to sequence position \( n + 1 \) if an RMA is assigned to sequence position \( k \). Thus makespan corresponds to \( C_{\max}(n+1)(k) \) and \( k + r = n + 1 \) or \( r = n - k + 1 \). Substituting \( r = n - k + 1 \) into \( C_{\max}(j)(k) \) given by Lemma 1b yields the result.

Similarly to Jr and Geiger [13], we have the following results:

Proposition 2. If \( \alpha_{\frac{n}{2} - q} \leq \alpha_{\frac{n+1}{2} - q} (1 + b \alpha_{\frac{n}{2}}) \prod_{i=1}^{q} (1 + b \alpha(i)) \), then \( C_{\max}(\frac{n}{2} + 1 - q) \leq C_{\max}(\frac{n}{2} - q) \) for \( q = 0, 1, ..., \frac{n-4}{2} \).

Corollary 2. If \( \alpha_{j} \geq 1 \) for all \( j \) and \( \geq 1 \), then \( C_{\max}(\frac{n}{2} + 1 - q) \leq C_{\max}(\frac{n}{2} + 1 - q) \) for \( q = 0, 1, ..., \frac{n-4}{2} \).

From the above results, the optimal policy can be given as follows:
Theorem 1. The optimal policy for scheduling an RMA under condition (2) with $\alpha_{[j]} \geq 1$ for all $j$ and $b \geq 1$ is as follows: If $n$ is an even integer and $< (1 + \frac{a}{b}) \prod_{i=1}^{n} (1 + b\alpha_{[i]}) - 2(1 + \frac{a}{b}) \prod_{i=1}^{\frac{n}{2}} (1 + b\alpha_{[i]}) + \frac{a}{b}$, assign the RMA to sequence position $k^* = \frac{n}{2} + 1$. If $n$ is an odd integer and $t < (1 + \frac{a}{b}) \prod_{i=1}^{n} (1 + b\alpha_{[i]}) - (1 + \frac{a}{b}) \prod_{i=1}^{\frac{n-1}{2}} (1 + b\alpha_{[i]}) - (1 + \frac{a}{b}) \prod_{i=1}^{\frac{n+1}{2}} (1 + b\alpha_{[i]}) + \frac{a}{b}$ assign the RMA to sequence position $k^* = \frac{n+1}{2}$ or $k^* = \frac{n+1}{2} + 1$. Otherwise, do not schedule the RMA.

Acknowledgments

This research was supported by the Foundation of Shenyang Aerospace University [201428Y], the Foundation of Education Department of Liaoning [L201753], and the Support Program for Innovative Talents in Liaoning University.

References
