Dynamics Analysis of 2PPPPS-R-2PPPPS Serial-Parallel Mechanism

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Abstract. In this paper, the stress condition of each hinge point of the new 2PPPPS-R-2PPPPS serial-parallel mechanism is analyzed comprehensively by establishing dynamic equations and Euler equations. Given the stress at output terminal of the mechanism that has given spatial position and pose, the movement parameters of principal axis, the movement parameters of connecting rod and the movement parameters of horizontal and vertical moving sliders, we can solve the inverse dynamics solution of the mechanism, and through computer programming, we can calculate and draw the intuitive and effective results.

Introduction

The series robots have low stability [1]. But it is easy to solve the dynamics forward solution, the kinematic coupling degree of each joint is smaller, and the control is simple; the working space of robot is larger than that of machine tool. Serial robots are more complex and costly. The parallel robot has the advantages of high stiffness, large load capacity, compact structure and high position accuracy [2], which are complementary with the tandem robot. However, it is difficult to solve the dynamics forward solution, the movement branches are easy to be coupled, the working space is relatively small [3], and the application in the actual production activity is less. Based on the advantages and disadvantages of the serial robot and the parallel robot, the 2PPPPS-R-2PPPPS serial-parallel robot is constructed (as shown in Fig.1), it has the advantages of serial and parallel robot [4,5]. In the form of practical examples, this paper calculate and draw the intuitive and effective results by using computer programming on the study object[6].

![Figure 1. The series-parallel mechanism and its spatial force diagram](image)

Dynamic Analysis

When the spatial position of each hinge point of the mechanism and the spatial position of the output spindle are known (shown in Fig.1) [7], the force and torque at the end of the output spindle of the mechanism can be expressed as $F$ and $M$ respectively, according to Newton's law of motion and Euler equation [8], the dynamics inverse solution of the mechanism shown in Figure 1 is solved[8].
Suppose that the length of the output moving platform branch \(a_1\) and \(a_2\) is \(d\), because of the special structure of the mechanism, the moving platform branch chain \(A_1l_1a_1l_1A_1\) is always in the plane \(p_{23}\), the Euler equation of the moving platform branch \(a_1\) in this plane with \(a_1\) as the fulcrum (as shown in Fig. 2) can be obtained as follows.

\[
\begin{align*}
F^x_{a_1} \cdot \cos \alpha^x_{p_{12}} \cdot \sin \beta^x_{a_1} \cdot d + F^y_{a_1} \cdot \cos \alpha^y_{p_{12}} \cdot \sin \beta^y_{a_1} \cdot d + F^z_{a_1} \cdot d \cdot \cos \alpha^z_{a_1} & = J_{a_1a_1} \cdot \omega^x_{a_1} \\
F^x_p \cdot \cos \alpha^x_{p_{12}} \cdot \sin \beta^x_p \cdot \frac{d}{2} + F^y_p \cdot \cos \alpha^y_{p_{12}} \cdot \sin \beta^y_p \cdot \frac{d}{2} + F^z_p \cdot \frac{d}{2} \cdot \cos \alpha^z_{a_1a_1} & = J_{a_1a_1} \cdot \omega^y_{a_1a_1} \\
F^x_p \cdot \cos \alpha^x_{p_{12}} \cdot \sin \beta^x_p \cdot \frac{d}{2} + F^y_p \cdot \cos \alpha^y_{p_{12}} \cdot \sin \beta^y_p \cdot \frac{d}{2} + F^z_p \cdot \frac{d}{2} \cdot \cos \alpha^z_{a_1a_1} & = J_{a_1a_1} \cdot \omega^z_{a_1a_1} \\
\end{align*}
\]

(1)

Figure 2. The moment balance analysis of moving platform branch \(a_1a_3\) with \(a_1\) as the fulcrum

In the formula (1), \(F^x_{a_1}, F^y_{a_1}, F^z_{a_1}\) means the three stress components of the moving platform hinge point \(a_1\) that are respectively along the \(x\), \(y\) and \(z\) axis of the space fixed coordinate system. \(F^x_p, F^y_p, F^z_p\) means the three stress components of the moving platform center point \(p\) that are respectively along the \(x\), \(y\) and \(z\) axis of the space fixed coordinate system. \(\alpha^x_{p_{12}}, \alpha^y_{p_{12}}\) means the angle between the plane \(p_{12}\) and the \(x\) and \(y\) axis of the space fixed coordinate system. \(\alpha^x_{a_1}, \alpha^y_{a_1}\) means the angle between the connecting rod \(l_1\) (the \(z\) axis of the fixed coordinate system) and the moving platform branch \(a_1a_4\). \(\beta^x_{a_1}, \beta^y_{a_1}\) means the angle between the moving platform branch \(a_1a_4\) and the projection of \(F^x_{a_1}, F^y_{a_1}, F^z_{a_1}\) in plane \(p_{14}\). \(J_{a_1a_1}\) means the moment of Inertia the moving platform branch \(a_1a_4\) around the point \(a_1\) that parallel to the normal \(n_{14}\) of the plane \(p_{14}\). \(\omega^x_{a_1a_1}\) means the angular acceleration component of the moving platform branch \(a_1a_4\) around hinge point \(a_1\) that parallel to the normal \(n_{14}\) of the plane \(p_{14}\).

Similarly, the moving platform branch chain \(A_2l_2a_2l_2A_2\) is always in the plane \(p_{23}\), so that the Euler equation of the moving platform branch \(a_2a_3\) in this plane with \(a_2\) as the fulcrum (as shown in Fig. 3) can be obtained as follows.

\[
\begin{align*}
F^x_{a_2} \cdot \cos \alpha^x_{p_{23}} \cdot \sin \beta^x_{a_2} \cdot d + F^y_{a_2} \cdot \cos \alpha^y_{p_{23}} \cdot \sin \beta^y_{a_2} \cdot d + F^z_{a_2} \cdot d \cdot \cos \alpha^z_{a_1a_3} & = J_{a_2a_2} \cdot \omega^x_{a_2a_3} \\
F^x_p \cdot \cos \alpha^x_{p_{23}} \cdot \sin \beta^x_p \cdot \frac{d}{2} + F^y_p \cdot \cos \alpha^y_{p_{23}} \cdot \sin \beta^y_p \cdot \frac{d}{2} + F^z_p \cdot \frac{d}{2} \cdot \cos \alpha^z_{a_1a_3} & = J_{a_2a_2} \cdot \omega^y_{a_2a_3} \\
F^x_p \cdot \cos \alpha^x_{p_{23}} \cdot \sin \beta^x_p \cdot \frac{d}{2} + F^y_p \cdot \cos \alpha^y_{p_{23}} \cdot \sin \beta^y_p \cdot \frac{d}{2} + F^z_p \cdot \frac{d}{2} \cdot \cos \alpha^z_{a_1a_3} & = J_{a_2a_2} \cdot \omega^z_{a_2a_3} \\
\end{align*}
\]

(2)

Figure 3. The moment balance analysis of moving platform branch \(a_2a_3\) with \(a_2\) as the fulcrum

In the formula (2), \(F^x_{a_2}, F^y_{a_2}, F^z_{a_2}\) means the three stress components of the moving platform hinge point \(a_3\) that are respectively along the \(x\), \(y\) and \(z\) axis of the space fixed coordinate system. \(F^x_p, F^y_p, F^z_p\) means the three stress components of the moving platform center point \(p\) that are respectively along the \(x\), \(y\) and \(z\) axis of the space fixed coordinate system. \(\alpha^x_{p_{23}}, \alpha^y_{p_{23}}\) means the angle between the plane \(p_{23}\) and the \(x\) and \(y\) axis of the space fixed coordinate system. \(\alpha^x_{a_2}, \alpha^y_{a_2}\) means the angle
between the connecting rod \( l_1 \) (the \( z \) axis of the fixed coordinate system) and the moving platform branch \( a_1a_1 \cdot \beta_{i_1}^n \beta_{i_1}^n \) means the angle between the moving platform branch and the projection of \( F_{a_1}^n \), \( F_{a_1}^n \) in plane \( P_{a_1} \). \( \beta_{i_1}^n \), \( \beta_{i_1}^n \) means the angle between the moving platform branch \( a_1a_1 \) and the projection of \( F_{a_1}^n \), \( F_{a_1}^n \) in plane \( P_{a_1} \). \( J_{a_2a_3} \), \( J_{a_2a_3} \) means the moment of inertia of the moving platform branch \( a_2a_3 \) around the point \( a_2 \) that parallel to the normal \( n_{23} \) of the plane \( P_{a_2} \).

Since the spatial position and pose of the output spindle of the mechanism is known, the Euler equation of the moving platform branch \( a_2a_4 \) with \( a_2 \) as the fulcrum around the output spindle axis \( n \) (as shown in Fig. 4) can be obtained, as follows.

\[
F_{a_1}^n \cdot \cos \alpha_{i_1}^n \cdot \sin \gamma_{a_1}^n \cdot \frac{d}{2} + F_{a_2}^n \cdot \cos \alpha_{i_2}^n \cdot \sin \gamma_{a_2}^n \cdot \frac{d}{2} + \quad (3)
\]

\[
F_{a_1}^n \cdot \cos \alpha_{i_1}^n \cdot \sin \gamma_{a_1}^n \cdot \frac{d}{2} + F_{a_2}^n \cdot \cos \alpha_{i_2}^n \cdot \sin \gamma_{a_2}^n \cdot \frac{d}{2} \cdot \quad = \quad J_{a_2a_4} \cdot \omega^n_{a_2a_4}
\]

\[\text{Figure 4. The moment balance analysis of moving platform branch } a_2a_4 \text{ around the output spindle axis } n\]

In the formula (3), \( F_{a_1}^n , F_{a_2}^n \) means the two stress components of the moving platform hinge point \( a_1 \) that are respectively along the \( x \) and \( y \) axis of the space fixed coordinate system. \( \alpha_{i_1}^n , \alpha_{i_2}^n \) means the angle between the \( x \) and \( y \) axis of the space fixed coordinate system and the plane \( P_1 \) that include these moving platform hinge point \( a_1, a_2, a_3 \) and \( a_4 \). \( \gamma_{a_1}^n , \gamma_{a_2}^n \) means the angle between the moving platform branch \( a_1a_4 \) and the projection of \( F_{a_1}^n , F_{a_2}^n \) in plane \( P_2 \). \( \gamma_{a_1}^n , \gamma_{a_2}^n \) means the angle between the moving platform branch \( a_2a_4 \) and the projection of \( F_{a_1}^n , F_{a_2}^n \) in plane \( P_2 \). \( J_{a_2a_4} \) means the moment of inertia of the moving platform branch \( a_2a_4 \) around the moving platform center point \( P \) that parallel to the output spindle axis \( n \). \( \omega^n_{a_2a_4} \) means the angular acceleration component of the moving platform branch \( a_2a_4 \) around the moving platform center point \( P \) that parallel to the output spindle axis \( n \).

According to the derivation process of formula (3), the Euler equation of the moving platform branch \( a_2a_3 \) with the moving platform center point \( P \) as the fulcrum around the output spindle axis \( n \) (as shown in Fig. 5) can be obtained, as follows.

\[
F_{a_1}^n \cdot \cos \alpha_{i_1}^n \cdot \sin \gamma_{a_1}^n \cdot \frac{d}{2} + F_{a_2}^n \cdot \cos \alpha_{i_2}^n \cdot \sin \gamma_{a_2}^n \cdot \frac{d}{2} + \quad (4)
\]

\[
F_{a_1}^n \cdot \cos \alpha_{i_1}^n \cdot \sin \gamma_{a_1}^n \cdot \frac{d}{2} + F_{a_2}^n \cdot \cos \alpha_{i_2}^n \cdot \sin \gamma_{a_2}^n \cdot \frac{d}{2} \cdot \quad = \quad J_{a_2a_3} \cdot \omega^u_{a_2a_3}
\]

\[\text{Figure 5. The moment balance analysis of moving platform branch } a_2a_3 \text{ around the output spindle axis } n\]
In the formula (4), $F_{a_1}^x$, $F_{a_1}^y$ means the two stress components of the moving platform hinge point $a_1$ that are respectively along the $x$ and $y$ axis of the space fixed coordinate system. $\gamma_{a_1}^x$, $\gamma_{a_1}^y$ means the angle between the moving platform branch $a_1a_2$ and the projection of $F_{a_1}^x$, $F_{a_1}^y$ in plane $P$. $\phi_{a_1}^x$, $\phi_{a_1}^y$ means the angle between the moving platform branch $a_1a_3$ and the projection of $F_{a_1}^x$, $F_{a_1}^y$ in plane $P$. $J_{a_1}$ means the moment of inertia of the moving platform branch $a_1a_3$ around the moving platform center point $P$ that parallel to the output spindle axis $n$. $\omega_{a_1}^{\omega_{a}}$ means the angular acceleration component of the moving platform branch $a_1a_3$ around the moving platform center point $P$ that parallel to the output spindle axis $n$.

The output moving platform consists of branch $a_1a_4$ and $a_2a_5$ connected by rotating pair, the Euler equation with the hinge point $a_1$ and $a_2$ as the fulcrum around the axis $a_1a_2$ (as shown in Fig. 6) can be obtained, as follows,

$$
(F_{a_1}^x \cdot \sin \gamma_{a_1}^x \cdot \sin \gamma_{a_1}^{\omega_{a}} + F_{a_1}^y \cdot \sin \alpha_{a_1}^x \cdot \sin \beta_{a_1}^{\omega_{a}} + F_{a_1}^x \cdot \sin \alpha_{a_1}^y \cdot \sin \gamma_{a_1}^{\omega_{a}}) \cdot \frac{d}{z} \cdot \sin \phi_{a_1}^{\omega_{a}} + 
(F_{a_1}^x \cdot \sin \gamma_{a_1}^x \cdot \sin \gamma_{a_1}^{\omega_{a}} + F_{a_1}^y \cdot \sin \alpha_{a_1}^y \cdot \sin \alpha_{a_1}^{\omega_{a}} + F_{a_1}^y \cdot \sin \alpha_{a_1}^y \cdot \sin \gamma_{a_1}^{\omega_{a}}) \cdot \frac{d}{z} \cdot \sin \phi_{a_1}^{\omega_{a}} + 
(F_{a_1}^x \cdot \sin \alpha_{a_1}^y \cdot \sin \gamma_{a_1}^{\omega_{a}} + F_{a_1}^y \cdot \sin \alpha_{a_1}^y \cdot \sin \beta_{a_1}^{\omega_{a}} + F_{a_1}^y \cdot \sin \alpha_{a_1}^y \cdot \sin \gamma_{a_1}^{\omega_{a}}) \cdot \frac{d}{z} \cdot \sin \phi_{a_1}^{\omega_{a}} + 
M_z \cdot \cos \phi_{a_1}^{\omega_{a}} = J_{a_1} \cdot \omega_{a_1}^{\omega_{a}};
$$

Figure 6. The moment balance analysis of moving platform with $a_1$ and $a_2$ as the fulcrum

In the formula (5), $F_{a_1}^z$, $F_{a_1}^x$, $F_{a_1}^y$ means the stress components of the moving platform hinge point $a_1$, $a_4$ and center point $P$ that are along the $z$ axis of the space fixed coordinate system. $\alpha_{a_1}^z$ means the angle between the plane $P_a$ and the $z$ axis of the space fixed coordinate system. $\gamma_{a_1}^{\beta_{a_1}}$, $\gamma_{a_1}^{\phi_{a_1}}$, $\phi_{a_1}^{\alpha_{a_1}}$ means the angle between the axis $a_1a_2$ and the projection of $F_{a_1}^x$, $F_{a_1}^y$, $F_{a_1}^z$ in plane $P_a$. $\beta_{a_1}^{\alpha_{a_1}}$, $\beta_{a_1}^{\phi_{a_1}}$, $\beta_{a_1}^{\gamma_{a_1}}$ means the angle between the axis $a_1a_3$ and the projection of $F_{a_1}^x$, $F_{a_1}^y$, $F_{a_1}^z$ in plane $P_a$. $\xi_{a_1}^\alpha$, $\xi_{a_1}^\beta$, $\xi_{a_1}^\gamma$ means the angle between the axis $a_1a_2$ and the projection of $F_{a_1}^x$, $F_{a_1}^y$, $F_{a_1}^z$ in plane $P_a$. $\phi_{a_1}^{\beta_{a_1}}$, $\phi_{a_1}^{\phi_{a_1}}$, $\phi_{a_1}^{\gamma_{a_1}}$ means the angle between the axis $a_1a_3$ and the projection of $F_{a_1}^x$, $F_{a_1}^y$, $F_{a_1}^z$ in plane $P_a$. $\alpha_{a_1}^z$, $\beta_{a_1}^{\alpha_{a_1}}$, $\beta_{a_1}^{\phi_{a_1}}$, $\beta_{a_1}^{\gamma_{a_1}}$, $\phi_{a_1}^{\beta_{a_1}}$, $\phi_{a_1}^{\phi_{a_1}}$, $\phi_{a_1}^{\gamma_{a_1}}$ means the angle between the axis $a_1a_2$ and the $y$ axis of the space fixed coordinate system. $M_y$ means the component of moment of couple $M$ acting on the output moving platform around the $y$ axis of the space fixed coordinate system. $J_{a_1}$ means the moment of Inertia of the moving platform around the axis $a_1a_2$. $\omega_{a_1}^{\omega_{a}}$ means the angular acceleration component of the moving platform around the axis $a_1a_2$.

Similarly, the Euler equation with the hinge point $a_1$ and $a_3$ as the fulcrum around the axis $a_1a_3$ (as shown in Fig. 7) can be obtained, as follows:

$$
(F_{a_1}^x \cdot \sin \gamma_{a_1}^x \cdot \sin \gamma_{a_1}^{\omega_{a}} + F_{a_1}^y \cdot \sin \alpha_{a_1}^x \cdot \sin \beta_{a_1}^{\omega_{a}} + F_{a_1}^z \cdot \sin \alpha_{a_1}^x \cdot \sin \gamma_{a_1}^{\omega_{a}}) \cdot \frac{d}{z} \cdot \sin \phi_{a_1}^{\omega_{a}} + 
(F_{a_1}^x \cdot \sin \gamma_{a_1}^x \cdot \sin \gamma_{a_1}^{\omega_{a}} + F_{a_1}^y \cdot \sin \alpha_{a_1}^y \cdot \sin \alpha_{a_1}^{\omega_{a}} + F_{a_1}^y \cdot \sin \alpha_{a_1}^y \cdot \sin \gamma_{a_1}^{\omega_{a}}) \cdot \frac{d}{z} \cdot \sin \phi_{a_1}^{\omega_{a}} + 
(F_{a_1}^x \cdot \sin \alpha_{a_1}^y \cdot \sin \gamma_{a_1}^{\omega_{a}} + F_{a_1}^y \cdot \sin \alpha_{a_1}^y \cdot \sin \beta_{a_1}^{\omega_{a}} + F_{a_1}^y \cdot \sin \alpha_{a_1}^y \cdot \sin \gamma_{a_1}^{\omega_{a}}) \cdot \frac{d}{z} \cdot \sin \phi_{a_1}^{\omega_{a}} + 
M_z \cdot \cos \phi_{a_1}^{\omega_{a}} = J_{a_1} \cdot \omega_{a_1}^{\omega_{a}};
$$

Similarly, the Euler equation with the hinge point $a_1$ and $a_3$ as the fulcrum around the axis $a_1a_3$ (as shown in Fig. 7) can be obtained, as follows:
In the formula (6), $\gamma_{a_1}^{a_2}, \gamma_{a_1}^{a_3}, \gamma_{P}^{a_2}$ means the angle between the axis $a_1,a_3$ and the projection of $F_{a_1}^{x}, F_{a_1}^{y}$ in plane $P_a$, $\beta_{a_1}^{a_2}, \beta_{a_1}^{a_3}, \beta_{P}^{a_2}$ means the angle between the axis $a_1,a_3$ and the projection of $F_{a_1}^{x}, F_{a_1}^{y}, F_{P}$ in plane $P_a$. $\xi_{a_1}, \xi_{a_1}, \zeta_{a_1}, \zeta_{P}$ means the angle between the axis $a_1,a_3$ and the moving platform branch $a_1,a_4$ and $a_2,a_3$, $\phi_{a_1}^{a_2}$ means the angle between the axis $a_1,a_3$ and the $x$ axis of the space fixed coordinate system. $M_a$ means the component of moment of couple $M$ acting on the output moving platform around the $x$ axis of the space fixed coordinate system. $J_a$ means the moment of Inertia of the moving platform around the axis $a_1,a_3$. $\omega_{a_1}^{a_2}$ means the angular acceleration component of the moving platform around the axis $a_1,a_3$.

Similarly, the Euler equation with the hinge point $a_3$ and $a_4$ as the fulcrum (as shown in Fig. 8) can be obtained, as follows:

$$
\begin{align*}
(F_{a_1}^{x} \cdot \sin \alpha_{e}^{x} + F_{a_1}^{y} \cdot \sin \alpha_{e}^{y} + F_{a_1}^{z} \cdot \sin \alpha_{e}^{z} + F_{a_1}^{v} \cdot \sin \alpha_{e}^{v} - \sin \alpha_{e}^{w} \cdot d \cdot \sin \phi_{a_1}^{a_2}) \\
(F_{a_1}^{x} \cdot \sin \alpha_{e}^{x} + F_{a_1}^{y} \cdot \sin \alpha_{e}^{y} + F_{a_1}^{z} \cdot \sin \alpha_{e}^{z} + F_{a_1}^{v} \cdot \sin \alpha_{e}^{v} - \sin \alpha_{e}^{w} \cdot d \cdot \sin \phi_{a_1}^{a_2}) \\
(F_{a_1}^{x} \cdot \sin \alpha_{e}^{x} + F_{a_1}^{y} \cdot \sin \alpha_{e}^{y} + F_{a_1}^{z} \cdot \sin \alpha_{e}^{z} + F_{a_1}^{v} \cdot \sin \alpha_{e}^{v} - \sin \alpha_{e}^{w} \cdot d \cdot \sin \phi_{a_1}^{a_2}) \\
M_x \cdot \cos \phi_{a_1}^{a_2} = J_a \cdot \omega_{a_1}^{a_2}
\end{align*}
$$

In the formula (7), $\gamma_{a_1}^{a_2}, \gamma_{a_1}^{a_3}, \gamma_{P}^{a_2}$ means the angle between the axis $a_1,a_4$ and the projection of $F_{a_1}^{x}, F_{a_1}^{y}, F_{P}$ in plane $P_a$, $\beta_{a_1}^{a_2}, \beta_{a_1}^{a_3}, \beta_{P}^{a_2}$ means the angle between the axis $a_1,a_3$ and the projection of $F_{a_1}^{x}, F_{a_1}^{y}, F_{P}$ in plane $P_a$. $\xi_{a_1}, \xi_{a_1}, \zeta_{a_1}, \zeta_{P}$ means the angle between the axis $a_1,a_3$ and the moving platform branch $a_1,a_4$ and $a_2,a_3$, $\phi_{a_1}^{a_2}$ means the angle between the axis $a_1,a_3$ and the $y$ axis of the space fixed coordinate system. $J_a$ means the moment of Inertia of the moving platform around the axis $a_1,a_4$. $\omega_{a_1}^{a_2}$ means the angular acceleration component of the moving platform around the axis $a_1,a_4$.

Similarly, the Euler equation with the hinge point $a_2$ and $a_4$ as the fulcrum (as shown in Fig. 9) can be obtained, as follows:

$$
\begin{align*}
(F_{a_1}^{x} \cdot \sin \alpha_{e}^{x} + F_{a_1}^{y} \cdot \sin \alpha_{e}^{y} + F_{a_1}^{z} \cdot \sin \alpha_{e}^{z} + F_{a_1}^{v} \cdot \sin \alpha_{e}^{v} - \sin \alpha_{e}^{w} \cdot d \cdot \sin \phi_{a_1}^{a_2}) \\
(F_{a_1}^{x} \cdot \sin \alpha_{e}^{x} + F_{a_1}^{y} \cdot \sin \alpha_{e}^{y} + F_{a_1}^{z} \cdot \sin \alpha_{e}^{z} + F_{a_1}^{v} \cdot \sin \alpha_{e}^{v} - \sin \alpha_{e}^{w} \cdot d \cdot \sin \phi_{a_1}^{a_2}) \\
(F_{a_1}^{x} \cdot \sin \alpha_{e}^{x} + F_{a_1}^{y} \cdot \sin \alpha_{e}^{y} + F_{a_1}^{z} \cdot \sin \alpha_{e}^{z} + F_{a_1}^{v} \cdot \sin \alpha_{e}^{v} - \sin \alpha_{e}^{w} \cdot d \cdot \sin \phi_{a_1}^{a_2}) \\
M_x \cdot \cos \phi_{a_1}^{a_2} = J_a \cdot \omega_{a_1}^{a_2}
\end{align*}
$$
In the formula (8), $\gamma_{a_i a_1}^{a_2 a_4}$ means the angle between the axis $a_1 a_4$ and the projection of $F_{a_1}^x, F_{a_4}^x, F_{p}^x$ in plane $P$. $\beta_{a_1 a_4}^{a_2 a_4}$ means the angle between the axis $a_2 a_4$ and the projection of $F_{a_1}^y, F_{a_4}^y, F_{p}^y$ in plane $P$. $\phi_{a_1 a_4}^{a_2 a_4}$ means the angle between the axis $a_1 a_4$ and the moving platform branch $a_1 a_4$ and $a_2 a_4$, $\phi_{a_1 a_4}^{a_2 a_4}$ means the angle between the axis $a_1 a_4$ and the $x$ axis of the space fixed coordinate system. $a_{a_1}$ means the moment of Inertia of the moving platform around the axis $a_{a_1} a_{a_4}$.

From the formula (1) to (8), we can see that there are 12 unknown variables $F_{a_i}^x, F_{a_i}^y, F_{a_i}^z$ ($i = 1,2,3,4$) in these eight expressions, due to the special structure of the mechanism, the relationship between some variables is the following,

$$F_{a_1}^z = F_{a_2}^z = F_{a_3}^z = F_{a_4}^z = F_{p}^z / 4.$$  \hspace{1cm} (9)

Thus it can be known that the formula (1) to (8) contains only 8 unknown variables, so formula (1) to (8) can be expressed in matrix form, as follows:

$$\mathbf{J} \cdot \mathbf{X} = \mathbf{b}.$$  \hspace{1cm} (10)

The force Jacobi matrix $J$ in formula (10) is a matrix composed of the coefficients of 8 unknown variables in formula (1) to (8), the $X$ is a column vector composed of variables $[F_{a_1}^x, F_{a_2}^x, F_{a_3}^x, F_{a_4}^x, F_{a_1}^y, F_{a_2}^y, F_{a_3}^y, F_{a_4}^y, F_{a_1}^z, F_{a_2}^z, F_{a_3}^z, F_{a_4}^z]^T$, the $b$ is a column vector consisting of known items of formula (1) to (8) without 8 unknown variables. The unique definite solution can be obtained. When the space position and posture of mechanism are given, the stress $F$ and moment of couple $M$ of the output spindle are known, the Jacobi matrix $J$ is nonsingular matrix.

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Reference


