Parametric Acoustic Receiving Array in Two-phase Media

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Abstract. Parametric acoustic receiving array can achieve higher space gain with relatively small (virtual) array aperture compare with ordinary line array. The amplitude of the secondary signals of parametric receiving array is proportional to the third power of the acoustic velocity of the medium, and two-phase media is much more practical than single medium with slow acoustic velocity in underwater acoustic engineering. Two-phase media model and equations for the secondary signals of parametric acoustic array are proposed and some calculation examples are given and analyzed.

Introduction

Parametric acoustic receiving array exploits nonlinear acoustic effects in fluids to form virtual end-fire array to realize directive reception of low frequency acoustic signal[1]. It needs only a pair of transmitting-receiving high frequency acoustic transducers to form a parametric receiving array to achieve directive reception of low frequency signal, and is suitable for some kind of applications in underwater acoustic engineering where the space for the apparatus is limited in some way.

Fig. 1 shows the principle of parametric receiving array: a strong high frequency signal (called pump signal, with frequency $f_0$) projected from the pump transducer meets the weak low frequency signal (signal to be detected, with frequency $f_s$) in the nonlinear interaction zone. On account of the nonlinear acoustic effects, the difference and sum frequency signals (secondary signals) are generated by the virtual source (from nonlinear interaction) through the whole nonlinear interaction zone. These secondary signals will be accumulated from the position of pump transducer to the position of the high frequency receiver which is on the pump wave axis and apart from the pump transducer by the distance $L$. $L$ is called the array length of parametric receiving array. $\theta$ is the angle between the direction of low frequency signal and the pump wave axis. The secondary signals...
received by the high frequency receiver are accumulated through the length L and have the property of an end-fire array with length L, which is directional. The low frequency signal to be detected can be resumed by demodulation of the secondary signal.

The amplitude of the secondary signals of parametric receiving array is proportional to the third power of the acoustic velocity of the medium \([2]\). If the acoustic velocity of the medium is much slower than that of the water, much larger secondary signals can be obtain and make it easier to be detected in underwater acoustic engineering \([3]\). Usually the length of parametric acoustic receiving array is as long as more than 100m in order to get large space gain, this makes it more suitable to use medium of slow acoustic velocity only in part of the whole length, and with the remain part still water. That means the parametric receiving array is in two-phase media. There is no suitable model for the prediction of the property of parametric receiving array in two-phase media right now.

**Model and Equations**

Fig. 2 is the diagram of a two-phase media parametric receiving array. The slow acoustic velocity medium is on the side of the pump transducer, expanding from the pump transducer to \(L_0\) along the pump wave axis. \(L_0\) is smaller than Reighly distance and the pump wave can be considered as plane wave in this medium. The pump wave is spherical wave in water as normal parametric acoustic array \([2]\). Then, the total secondary signal received by the high frequency receiver is combined by two parts: those generated in the slow acoustic velocity medium traveled to the receiver and the those generated in water traveled to the receiver.

There are two kinds of virtual source: \(q_0\) in water and \(q_1\) in the slower acoustic velocity medium. The low frequency signal is assumed to be plane wave signal, and only signals on the pump wave axis need to be considered \([2]\), from the equation (3) in reference \([2]\) we have

\[
q_0 = \frac{(\omega_1 + \omega_2)\beta}{\rho_0 c_0^4} p_1 p_2 \rho_2^2 \rho_0^2 e^{-\alpha L_0 - \alpha' r - j(k_1 L_0 + k_2 r + k_{12} R)} / (r - L_0) \quad r > L_0
\]

\[
q_1 = \frac{(\omega_1 + \omega_2)\beta}{\rho_0^2 c_0^4} p_1' p_2' e^{-\alpha' r - j(k_1' r + k_{12}' R)} \quad r \leq L_0
\]

Here \(\omega_1\) and \(\omega_2\) is the angular frequency of pump and low frequency signal, respectively. \(\rho_0, c_0, \rho_0', c_0'\) is the density and acoustic velocity in water, and the density and acoustic velocity in the slow acoustic velocity medium, respectively. \(\alpha, \alpha'\) is the absorption coefficient in water and in the slow acoustic velocity medium, respectively. \(k_1, k_2, k_1', k_2'\) is the wave number of pump and low frequency signal in water, and the wave number of pump and low frequency signal in the slow acoustic velocity medium, respectively. \(r\) and \(R\) is the direction vector of pump and low frequency signal, respectively. \(R = r \cos(\theta)\). \(P_1\) is the source level when no slow acoustic velocity medium exists.
and A is the transmission loss when the pump wave traveling from slow acoustic velocity medium to water.

**a) Secondary signal generated by the virtual sources in the slow acoustic velocity medium.**

P'\(1\) is the amplitude of the pump wave on the surface of a piston pump source. When there is no slow acoustic velocity medium adhere to the transducer, the amplitude on the surface is \(P_s = P_0 \lambda / S\). Considering the acoustic power projected is constant, and

\[
E = \frac{P_1^2}{\rho_0 c_0} = \frac{P_{s1}^2}{\rho_0 c_0} \frac{\rho_0 c_0}{\rho_0 c_0} \tag{3}
\]

E is the acoustic power, Then

\[
P'_1 = (P_0 \lambda / S) \frac{\rho_0 c_0}{\rho_0 c_0} \tag{4}
\]

P_2 and P'_2 is the amplitude of the low frequency signal in water and in the slow acoustic velocity medium, respectively. For plane pump wave, the particle velocity at point \((r, 0)\) is

\[
\delta u = \delta_1 \times \delta r \tag{5}
\]

From point \((L_0, 0)\) the pump signal will be expanding spherically, and the amplitude of the secondary signals generated by the virtual source in the slow acoustic velocity medium will reduced to \(1 / (L - L_0)\) of the original when travelling to the point \((L, 0)\). The acoustic absorption and the phase change of the signals should be considered separately in these two media. Then

\[
\delta u_L = A \frac{\delta u}{L - L_0} e^{-(a'/(L_0 - r) - a(L - L_0) - j(k'_1(L_0 - r) + k_1(L - L_0)))} \tag{6}
\]

Here A is the transmission loss of the sum frequency signal (the sum frequency is very close to the pump wave frequency, this loss is almost no difference), and \(k_1, k'_1, k_1, k'_1\) is the wave number of sum frequency signal in water and in the slow acoustic velocity medium respectively. Only sum frequency is considered here because the principle for the difference frequency signal is the same. Then, the contribution of the sum frequency signal at the point of receiver from the virtual sources in the slow acoustic velocity medium is

\[
P_0 = \rho_0 c_0 \int_0^{L_0} \delta u_L = \rho_0 c_0 \int_0^{L_0} \left[ A \frac{\delta u}{L - L_0} e^{-(a'/(L_0 - r) - a(L - L_0) - j(k'_1(L_0 - r) + k_1(L - L_0)))} \right] dr \tag{7}
\]

\[
= \rho_0 c_0 \int_0^{L_0} \frac{\kappa (\omega_0 + \omega_1) \beta}{(\rho_0 c_0^2 \rho_0 c_0^2) (L - L_0)} \int_0^{L_0} P_1 P'_1 e^{-a'/(L_0 - r) - a(L - L_0) - j(k'_1(L_0 - r) + k_1(L - L_0))} P_2 e^{-a'/(L - r) - a(L - L_0) - j(k_1(L - r) + k'_1(L_0 - r))} \int_0^{L_0} e^{j(k'_1(L_0 - r) + k_1(L - r))} \delta r \tag{8}
\]

\[b) \text{Secondary signal generated by the virtual sources in water}\]

Since the pump wave in water is spherical wave, the particle velocity at point \((r, 0)\) when \(r > L_0\) is:

\[
\delta u' = \frac{1}{2} q_0 \times \delta r \tag{8}
\]

The signals generated by the virtual sources in water are expanded spherically. The expanding radium at point \((r, 0)\) when \(r > L_0\) is \(r - L_0\) while the expanding radium at point \((L, 0)\) is \(L - L_0\), then the amplitude of the signal generated at point \((r, 0)\) will reduced to \((r-L_0) / (L-L_0)\) of the original when travelling to the point \((L, 0)\), then
The contribution of the sum frequency signal at the receiver from the virtual sources in water is

\[
\delta u'_{\perp} = A \frac{(r - L_0)}{L - L_0} e^{(-a_1(L-r)-\beta_1(L-r))} \]

(9)

Finally, the sum frequency signal generated by the parametric receiving array shown in fig. 2 will be

\[
P_d = P_n + P_f
\]

(11)

**Calculation Example**

Rubber with density 0.1, acoustic velocity 1000m/S, absorption coefficient 1dB/m at frequency 100 kHz and nonlinear coefficient 4.97, is chosen for the calculation. Although the acoustic velocity in rubber is much slower than that in water, it is not proper to choose a very large \(L_0\). The strong power absorption of the rubber may cause the increased absorption effects greater than the increased nonlinear effects if \(L_0\) is too large, and the secondary signal may decrease instead of increase. Except the secondary signal is still high enough even if \(L_0\) is very large, and the ratio of the pump signal and the secondary signal may decrease because of the higher nonlinear transmission efficiency in rubber to make it easier to extract the weak secondary signals from the jam caused by the strong pump signal. The transmission loss for the secondary signal traveling from rubber to water is

\[
t_1 = 2 \times \frac{\rho_0 c_0'}{\rho_0 c_0'} = 0.75
\]

The pump source level is chosen to be 230dB re. 1uPa and aperture is 450x225mm2. For the pump frequency of 100 kHz, \(P_s=213\)dB, \(P_1'=211\)dB, and Reighly distance is 6.7m. Sound level of the low frequency signal is 40dB and frequency is 2 kHz. \(L_0=5\)m (0.75 Reighly distance).

Table 1 shows the comparison between parametric receiving array in water and in rubber-water two-phase media.

<table>
<thead>
<tr>
<th>(L_0) (m)</th>
<th>L (m)</th>
<th>Sound level of sum frequency signal (dB)</th>
<th>Ratio of pump and sum frequency signal (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>In water</td>
<td>In two-phase media</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>23</td>
<td>31</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>26</td>
<td>24</td>
</tr>
<tr>
<td>20</td>
<td>20</td>
<td>26</td>
<td>20</td>
</tr>
<tr>
<td>100</td>
<td>100</td>
<td>26</td>
<td>18.5</td>
</tr>
</tbody>
</table>

From table 1, it is seen that if the rubber length \(L_0\) is much small compare to the array length \(L\), there is no advantage for the rubber-water two-phase media. Thus, for this kind of high absorption media like rubber, it is not proper to used them in long aperture parametric receiving array. They may be used form array of parametric receiving arrays [4]. For long-aperture parametric receiving array, much higher nonlinear transmission efficiency and much lower acoustic absorption is necessary, and bubbling water with proper void fraction may be the proper one for more investigation [5].
Reference


