The Design on Lunar Probe Soft Landing and Its Optimal Strategy

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Abstract. The main research in the paper tells us an optimal control strategy of Chang’e III in the process of the lunar probe soft landing. After entering into the orbit, the process of soft landing is divided into 5 stages including deceleration, quick adjustment, obstacle, avoidance and precise obstacle. According to the Newton’s Second Theorem and coordinate transform, the equation on each stage can be established. What’s more, on the basis of the Bellman Optimality Theory and according to the maximum principle, the extremum that meets the boundary condition can be calculated. Eventually, the optimal control strategy can be obtained and the calculation of substitution will be the optimal track.

Introduction

While Chang’e-III was in high-speed flight, to ensure the precision of the landing point in a predetermined area, we need a particular design for trajectories of Lunar Soft-Landing and its landing control strategy. The soft landing process can be divided into six stages, so one need to know the state at each critical point and minimize the fuel consumption in the process of soft landing to the greatest extent.

Model Hypothesis

(1) As the landing process of Chang-e III is relatively short, we ignore some of the elements such as the moon's gravity and the gravitational perturbation, so that a two-body model can be applied.
(2) The moon is a standard sphere of uniform quality.
(3) Do not consider the impact of the moon's rotation on Chang-e III .
(4) In the soft landing process of Chang-e III, the moon is always moving into the same orbit and the orbit of Chang-e III traveling around the moon is a polar orbit.

The Details

Assume that after Chang-e III lands on the ready track, the main thrust direction of the engine deceleration will provide a speed in the opposite direction. Soft landing process is a multistage decision issue that can be divided into following phases: Main reducer section \([ t_0, t_1 ]\), Rapid adjustment phase \([ t_1, t_2 ]\), Rough obstacle avoidance phase \([ t_2, t_3 ]\), Precision obstacle avoidance phase \([ t_3, t_4 ]\) and down stage at an average lower speed, in which, \( t_0 = 0 , t_5 = 735 \) , unit in second.

Search for an optimal control strategy which meets the boundary conditions at every stage.

\[
\left\{u_i(t) \right\}_{t_{i-1}} \leq t \leq t_i, i = 1,2,3,4,5
\]

Make total index \( J[u_i(t)] = \int_{t_{i-1}}^{t_i} -u_i(t) dt \) obtain the minimum value, and find the optimal trajectory of Chang'e III on soft landing correspondingly. \( \left\{S_i'(t) \right\}_{t_{i-1}} \leq t \leq t_i, i = 1,2,3,4,5 \).

Firstly, several coordinate systems are defined. The core of moon refers to the frame \( o_x, y, z \); the coordinates of the point O is located in the center of the moon, the axis \( z \) points to the initial soft landing point from the center of the moon, and the axis \( x \) is in the plane of the orbit of the moon and
ahead to the forward direction. Rectangular axis \(y_r, x_r\) and \(z_r\) constitute a coordinate system which is only applied in the calculation of descent trajectory of soft landing.

The descent orbit refers to the coordinate system \(\alpha x_o y_o z_o\): The coordinate origin \(o\) was located in the center of the land. Axis \(x_o\) was designed at the local level and pointed to the direction of the lander. Axis \(y_o, x_o\) and \(z_o\) constitute a coordinate system.

The coordinate system \(\alpha x_b y_b z_b\) of the lander: Coordinate origin \(o\) was located in the centroid of Lander. Axis \(x_b\) is in the extended line of thrust vector braking force. The thrust is in a forward direction. According to the installation of the equipment, axis \(y_b\) and \(z_b\) are set up respectively which constitute a coordinate system with axis \(x_b\). The relationship between the sketch map of coordinate system and the positions of lander and thrust vector are shown in Figure 1.

![Figure 1. Axis figure.](image)

After solving the known formula, the placement coordinates can be obtained as below:

\[
\begin{align*}
x_r &= 826.87 \text{km} \\
y_r &= 236.93 \text{km} \\
z_r &= -906.60 \text{km}
\end{align*}
\]

We can obtain the coordinate of any point \((x, y, z)\) of the line in the selenocentric inertial system to be \((0.66x, 0.69y, 0.17z)\)

Decision variables are \(u_0(t) = \sqrt{u^2_{0,1}(t) + u^2_{0,2}(t) + u^2_{0,3}(t)}\)

Boundary conditions: \(S_{f}(t_f)\) is known,

\[|v_0(t_f)| = 57,0 < \alpha(t_f) < \frac{\pi}{2}, 0 < \beta(t_i) < \frac{\pi}{2}, m(t_i) > 0\]

The allowed decision collection is

\[\Omega = \{1500 \leq |u_0(t)| \leq 7500\}\]

Stage performance was marked at

\[J_{1} = \int_{0}^{t} |u_0(t)| \, dt\]

In literature [7], the formula gives the three-dimensional soft landing process in rocket descending orbit reference coordinate system, on the premise of not considering perturbation effect. Since the thrust direction provided by main deceleration engine is opposite to the direction of speed, the thrust azimuth angle \(\phi\) and thrust elevation angle \(\theta\) between \(u_{0,r}(t)\) and coordinate axis \(o x_0 y_0 z_0\) should meet the following condition.
\[ \theta = \arcsin \sqrt{v_{0,1}^2(t) + v_{0,2}^2(t) \over v_0(t)}, \ \phi = \pi \]

As a result of this, the state equation of Chang’ E-III in the final decelerating stage is

\[
\frac{d[S_r(t)]}{dt} = v_{0,3}(t),
\]

\[
\frac{d[\alpha(t)]}{dt} = \frac{u_{0,2}(t)}{S_r(t) \sin \beta(t)},
\]

\[
\frac{d[\beta(t)]}{dt} = \frac{u_{0,1}(t)}{|S_r(t)|},
\]

\[
\frac{du_{0,1}(t)}{dt} = -\sqrt{v_{0,1}^2(t) + v_{0,2}^2(t) \over v_0(t)} \left[ \frac{v_0(t)}{m(t)} - u_{0,1}(t)u_{0,3}(t) / |S_r(t)| + v_{0,2}(t) / (|S_r(t)| \tan \beta(t)) \right],
\]

\[
\frac{du_{0,2}(t)}{dt} = -u_{0,2}(t)u_{0,3}(t) / |S_r(t)| - v_{0,2}(t)v_{0,3}(t) / (|S_r(t)| \tan \beta(t)),
\]

\[
\frac{du_{0,3}(t)}{dt} = \left[ \frac{v_0(t)v_{0,3}(t)}{|v_0(t)|m(t)} - \mu_m / |S_r(t)|^2 + (u_{0,1}^2(t) + u_{0,2}^2(t)) / |S_r(t)| \right],
\]

\[
\frac{dm}{dt} = -v_0(t).
\]

In precise obstacle avoidance phase, it is required to build a vertical descending dynamic model. In the boundary conditions, \( h(t_4) \) is known, \( v(t_4)=0, \) \( m(t_4) \) is free, and \( S_{0,1}(t_4) \) and \( S_{0,2}(t_4) \) are known. Except \( S_{0,1}(t_4) \) and \( S_{0,2}(t_4) \), the established state equation accords with the course obstacle avoidance state equation.

The state equation is consistent with the equation of the state for the coarse barrier.

Optimal principle: For the decisive problems under the condition of multi-stage are concerned, regardless of the previous outcome, for the current state of the formation, the rest of the decisions are bound to constitute the best strategy for each. Use the terminal condition starting from the last equation and push forward against the basic equations to obtain the strategical decisions and optimal functions in each state.

Maximum principle: The state equation is considered as

\[ x(t) = f(x, u) \]

Functional target for the mixed performance indicates that

\[ J[u(t)] = \Phi(x(t)) + \int_{t_0}^{t_1} f_0(x(t), u(t))dt \]

The maximum value of the objective functional can be achieved at the end constraint relationships as follows:

\[ N_i (t_1, x_1(t_1), \ldots, x_n(t_1)), \quad 0 \leq i \leq 1, \quad r \leq r \leq n \]

Introduce Langrange multipliers and penalty methods
\[
\dot{\lambda}(t) = (\lambda_1(t), \lambda_2(t), \ldots, \lambda_n(t))^T
\]

Functional is:

\[
\mathcal{J}(x, \mu, \lambda) = \sum_{i=1}^r \mu_i N_i(t_i, x(t_i)) + \int_0^t [f_0(t, x, u, \lambda) + \dot{\lambda}^T(t)(f(t, x, u) - \dot{x})]dt
\]

The above problems concerned with optimal control are converted to the problems of non-state constraint and terminal constraint. For convenience, it can be made like

\[
H(t, x, u, \lambda) = f_0(t, x, u, \lambda) + \dot{\lambda}^T(t)f(t, x, u)
\]

\[
H(t, x, u, \lambda) = H(t, x, u, \lambda) - \lambda^T(t)\dot{x}
\]

If Continuous partial derivatives \((t, x)\) and \(f(t, x, u)\) are concerned about \(t\) and \(x\), the optimal controlling can be achieved at the maximum performance and status of the above-mentioned constraints as if \(u(t)\) with \(t_1\). Therefore, \(x(t)\) would be the correspondingly optimal trajectory, thus there must be a vectorial function \(t\) which is used to fulfill auxiliary equation:

\[
\dot{\lambda}(t) = -\frac{\partial H}{\partial x}
\]

with terminal conditions

\[
\lambda(t_1) = \frac{\partial}{\partial x(t_1)}[\Phi(t_1, x(t_1)) + \sum_{i=1}^r \mu_i N_i(t_1, x(t_1))]
\]

The controlling function \(u(t)\) meets \(H(t, x, u, \lambda) \max H(t, x, u, \lambda) (t, u) (t)\)

The value of the Hamiltonian function at the termination time of \(t_1\) is

\[
H(t_1, x^*(t_1), \lambda^*(t_1), u^*(t_1)) = -\frac{\partial}{\partial t} \Phi(t_1, x^*(t_1))
\]

Controlling variables ought to meet the conditions of

\[
H(t, x^*, \lambda, u^*) = H(t, x^*(t_1), \lambda(t_1), u^*(t_1)) + \int_0^t \frac{\partial}{\partial t} H(s, x^*(s), \lambda(s), u^*(s))ds
\]

Termination time \(t_1\) ought to meet the conditions of

\[
(\frac{\partial \Phi}{\partial t} + \sum_{i=1}^r H_i \frac{\partial N_i}{\partial t})\bigg|_{t=t_1} = 0
\]

The theory of the above problems is solved by optima index. It can be known from the last stage that the optimal controlling problems can be recognized by the performance index and the maximum principle. In order to make \(J^*_4(u)\) meet the optimal control, first of all, it’s necessary to make Chang’e III move in an uniform variable rectilinear motion under main thrust of 1500 at the least during \([0, \ 4]\); the moment when the switch function is 0 is right the time when Chang’e III main thrust converts from minimum 1500 to maximum 7500, namely, it moves in an uniform decelerated motion under main thrust of 7500 during the stage of \([\ 4, t_4]\).

By using the state equation and boundary value terms of the slowly descent stage, the relationship between the switching function and the \(t_4\) can be determined at 0.

According to the same method, the backward recursion is to be considered as

\[
J^*_3(u) = \max_{1500 \leq u_{0,3}(t) \leq 7500} \left( -\int_0^t u_{0,3}(t)\,dt + J^*_4(u) \right)
\]

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\[ J_2^*(u) = \max_{1500 \leq u_1 \leq 23,0750} (-\int_{u_2}^{u_2} u_{0,3}(t)dt + J_3^*(u)) \]

\[ J_1^*(u) = \max_{1500 \leq u_1 \leq 23,0750} (-\int_{u_1}^{u_2} u_{0,3}(t)dt + J_2^*(u)) \]

\[ J_0^*(u) = \max_{1500 \leq u_1 \leq 23,0750} (-\int_{u_1}^{u_1} u_{0,3}(t)dt + J_1^*(u)) \]

Optimal strategy can be obtained in turn by the terms of

\[ \{u^*_i(t)\}_{t_{i-1}}^{t_i} \leq t \leq t_i, i = 1,2,3,4,5 \].

By using the state equation, the solution can be obtained, and the optimal trajectory is gained as below:

\[ \{S^*_i(t)\}_{t_{i-1}}^{t_i} \leq t \leq t_i, i = 1,2,3,4,5 \].

Finally, the simulation flight curve of the main deceleration is given by Chang’e-III:
Conclusion

In this paper, we mainly study the design of obit for soft landing about trajectory and the optimal control strategy of Chang'e-III. After it goes into the orbit, the process of soft landing mainly includes deceleration, quick adjustment, coarse obstacle avoidance, precise obstacle avoidance, fall at a slow speed. Newton's Second Theorem and the coordinate transform is utilized to establish the state equation of each stage. According to the Bellman Optimality Theory, we follow back and use the maximum principle to calculate the extremum which meets the boundary conditions correspondingly, and obtain the optimal control strategy. Once substituting and solving these, the optimal trajectory process can be got.

References


