REITs Portfolio Optimization: A Nonlinear Generalized Reduced Gradient Approach

Rita Yi Man Li and Amos Chan

HKSYU Real Estate and Economics Research Lab / Sustainable Real Estate Research Center, Hong Kong Shue Yan University, Hong Kong

Keywords: REITs, Optimization, Generalized reduced gradient model.

Abstract. Many investors would like to know which countries’ REITs they should invest, and the respective industrial REITs in those countries to maximize their profits. To construct REITs the optimal portfolios for Asia-Pacific/North-America/Global Portfolio, we gathered REITs daily data and categorised it according to different industries. Variance and covariance matrix between different industries of the target countries and the REITs performance between different countries are used to study the relationship between them. The global optimal portfolio is investigated by analyzing REIT data of Asia –Pacific and North America countries. By adopting Nonlinear Generalized Reduced Gradient approach, this paper investigates the optimal portfolio of REITs.

Introduction

A Real Estate Investment Trust (REIT) invests in real estate through property acquisition or mortgages, and trades in the stock exchange markets. REITs provide investors with a liquid stake in real estate. They receive special tax reductions and offer high dividend (Chaudhry, 2004). The first listed property trusts outside the Europe and the US were launched in Australia in 1971. Nevertheless, REITs were not popular until the beginning of the 21st century where they spread across Asia since its first launch of the Japanese REITs in 2001 (Ambrose and Linneman 2001). The REITs have unique characteristics such as tax transparency (Li, et. al., 2017).

REITs can be classified into equity, mortgage and hybrid. Their performances were mainly determined by the performance of the underlying direct property assets (Grupe, 1999). Many of the previous research throws light on the factors that affect REITS’ performance and shed lights on market capitalisation / size of the REITs that affect the yield and the interest rates’ impact on return. For example, Australian REIT performance were affected by REITs’ gearing, market-to-book value ratio and capitalisation (Yong, et al., 2009). Yong et. al. (2009) suggested that REITs’ returns and sizes are negatively correlated. Long-run interest rate exhibited a negative relationship with the short-run return (Allen 2000). Chaudhry (2004), Hamelink and Hoesli (2004) stated that larger REITs were more geographically diversified but less diversified across the types of properties, leading to an inverse relationship between size and return (Yong, Allen and Lim , 2009). Ratcliffe (2007)’s research indicated an opposite result where there was a positive insignificant relationship with the short-term interest rates. Gore and Scott (1998) found that fund from operations was closely related with stock returns. Fields (1998) concluded that the REITs’ claim of fund from operations advantage was not mature.


The theory of asset pricing attributed to the efficient market theory, one of the most important theories in modern finance (Palan, 2004). The theory suggested that stock market efficiency always lead the shares to share prices, incorporating and reflecting all significant information. Thus, it is impossible to gain above-normal returns. Shkeifer (2000) mentioned that specific conditions affected markets efficiency. Market efficiency is therefore achieved if any one of the following criteria are met, the rationality of market participants, independent deviations from rationality and arbitrage. Accordingly, an efficient market applies when the stock prices reflect all the available information in the market.
Sharpe (1995) defined an efficient market as security’s price equals its investment value. Consequently, the investment value refers to a security’s underlying fair value an efficient market (Sharpe, 1995). As REITs derive most of their cash flow and earnings from the real estate assets, it is reasonable to hypothesize that they are co-integrated with the underlying real estate market.

All in all, REIT Portfolio optimization has not been studied before despite its practical value to investors. This study investigated the optimal REIT portfolio according to the region and the types of the industries via Generalized Reduced Gradient (GRG) Approach.

**Portfolio Optimisation**

The most common risk-adjusted benchmark is the sharp ratio, which provides a measure of reward per unit of risk (Higgins, 2008). However, that is usually difficult to interpret, especially amongst the layman investors (Bernstein, 2000). Most of the previous studies of optimal real estate allocations rely on the traditional mean-variance optimization according to the short-term returns (MacKinnon, 2009). Investors choose the most profitable investment to build their investment portfolios. Likewise, investment trust managers also wish to know which investment needs to enhance the profitability to attract more investors. Markowitz (1952) introduced the portfolio-selection technique which is known as the modern portfolio theory. The mean variance is one of the most important theories in finance studies where the Markowitz Efficient Frontier refers to the set of portfolios in which maximum expected returns is reached at a given level of risk (Lee et. al, 2010). Portfolio optimisation analysis is generally studied based on return, risk and variance of portfolios. Markowitz was the pioneer who show how to reduce the variance of the investment portfolio via diversification (Lee et. al, 2010).

We investigated the optimal REIT portfolio of Asia-Pacific and North-America and found the optimal weighting for each country. Additionally, we used their optimal portfolio’s maximum return to construct an optimal portfolio for each of the countries. Lastly, we constructed an optimal portfolio for the global REIT. Investors could then optimise their REITs’ price by investing their money in each country according to the optimal portfolio weighting, and they could follow the industrial weighting presented to invest in the best performed industrial REIT in those countries. To find the optimal portfolio in each area, we used the Generalised Reduced Gradient nonlinear optimisation method.

**Generalised Reduced Gradient Methods**

Generalised Reduced Gradient methods are algorithms for solving non-linear programs of general structure. Lasdon et al. (1974) discusses the basic principles of GRG and constructs a specific GRG algorithm. The logic of a computer program implementing this algorithm is presented by means of flow charts and discussion. A numerical example is given to illustrate the functioning of this program. The nonlinear program to be solved is assumed to have the following form:

\[
\begin{align*}
\text{minimize } & f(X) \\
\text{subject to } & g_i(X) = 0, \ i = 1, \ldots, m \\
& l_i \leq X_i \leq u_i, \ i = 1, \ldots, n
\end{align*}
\]

where \( X \) is n-vector and \( l_i, u_i \) are given lower and upper bounds. We assume \( m < n \) since, in most cases, \( m > n \) implies an infeasible problem or one with a unique solution. (1)-(3) is general, since inequality constraints are transformed to equalities (2), by adding slack variables. The basic idea of GRG is to use the equalities (2) to express \( m \) variables, called the basic variables, in terms of the remaining \( n-m \) non-basic variables. Let \( \bar{x} \) be the feasible point and \( y \) be the vector of basic variables and \( x \) the non-basic at \( \bar{x} \), so that the equalities (3) can be rewritten as

\[
x = (x, y), \bar{x} = (\bar{x}, \bar{y})
\]
\[ g(y, x) = 0 \quad (5) \]

where

\[ g = (g_1, ..., g_m) \quad (6) \]

Assume that the objective \( f \) and constraint functions \( z \) are differentiable, if equation (5) has a solution \( y(x) \) for all \( x \) in some neighborhood of \( \bar{x} \), it is sufficient that the \( \partial g / \partial y \), evaluated at \( \bar{x} \), is non-singular. If it is, then the objective may be expressed as a function of \( x \) only:

\[ F(x) = f(y(x), x) \quad (7) \]

The nonlinear program is transformed, at least for \( x \) close to \( \bar{x} \), to a reduced problem with only upper and lower bounds:

\[
\begin{align*}
\text{minimize} & \quad F(x) \\
\text{Subject to} & \quad l_{NB} \leq x \leq u_{NB}
\end{align*}
\]

where \( l_{NB} \) and \( u_{NB} \) are the vectors of bounds for \( x \). GRG solves the original problem (1)-(3) by solving (8)-(9). Such problems may be solved by simple modifications of unconstrained minimisation algorithms. Consider the feasible point \( \bar{x} \) with basic variables \( y \) and non-basic variables \( x \), we then solve the reduced problems (8)-(9). With regard to the reduced problem (8)-(9) to yield useful results, \( x \) must be free to vary about the current point \( \bar{x} \). The bounds (9) restrict \( x \), but it is easy to move \( x \) in directions to keep these bounds satisfied. The bounds on the basic variables, however, pose a more serious problem. If some components of \( \bar{y} \) are at their bounds, even a slight change in \( x \) from \( \bar{x} \) causes some bounds violation. To avoid that from happening and safeguard the presence of the function \( y(x) \), we assume that at any point \( X \) satisfying (2)-(3), there exists a partition of \( X \) into \( m \) basic variables \( y \) and \( n-m \) non-basic variables \( x \) such that

\[
\begin{align*}
l_B & < y \leq u_B \\
\text{where} \quad l_{y} \text{ and } u_{y} \text{ are the vector of bounds on } y \text{ and } x
\end{align*}
\]

\[ B = \partial g / \partial y \text{ is nonsingular} \quad (11) \]

To evaluate the objective (7), we need to know the values of the basic variables \( y(x) \). Apart from the linear and a few nonlinear cases, the function \( y(x) \) cannot be calculated in closed form. However, \( y(x) \) can be computed for any given \( x \) by an iterative method to solve the equalities (5). A procedure that solve the reduced problem starting from \( x^o \), is:

(0) Set \( I = 0 \).

(1) Substitute \( x^i \) into (5) and determine the corresponding values for \( y_i \), by an iterative method for solving nonlinear equations.

(2) Determine a direction of motion, \( d_i \), for the non-basic variables \( x \).

(3) Choose a step size \( \alpha_i \), such that

\[ x_{i+1} = x_i + \alpha_i d_i \]

This is often done by solving the one-dimensional search problem

\[ \text{minimize } F(x_i + \alpha d) \]

With a restricted such that \( x_i + \alpha d \) satisfies the bounds on \( x \). This one-dimensional search requires repeated applications of step (1) to evaluate \( F \) for various \( \alpha \) values.

(4) Test the current point \( x_i = (y_i, x_i) \) and return to (1). If not optimal set \( I = I + 1 \), then return to (1).

If the values in step (1) of one or more components of \( y_i \) exceed their bounds, the iterative procedure must be interrupted. For simplicity, we assume only one basic variable violates a bound. This variable must be non-basic and some component of \( x \), which is not in a bound, is basic. After this change of basis, we have a new function \( y(x) \), a new function \( F(x) \), and a new reduced problem.
The initial point \( x \) is on the curve \( g_2(X) = 0 \). We have taken the basic variables as \((x_5, x_6, x_7)\), although the only variable that cannot be basic is \( x_4 \), since it is at lower bound of zero. The objective of the first reduced problem is \( \mathbf{x}^T \mathbf{v} \), which is just the objective \( f \) as measured on the curve \( \mathbf{z} = 0 \). It is possible that the algorithm minimizing \( x_5 \) might release \( x_5 \) from its lower bound of zero, in which case we would move interior to \( x_5 = 0 \). Assume this does not happen, we move along \( x_5 = 0 \) as indicated by the arrow until we hit the curve \( x_5 = 0 \). At this point the slack for \( x_5 \) goes to zero. Since it is basic, it must leave the basis to be replaced by one of the non-basics, \( x_3 \) or \( x_4 \). When \( x_5 \) is zero, \( x_5 \) becomes basic. We then have a new objective \( f(x_5, x_4) \), with \( x_3 \) and \( x_4 \) at lower bound of zero. The algorithm optimizing \( f_2 \) will determine that, if either \( x_3 \) or \( x_4 \) is released from its lower bound, \( f_2 \) can be decreased. Assume \( x_4 \) is released from its bound (\( x_3 \) and \( x_4 \) might both be released from their bounds). Then the algorithm minimises \( f_4 \), which is simply \( f \) as measured along the curve \( x_4 = 0 \). Motion is towards the \( x_4 \) axis. Upon reaching it, \( x_4 \) becomes zero, and another basis change occurs, with \( x_3 \) becomes non-basic and \( \mathbf{x} \) becomes basic. Optimization of the new function \( f_2 \) will terminate at the constrained optimum of \( f \).

**Data and Results of the Optimal Portfolio**

The REITs price data of each country is classified according to industries, REITs’ average industrial return and the standard deviation of the industrial REIT performance. Data from 2/19/2014 to 2/19/2016 is collected from Datastream. The lowest industrial standard deviation of each country is found and set as the constraint for the optimal portfolio. We ran the variance covariance matrix with the daily industrial REIT performance to find the variances and covariance associated with different industries (Wasserman, 2004). In the covariance matrix, the off-diagonal elements contain the covariance of each pair of variables. The diagonal elements of the covariance matrix contain the variances of each variable. The variance measures how much the data is scattered with the mean. The variance is equal to the square of the standard deviation (Higham, 2002). The optimal portfolio concept falls under the modern portfolio theory (Shleifer, 2000) which assumes that investors minimise risk while striving for the highest return (Delcoure & Dickens, 2004). We then utilise the Generalized Reduced Gradient nonlinear optimization model to find the industrial weighting that gives the minimum risk with the highest return. After getting the optimal portfolio for each country, the return rate of each countries optimal portfolio was measured to form an optimal portfolio.

**Global, Asia-Pacific and North America REITs’ Optimal Portfolio**

The Global optimal portfolio has an expected return of 0.02372%, with portfolio standard deviation of 1.91393%, and Sharpe ratio of 0.012393637. This portfolio gives the highest expected return. It has the same portfolio standard deviation as the Asia Pacific optimal portfolio. Investors should invest 79.411% of their money into Taiwan’s REITs, 11.3959% Malaysia, 0.6706% Thailand, 7.135% Korea, and 1.4004% Canada. Moreover, some have a high standard deviation in REITs performance as compared to the portfolio constraint (1.91393%) and are not the best options to be included in the Asia-Pacific optimal portfolio. Although Korean REITs do not have a good performance, their national portfolio standard deviation is the second lowest amongst the global REIT market. Thus, it lowers the portfolio’s standard deviation. Moreover, the covariance between Korean REITs and Taiwan is -0.00000036, the two REITs move in opposite directions. The risk might be lowered given the same amount or potential return (Sharpe, 1966). There are other REITs that have negative covariance with the Taiwan REITs, but their performance is very poor, with limited expected return or extreme high standard deviation, which is why they are not chosen in the portfolio. Although, Canada has the best performance amongst all the REITs, its standard deviation (45%) is higher than the portfolio constraint, thus we only include 1.4004% of Canada REITS in the portfolio.

Asia-Pacific optimal portfolio has an expected return of 0.01532%, with portfolio standard deviation of 1.91393%, and Sharpe ratio of 0.008005821. Investors should invest 83.09% of their money into Taiwan’s REIT, 15.005% Malaysia, 1.360% Thailand, and 0.535% Hong Kong. That excludes Japan, Australia, China, Korea or Singapore due to their poor REIT performance.
Furthermore, some have a super high standard deviation in their REIT performance compared to the portfolio constraint (1.91393%), so they are not selected to be included in the Asia-Pacific optimal portfolio.

The North America optimal portfolio would give an expected return of 1.07571%, with portfolio standard deviation of 45.48983%, and Sharpe ratio of 0.023647323. This portfolio gives the highest Sharpe ratio, thus provides a better return for the same risk (Sharpe, 1966). However, the portfolio standard deviation is the highest; therefore, this is a risky portfolio than the other two. Investors should invest 100% of their money in Canada’s REIT, and 0% in the US. We only invest in Canada because Canada’s REIT performance is the best in North America with 1.076% national portfolio return rate. Additionally, its national portfolio standard deviation (45.5334%) is lower than that in the US (208.506%), thus we should not invest in the US REITs.

**Optimal Portfolio of REITs in Taiwan, Malaysia, South Korea and Thailand**

Taiwan REITs include office, hotel and retail. The optimal investment portfolio includes 24.5427% office REITs, 47.1493% hotel REITs, and 28.3080% retail REITs. Following the weightings, this is an expected return of 0.005326%, with portfolio standard deviation of 0.582466%, and Sharpe ratio of 0.00914388. The Sharpe ratio provides a means to examine the performance of an investment by adjusting its risk. It measures the risk premium per unit of deviation in an investment asset or a trading strategy, typically referred to as the deviation risk measure, thus a higher Sharpe ratio provides a better return for the same risk (Sharpe, 1966). Malaysia’s REIT include office, hotel, retail, industrial, and diversified REITs. Investors should invest 97.971383% hotel REITs, and 2.028617% industrial REITs. That would give an expected return of 0.0531039%, with portfolio standard deviation of 0.7915288%, and Sharpe ratio of 0.067090286. Taiwan’s REIT includes residential and office REITs. Investors should invest 34.156884% residential REITs, and 65.843116% office REITs, which would give -0.043220% expected return, with portfolio standard deviation of 1.817312%, Sharpe ratio of -0.023782311. The Korean REITs can diversify the risk in the portfolio, as its price move in the opposite position to the Taiwan REITs. Thailand REITs include residential, industrial, retail, hotel, office REITs and diversified REITs. Investors should invest 100% residential REITs with the highest return.

![Figure 1. Global optimal portfolio (left), Asia Pacific optimal portfolio (middle) North America optimal portfolio (right).](image1)

![Figure 2. Optimal portfolio for Taiwan, Malaysia, Korea, Thailand (from left to right).](image2)
Optimal Portfolio in the US and Canada

The US’s REIT include residential, industrial, retail, hotel, office and diversified REITs. Investors should invest 80.225% in hotel REITs, 15.325% in diversified REITs, 4.450% in residential REITs. Following the weightings, this would give an expected return of -1.0757118951%, with portfolio standard deviation of 11.3942164338%, and Sharpe ratio of 0.094408589. Canada’s REITs include residential, industrial, retail, hotel, office and diversified REITS. Investors should invest 50.338% in diversified REITs, 38.508% in hotel REITs, 6.283% in residential REITs, and 4.868% in office REITs. Following the weightings, this would give an expected return of -1.0757118951%, with portfolio standard deviation of 11.3942164338%, and Sharpe ratio of 0.094408589.

Conclusions

In this paper, we investigated the optimum portfolio of REIT price return of Asia-Pacific, North America and a global portfolio with Asia-Pacific and North America countries with the nonlinear Generalised Reduced Gradient methods. Empirical results showed that the performance of global optimal portfolio is better than using Asia-Pacific countries or North American countries alone. Therefore, to optimise the REIT price portfolio, investors should invest in both Asia-Pacific and North-America countries. Finally, the optimal portfolio weight suggests investing the largest portfolio proportion in Taiwan REITs, followed by Malaysia REITs, Korea REITs, Thailand REITs, and Canada REITs.

References


