Novel Image Segmentation Algorithm Based on Automatic GVF Snake Model

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\textbf{Keywords:} Image segmentation, Gradient vector field, Snake model, SUSAN algorithm, Initial contour, Fitting outsourcing polygon.

\textbf{Abstract.} To address issues that GVF Snake model needs to initialize the contour manually so that image segmentation cannot be automatically handled, the effect of segmentation is also related to the initial contour, and its efficiency and accuracy of the model are not ideal, a novel algorithm for image segmentation based on automatic GVF Snake model is proposed in this paper. We introduce the concepts of "flat redundant point" and "fitting outsourcing polygon". In our algorithm the discrete feature edge points set of target image is first obtained by SUSAN corner detection algorithm. Secondly, finding the fitting outsourcing polygon of discrete edge points set, Finally, the GVF Snake curve is evolved by using the fitting outsourcing polygon as the initial contour curve to complete the image segmentation. It is not only proved that the fitting outsourcing polygon as the initial contour curve can converge to the real target image edge by the algorithm in theory, but the experimental results also show that the proposed algorithm can effectively improve the segmentation accuracy, greatly reduce the number of iterations of the model, and improve the efficiency of the algorithm.

\textbf{Introduction}

Since Active contour model, or Snake model, was first introduced by Kass et al. in 1987\textsuperscript{[1]}, it has been extensively utilized in image analysis and computer vision. The main principle of Snake model is to find the boundary of target image by minimizing the curve energy function. A traditional active contour model is represented by an elastic curve \(X(s)=(x(s), y(s)), s \in [0, 1]\), it moves through the spatial domain of an image to minimize the following energy function:

\[
E = \int_0^1 (E_{\text{int}}(X(s)) + E_{\text{ext}}(X(s)))ds
\]

where the internal energy \(E_{\text{int}}(X(s)) = (\alpha |X'(s)|^2 + \beta |X'(s)|^3)^{1/2}\) ensures the contour continuous and smooth during deformation. the external energy \(E_{\text{ext}}(X(s))\) attracts the snake to the desired image features.

However, the traditional Snake model is sensitive to initialization, limited to capture range, and poor convergence to boundary concavities [2]. To deal with these problems, many improved methods have been put forward [3-7]. The gradient vector flow (GVF) Snake [3] has a relative large capture range and can extract some concavities. However, GVF has difficulty in extracting target boundary when the initial contour curve is far from the target. Moreover, the number of iterations increase rapidly, and the efficiency reduce greatly. Therefore, the initial contour should be set as close as possible to the true contour. There are several main methods of setting the initial contour are as follows: 1) manual interactive setting; 2) sequence image difference; 3) other methods [8-12]. In some cases, it is very difficult or even impossible to manually determine the initial contour. At
present, using the segmentation result based on other methods as the initial contour has been one of the most successful method. Hsu et al. [8] used the genetic algorithm to find automatically the parameters of Canny edge detector for face detection, and then use the Poisson GVF model to extract face contours. Zhou et al. [9] obtained the initial segmentation curve by designing a texture detector and then improve the pre-segmentation results by using an adaptive active contour algorithm. Liu et al. [10] selected the appropriate threshold for different image to divide it into two regions: target and background, remove the small objects by mark, and take the maximum object boundary as the initial contour of the GVF Snake model to segment the image. Kovacs et al. [11] defined a new external force, they detected the related local structure of each feature key point, obtained convex hull of extended feature points set as the initial contour of snake. Moreover, used the key points set to compute the new feature edge mapping. They achieved a good segmentation effect when the test image had a poor contrast, high curvature, and complex boundaries. Zhou et al. [12] used the convex hull of the edge points set detected by SUSAN operator as the initial contour of Snake model. But it deformation at the concave edge slowly, requires many iterations and has less efficiency.

In this paper, based on the analysis of GVF, a novel algorithm for image segmentation based on automatic GVF Snake model is proposed. We put forward the concepts of "flat redundant point" and "fitting outsourcing polygon". Theoretical analysis and experimental results show that our method not only overcomes the shortcomings of GVF effectively, but also improves the anti-noise ability and has better real-time performance and segmentation effects than other initialization methods.

GVF Snake Model

Gradient vector flow was proposed by Xu and Prince as a new external force for active contour model to overcome the issues of the traditional snake. The GVF is a vector field $V=(u(x,y), v(x,y))$ obtained by minimizing the following energy functional:

$$
\varepsilon(V) = \int \left[ \lambda (u_x^2 + u_y^2 + v_x^2 + v_y^2) + |\nabla f|^2 |V - \nabla f|^2 \right] dx dy
$$

where $f$ is the edge map. $\lambda$ is a regularization parameter (more noise, increase $\lambda$).

Using the calculus of variations, GVF is obtained by solving the following Euler equation:

\[
\begin{align*}
\lambda \nabla^2 u - (u - f_x)(f_x^2 + f_y^2) &= 0, \\
\lambda \nabla^2 v - (v - f_y)(f_x^2 + f_y^2) &= 0.
\end{align*}
\]

Analysis of GVF force field can be found that the $\nabla f$ is a vector that directed to the true contour, the force field $V$ is on the true contour and its direction points to the true contour, and the region is defined as an "effective area" in the literature [13], when the initial contour is set in this region, the curve will converge to the true contour under the action of GVF field.

GVF expands the capture range and relaxes the requirements for setting the initial contour. But the capture range of contour depends on the scope of GVF field, and the expansion of GVF field is based on the number of iterations, its computation cost will increase sharply. Moreover, its computational complexity is also great when the initial contour is far away from the "effective area". In addition, "false edge" will be formed to prevent the curve from approaching the real edge because of the local minimum of energy. It is obviously that the initial contour setting is directly related to the efficiency of the whole algorithm and the accuracy of the final segmentation results.

New Automatic GVF Snake Model

In this paper, we use the SUSAN operator to detect the points set of feature edge to form a new fitting outsourcing polygon as the initialize contour, and then use GVF force field to get the final image segmentation results. The process of our algorithm mainly includes the following three steps:
Step 1: Using SUSAN corner detection algorithm to obtain the discrete feature edge points set of target image.

Step 2: Finding the fitting outsourcing polygon of discrete edge points set.

Step 3: The GVF Snake curve is evolved by using the fitting outsourcing polygon as the initial contour curve to complete the image segmentation, as shown in Figure 1.

Figure 1. Automatic GVF Snake model.

**SUSAN Edge Detection**

SUSAN detection operator is a famous corner detecting operator proposed by Smith et al. in 1997. It can effectively detect the edge and corner of target [14]. This method defines a concept of each image point, univalue segment assimilating nucleus (USAN), which associated with it a local area of similar brightness. Its principle is to calculate the USAN value of each pixel in the image and compare it with a threshold $T$, the compared result can be used for a judgement whether the pixel is edge point.

$$
C(x_0, y_0; x, y) = \begin{cases} 
1 & \text{if } |f(x_0, y_0) - f(x, y)| \leq T \\
0 & \text{if } |f(x_0, y_0) - f(x, y)| > T
\end{cases}
$$

(4)

where $(x_0, y_0)$ is the coordinate of nucleus in image, $(x, y)$ is the coordinate of other pixel, $C$ is the comparison result of gray value.

There are some features for the USAN value of each pixel. It takes the maximum when the nucleus pixel locates in grayscale uniform area; it will decrease when the nucleus is closing to the edge; it takes the minimum when the nucleus is a corner point. Because these characteristics of USAN can mark the position of edges and corners in the image accurately, edge and corner points can easily be detected. The threshold $T$ not only can filter some noise but also take small calculated cost. Besides these, the property of SUSAN operator is not sensitive for the size of template, and its parameters is easy to choose, and it can easily implement automatically.

**The Definition of Flat Redundant Point and Fitting Outsourcing Polygon**

**Definition 1** Let $P_{i-1}$, $P_i$ and $P_{i+1}$ are three points in the plane that do not coincide with each other. If the value of angle $\theta$ between the vectors $\overrightarrow{PP_{i-1}}$ and $\overrightarrow{PP_{i+1}}$ is within the range of $T_\theta$ and $\pi$, then the point $P_i$ is considered to a flat redundant point corresponding to $T_\theta$.

In definition 1, $T_\theta$ is usually greater than or equal to $5\pi/6$ to ensure the fitting effect. A vector dot product method is used to determine whether a point is a flat redundant point. Take any point $P_i$, points $P_{i-1}$ and $P_{i+1}$ are before and after adjacent point of $P_i$, respectively. As shown in Figure 2.

Figure 2. Determine the redundant points.
The angle $\theta$ between the vectors $\vec{P}_iP_{i+1}$ and $\vec{P}_{i+1}P_i$ is calculated as following equation (5):

$$\theta = \arccos \frac{\vec{P}_iP_{i+1} \cdot \vec{P}_{i+1}P_i}{\|\vec{P}_iP_{i+1}\| \|\vec{P}_{i+1}P_i\|}$$  

(5)

The following judgment formula is used to ensure $\theta$ is within the range of $T_\theta$ and $\pi$.

$$C = \begin{cases} 
1, & \cos \theta \leq \cos T_\theta, \\
0, & \cos \theta > \cos T_\theta. 
\end{cases}$$  

(6)

where $C$ is the results of the judgment, $1$ means $P_i$ is the flat redundant point, and $0$ means not.

**Definition 2** Given a points set $M$, the points $P_i (i=1, 2, ..., m$, where $m$ is the number of elements of $M$) are connected in a counterclockwise order to form a closed polygon $L$. According to the definition 1 to remove the flat redundant points on the polygon $L$, maintain the original order of the connection and obtain a new closed polygon, which is called fitting outsourcing polygon.

According to the literature [13], it can be seen that no matter how complex the true contours of the image are, the GVF force field is vertical to the true contour in certain regions. If the initial contour is set in the "effective area", the curve can converge to the true contour by the GVF force field. We can prove that the curve can converges to the real contour accurately when using the "fitting outsourcing polygon" which defined in this paper as the initial contour curve. the following theorems are obtained:

**Theorem 1.** Using the fitting outsourcing polygon of the edge points set as the initial contour curve of the GVF Snake model to segment an image, it can ensure that this curve converges to the true contour of the image.

**Proof:** Let the $X$-direction sampling precision of the edge feature points set $S$ be $p$ pixels, the sampling accuracy in the $Y$-direction is $q$ pixels, $10 \leq p, q \leq 15$, the boundary points set after sampling is $M$, $P_i$ is the element in $M$. According to the requirements of definition 1, the points set after removing the flat redundant points in $M$ is called $Q$, and the fitting outsourcing polygon is formed by connecting $Q$ in counterclockwise order.

Assuming that $P_{i-1}, P_i, P_{i+1}$ are the three adjacent points in $M$ (they do not coincide with each other), if $P_i$ is not a flat redundant point, keep this point and obtain the fitting straight-lines $P_iP_{i-1}, P_iP_{i+1}$; if $P_i$ is a flat redundant point, remove it and get the fitting straight-line $P_{i-1}P_{i+1}$. These two cases can be demonstrated by the following methods respectively.

1) The fitting straight-lines are $P_iP_{i-1}$ and $P_iP_{i+1}$. Since points $P_{i-1}, P_i, P_{i+1}$ belong to the set $R$, it means that $P_i$ is on the real contour. Therefore, $P_iP_{i-1}$ and $P_iP_{i+1}$ are located in the “effective area”.

2) The fitting straight-line is $P_{i-1}P_{i+1}$. We just need to prove that the distance from $P_i$ to $P_{i-1}P_{i+1}$ is within the range of “effective area”. From the definition1 we can see that $\theta \in [T_\theta, \pi]$, and the sampling precision are $p$ and $q$ respectively, $\|\vec{P}_{i-1}P_i\| \leq 2\sqrt{p^2 + q^2}$. In the triangle $P_{i-1}P_iP_{i+1}$, let the length of $P_{i-1}P_{i+1}$ is $a$. It can be seen from Figure 3 that when $a$ is fixed, the value of $d$ increases with the decrease of $\theta$. When $\theta$ is the same and $P_i$ is in the center line of $P_{i-1}P_{i+1}$, $d$ obtains the maximum value; It can be seen from Figure 4 that when $\theta$ is fixed, the value of $d$ increases as $a$ becoming larger. Therefore, the distance $d$ between $P_i$ and $P_{i-1}P_{i+1}$ is maximized when $\theta$ takes the minimum and $a$ takes the maximum and $P_i$ is in the center line of $P_{i-1}P_{i+1}$.

![Figure 3](image1.png)  
Figure 3. Length of a remain unchanged.  
![Figure 4](image2.png)  
Figure 4. Angle $\theta$ remains unchanged.  
![Figure 5](image3.png)  
Figure 5. distance from point to line.  

Hence, as shown in Figure 5, we can get the formula (7):
d = \left| P_{i+1}P_{i+1} \right| / 2 \tan(\theta / 2) \tag{7}

Then when \( \theta = 150^\circ \), \( a = 30\sqrt{2} \), \( p = 15 \), \( q = 15 \), \( d \) take the maximum, \( d = 5.68 \). Because \( d \in (0, 5.68) \), "fitting outsourcing polygon" curve is in the "effective area" of the GVF force field. With the "fitting outsourcing polygon" as the initial curve and using GVF force field for curve evolution, it can be guaranteed that the curve converges to the true contour eventually. Proof finished.

Algorithm Description

Step 1: Obtain the data coordinates and number \( n \) of the discrete feature points set.

Step 2: \( X \)-direction sampling. At the first, find the maximum and minimum points of the \( X \)-direction coordinate in the obtained discrete points set: \( x_{\text{max}}, x_{\text{min}} \). Divide all the points into \( m_{x} \) intervals according to the size of their \( X \) coordinates, where \( m_{x} = (x_{\text{max}} - x_{\text{min}}) / p \) and \( p \) is the sampling width. Then find the maximum and minimum points of the \( Y \)-direction coordinate in the interval \( i \), they are called \( k[i].\text{miny} \) and \( k[i].\text{maxy} \) respectively, where \( i = 1, 2, 3, ..., m_{x} \). As shown in Figure 6. For convenience, the point in Figure 6 is actually a pixel. Finally, extreme points \( k[i].\text{miny}, k[i].\text{maxy} \), \( x_{\text{min}} \) and \( x_{\text{max}} \) in each interval are stored in the coordinate array \text{point}_X\text{sampling}[].

Step 3: Sampling in the \( Y \)-direction like step 2. Find the maximum and minimum points of the \( Y \)-direction in the obtained discrete points set: \( y_{\text{min}} \) and \( y_{\text{max}} \), the sampling width is \( q \), and sampled points are stored in the coordinate array \text{point}_Y\text{sampling}[]. The sampling results are shown in Figure 7.

Figure 6. Sample point set on the X direction. Figure 7. Sample points set on the Y direction.

Step 4: Combine the sampling results of steps 2 and 3. Sort the points that in array \text{point}_X\text{sampling}[] and array \text{point}_Y\text{sampling}[] and store them into array \text{pointAll}[], as shown in Figure 8, where all sampling points make up a set \( M \). According to the formula (8), we can find the center of mass position \( P(x_{0},y_{0}) \) of points set. The red and blue dots coincide with each other partially in Figure 8, it means that they are coincidental, to facilitate the observation, represented by partial coincidence.

\[
x_{0} = \frac{1}{n} \sum_{(x, y) \in M} x \quad y_{0} = \frac{1}{n} \sum_{(x, y) \in M} y
\tag{8}
\]

where \( n \) is the number of points.

Figure 8. Sampled points set M and barycenter P(x0,y0). Figure 9. Virtual coordinate system.
Step 5: Taking the center of mass \(P(x_0, y_0)\) as virtual origin of coordinate, the image is divided into four quadrants, as shown in Figure 9. Points of each quadrant are connected in counterclockwise order, respectively. Taking the first quadrant as an example, points of the first quadrant are set as \(A_1, A_2, A_3, \ldots, A_m\). The point \(P\) is used as the starting point, vectors \(PA_1, PA_2, PA_3, \ldots, PA_m\) consist of \(P\) and other points of first quadrant to calculate the angular tangent \(\tan\alpha\) of each vector. According to the size of \(\tan\alpha\) in descending order, when the size of \(\tan\alpha\) is same, only one of them is retained, and all the points of first quadrant is obtained in the counterclockwise order to form a new sequence. Points set of second, third and fourth quadrants are completed in the counterclockwise order in turn to obtain a new sequence: \(K1, K2, K3, K4\). Points in each quadrant are connected in the order of sequences \(K1, K2, K3, K4\), and connecting the starting points and end points of first and second quadrants, second and third quadrants, third and fourth quadrants to obtain a closed polygon, as shown in Figure 10.

Step 6: To determine whether the intermediate point of three adjacent points is a flat redundant point by definition 1, if yes, delete it. Thus, a new points set \(Q\) is obtained, and \(Q\) is a subset of \(M\). The polygon obtained by connecting points of \(Q\) in counterclockwise order is used as the initial contour curve. As shown in Figure 11.

![Figure 10. Sort and connect R with clockwise.](image1)

![Figure 11. Initial contour curve.](image2)

**Computational Complexity**

Analyzing the time complexity of our model, it is found that the time of algorithm is mainly used to find the initial contour, calculate GVF force field and evolution of the contour curve. In our algorithm, the time complexity of SUSAN edge detection algorithm and fitting outsourcing polygon algorithm are \(O(mn)\) and \(O(n)\) respectively (\(n\) is the number of elements in points set). In addition, since the initial contour of our method is close to the true contour, it is known from theorem 7 that their distance just about several pixels. Therefore, required GVF field capture range is relatively small, and the number of iterations of GVF field and the convergence time of contours are reduced correspondingly. In the original GVF snake model, the number of iterations of GVF field \(V\) is about tens to hundreds, and the evolution of the contour curve is about tens to hundreds. But in this paper, calculating vector flow field just need several to dozens of times, the evolution of the contour curve is tens to hundreds. Compared with algorithms of setting the initial contour curve by manual interaction, the automatic initializing contour curve algorithm of this paper is more efficient. In summary, the proposed method has a significant improvement in convergence efficiency than original GVF Snake model.

**Experimental Results and Analysis**

In this section, we show the performance of the proposed method by presenting examples on various synthetic and real image, all examples are implemented in Windows Microsoft XP system, machine THINKPAD E430, 2GB memory and based on Matlab R2009a environment.

Firstly, a comparative analysis is performed on synthetic image with U-shaped in Figure 12. The parameter of GVF is \(\lambda=0.2\), and the number of iterations of vector flow field is 30. Figure 12(a) is the original image, Figure 12 (b) is the initial contour which set by artificial way, Figure 12(c) shows the result of GVF Snake model, and its number of iterations is 40. Figure 12(d) is the initial contour of fitting outsourcing polygon which is obtained by the proposed method. And the result of this paper is
shown in Figure 12(e) and it is iterated 20 times. The experimental results show that the initial contour obtained by proposed algorithm is closer to the true contour of the image, so that the number of iterations is reduced greatly, and the effect of convergence is improved obviously in the case of depression.

![Image](image1.png)

**Figure 12. Experiments on image U.**

Secondly, the leaf image is tested in Figure 13. the parameter of GVF is also \( \lambda = 0.2 \), and the number of iterations of vector flow field is 10. Figure 13(a) is the original image of leaf, Figure 13(b) is the initial contour which set by artificial way, Figure 13(c) shows the result of GVF Snake model, and its number of iterations is 100. the initial contour of fitting outsourcing polygon and result of this paper are shown in Figure 13(d) and (e) respectively, it is iterated 40 times. As shown in Figure 13, we can handle image segmentation automatically, and not only reduce the number of iterations greatly, but also can converge to details of image well.

![Image](image2.png)

**Figure 13. Experiments on leaf image.**

Finally, Figure 14 shows experimental results tested on flower image. the parameter of GVF is \( \lambda = 0.25 \), and the number of iterations of vector flow is 25. Figure 14(a) is the original image of flower, Figure 14(b) is the initial contour which set by artificial way, Figure 14(c) shows the result of GVF Snake model. the initial contour of fitting outsourcing polygon and result of this paper are shown in Figure 14(d) and (e) respectively. As shown in Figure 14, the contour curve can converge to the depression between the petals well, our algorithm improves the segmentation accuracy greatly.

![Image](image3.png)

**Figure 14. Experiments on flower image.**

**Conclusion**

In this paper, we have proposed a novel algorithm for image segmentation based on automatic GVF Snake model. This algorithm uses SUSAN algorithm to detect the edge of the image, and calculates the fitting polygon for the sampling edge points automatically, which is used as the initial edge contour to realize the automatic segmentation of the image. We have not only proved that the fitting outsourcing polygon as the initial contour can converge to the real target edge of the image by this algorithm in theory, and the experimental results also show that the "fitting outsourcing polygon" makes the ability of locate depression area has been improved significantly, and improves the
segmentation efficiency and accuracy obviously. At the same time, the convergence time of the GVF field and the number of iterations of the whole model is reduced greatly.

Acknowledgments

This paper is partially supported by the Basic and Advanced Research Project of CQCSTC (Grant No:cstc2017jcyjBX0037).

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