Study on Tension Interval Estimation of Bridge Suspenders with Complex Boundary Conditions

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Abstract. For the complex boundary conditions (BCs), the tension estimate formulas of bridge suspenders aren’t directly given as their frequency functions closely related with the boundary condition. Based on three typical BCs, hinged-hinged BCs (HH), clamped-hinged BCs (CH) and clamped-clamped BCs (CC), three typical regions are reduced, namely, I typical regions: HH-CH, II typical regions: HH-CC and III typical regions: CH-CC. Further, based on Rayleigh theorem, the frequency value of the hinged-hinged boundary conditions are lower than that of the clamped-hinged boundary conditions, the clamped-hinged conditions lower than the clamped-clamped conditions for the same order frequency. Making full use of the united tension formulas of bridge suspenders for three typical BCs [1], the tension of bridge suspenders for the complex BCs can be roughly estimated by the interval estimation, I typical regions: \( t_\text{min} \leq \tilde{t} \leq t_\text{max} \), II typical regions: \( t_\text{min} \leq \tilde{t} \leq t_\text{max} \) and III typical regions: \( t_\text{min} \leq \tilde{t} \leq t_\text{max} \). In the practical bridge engineering, the tension estimation can be firstly calculated by this method in order to check whether the tension test is correct.

Introduction

In the process of the bridge construction and operation, it is very necessary to know well the load conditions of the bearing suspenders, further to grasp the structural characteristics of cable-stayed bridge. At present, the vibration test method is undoubtedly the popular and practical technology to estimate the tension of the suspenders, which owns lots of superiorities, such as more economical, more effort, more timesaving than other methods, and is very suitable for the tension identification in the operation process, as the frequencies of the vibration method are easily obtained in the practical bridge structures.

Firstly, in the complex process of identifying the cable tension of the bridge suspenders for the clamped-clamped BCs and clamped-hinged BCs and, the data overflow may be caused by the exponential or hyperbolic functions in the frequency function. In order to handle the numerical problem and simplify the identification of cable tension, the exponential or hyperbolic functions are approximated according to the numerical properties of the exponential, hyperbolic and trigonometric function, the approximate frequency function and the approximate solving condition are deduced with the classical frequency formulas of the bridge suspenders and Rayleigh’s theorem [2-4].

And the frequency formulas of cables for HH and CC BCs are constructed using the periodicity of the approximate frequency function [5,6]. Based on the nonlinear iteration method and the numerical fitting theory, the frequency formulas of bridge suspenders are deduced and constructed as the united expression for two typical kinds of boundary condition. The numerical comparative study shows both of the relative error of the fitting formulas of two kinds of boundary conditions aren’t beyond ±4.0%.

Furthermore, for two typical kinds of boundary conditions, the tension formulas of bridge suspenders are deduced and the united expression are constructed by a new nondimensional parameters \( \eta \), which is combined with the natural frequency formulas of the hinged-hinged beam.
The numerical study shows both of the relative error of the fitting formulas of two kinds of boundary conditions aren’t beyond ±5.0%. The fitting formulas provide a fast identification method for the suspender tension estimation for two typical kinds of boundary conditions.

In the practical bridge engineering, some practical factor should be consider, such as the concrete tube, anchoring end and the practical length of cable. These problems are preliminarily studied by the parameter identification based on the optimization theory in the reference [7,8], but there are some confused problems. In the finite element model of the bridge suspenders made up of the bending stiffness $EI$, the line density $\rho A$, the cable length $l$ and the tension $T$, all of these are naturally chosen as the identification parameter. Although the finite element model explicitly contains these parameters, the four parameters can be transferred two independent parameters $\alpha$ and $\beta$, namely, the frequency functions are only related to the independent two parameters $\alpha$ and $\beta$ [9,10]. Once the numbers of the identification parameter and the change interval of that are determined, which can efficiently improve the identification method. But it is inevitable the identifying process still needs the repeating trial and error.

In this paper, the interval estimation are constructed to study the tension estimate of bridge suspenders for the complex boundary conditions with tedious repeat procedures. Firstly, three typical regions are reduced, namely, I typical regions: the HH-CH, II typical regions: HH-CC and III typical regions: CH-CC. Further, based on Rayleigh theorem, the tension of bridge suspenders for the complex BCs are estimated by the interval estimation.

**Three Typical Regions of the Suspender for Boundary Conditions**

If the bending stiffness $EI$, the line density $\rho A$, the cable length $l$, the typical boundary conditions of the bridge suspenders are given and the practical frequencies are tested, the tension are easily estimated by the approximate formulas [1,10].

However, for the practical bridge suspenders, the boundary conditions (BCs) can’t be ideally modeled by the hinged-hinged BCs (HH), clamped-hinged BCs (CH) and clamped-clamped BCs (CC), in fact, the practical boundary conditions of cables may lie the HH-CH, or HH-CC, or CH-CC. Based on these, three typical regions are constructed, as follow.

I typical regions: HH-CH

II typical regions: HH-CC

III typical regions: CH-CC

Any boundary conditions of the practical bridge suspenders can be categorized as one of the three regions base on its practical conditions.

**Interval Estimation Based on the Rayleigh Theorem**

Base on Rayleigh theorem, for the same order frequency, the frequency value of the hinged-hinged boundary conditions are not higher than that of the clamped-hinged boundary conditions, the clamped-clamped boundary conditions not higher than the clamped-clamped conditions.

$$\omega_{GG,i} \geq \omega_{GJ,i} \geq \omega_{JJ,i},$$

respectively represent the frequencies of the bridge suspenders under the hinged-hinged, clamped-hinged, clamped-clamped boundary conditions.

As we know, the bigger the natural frequency is, the higher the tension of bridge suspenders. Therefore, if the same test frequencies $\omega_{c,i}$ are respectively substituted into the three frequency functions under the three typical boundary conditions. Three tension can be got by the frequency functions. Further, using (1), the tension inequation can be constructed, as follow.

$$T_{JJ} \geq T_{GJ} \geq T_{GG}$$  \hspace{1cm} (2)
$T_{JH}$, $T_{GH}$, $T_{GG}$, respectively represent the tension of the bridge suspenders under the hinged-hinged, clamped-hinged, clamped-clamped boundary conditions.

The boundary conditions of the bridge suspenders can be easily confirmed between above three typical regions, further, the tension value can estimated, as follow.

I interval estimation:

$$T_{JG} \leq \dddot{T} \leq T_{JH}$$  (3)

II interval estimation:

$$T_{GG} \leq \dddot{T} \leq T_{JG}$$  (4)

III interval estimation:

$$T_{GG} \leq \dddot{T} \leq T_{JG}$$  (5)

The $T_{JH}$, $T_{GH}$, $T_{GG}$, also can be got by resolving the frequency functions[2,5], except for the hinged-hinged boundary conditions. If these values are directly calculated by the analytic formula or approximate formulas, the interval estimation method becomes more efficient.

**United Tension Formulas of Short Bridge Suspenders**

Usually, the natural frequencies of the suspender are normalized by that of the string in the reference [9-11]. Recently, for the first order frequency, the new nondimensional parameter η are introduced, that is to say, the natural frequencies of the suspender are nondimensionally handled by the natural frequencies of the Euler–Bernoulli beam under the hinged-hinged boundary condition[1].

$$\eta = 4 f^2 \rho^2 \frac{L^4}{EI}$$

Using (6), the united formulas of the tension suspender is

$$T = \frac{4 f^2 \rho^2 L^4}{(i + \mu_1)^2} \frac{EI}{f^2 (i + \mu_1)^2}$$  (7)

When the frequency order $i$ equal to 1, the relation between the nondimensional parameter $\eta_1$, and $\mu_1$ are respectively given in the Table.1 and Table.2 for the clamped-hinged, clamped-clamped boundary conditions.

**Table 1.** The relationship between $\eta_1$ and $\mu_1$ under the clamped-hinged boundary condition ($i=1$).

<table>
<thead>
<tr>
<th>$\mu_1$</th>
<th>$\eta_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.34e-1+3.10 e-2×exp(-$\eta_1$/25.32)+3.27 e-3×exp(-$\eta_1$/11.77)</td>
<td>24.087, 4.00e1</td>
</tr>
<tr>
<td>6.43e-2+1.86e-1×exp(-$\eta_1$/21.30)+1.62e-1×exp(-$\eta_1$/89.90)</td>
<td>4.00e1, 1.40e2</td>
</tr>
<tr>
<td>1.62e-2+1.02e-1×exp(-$\eta_1$/148.25)+4.66e-2×exp(-$\eta_1$/1029.81)</td>
<td>1.40e2, 3.50e3</td>
</tr>
<tr>
<td>3.02e-3+1.52e-2×exp(-$\eta_1$/4557.168)+7.61e-3×exp(-$\eta_1$/30478.64)</td>
<td>3.50e3, 9.00e4</td>
</tr>
<tr>
<td>0.00</td>
<td>[9.00e4, $\infty$)</td>
</tr>
</tbody>
</table>

**Table 2.** The relationship between $\eta_1$ and $\mu_1$ under the clamped-clamped boundary condition ($i=1$).

<table>
<thead>
<tr>
<th>$\mu_1$</th>
<th>$\eta_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.15e-1+8.3453 e-1×exp(-$\eta_1$/48.07075)</td>
<td>[50.72, 1.10e2]</td>
</tr>
<tr>
<td>1.4489 e-1+5.7276 e-1×exp(-$\eta_1$/82.92)</td>
<td>[1.10e2, 2.00e2]</td>
</tr>
<tr>
<td>8.379 e-2+2.9622 e-1×exp(-$\eta_1$/201.33)</td>
<td>[2.00e2, 6.00e2]</td>
</tr>
<tr>
<td>1.945e-2+1.1988e-1×exp(-$\eta_1$/474.54)+5.309 e-2×exp(-$\eta_1$/3077.71)</td>
<td>[6.00e2, 1.00e4]</td>
</tr>
<tr>
<td>5.47e-3+2.165e-2×exp(-$\eta_1$/7109.58)+1.289e-2×exp(-$\eta_1$/39417.23)</td>
<td>[1.00e4, 1.00e5]</td>
</tr>
<tr>
<td>0.00</td>
<td>[1.00e5, $\infty$)</td>
</tr>
</tbody>
</table>
In a word, the tension can be roughly estimated by this methods, as follow.

1. Base on the practical condition of the suspenders, the tension identification type are categorized by three typical regions.
2. The same test frequencies $\omega_{c,i}$ are respectively calculated by the united formals (6) base the typical regions.
3. The tension estimated by three interval estimation.

Conclusion

In this paper, the interval estimation are constructed to study the tension estimate of bridge suspenders for the complex boundary conditions. Firstly, three typical regions are reduced, namely, I typical regions: the HH-CH, II typical regions: HH-CC and III typical regions: CH-CC. Further, based on Rayleigh theorem, the tension of bridge suspenders for the complex BCs are estimated by the interval estimation. In the practical bridge engineering, the tension estimation can be firstly calculated by this method in order to check whether the tension test is correct.

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References


