Fundamental Diagram of Bicycle Traffic Based on Logistic Model

Cheng XU\textsuperscript{1,2,\ast}, Xin WANG\textsuperscript{3} and Xiao-nan YU\textsuperscript{1}

\textsuperscript{1}Department of Traffic Management Engineering, Zhejiang Police College, Hangzhou 310053 China
\textsuperscript{2}College of Civil Engineering and Architecture, Zhejiang University, Hangzhou 310058 China
\textsuperscript{3}Department of International School, Zhejiang Police College, Hangzhou 310053 China
\textsuperscript{\ast}Corresponding author

Keywords: Fundamental diagram, Bicycle traffic, Electric bicycle, Logistic model.

Abstract. Bicycle has become one of the important commuting travel ways to residents who is living in developing countries including China. Therefore, it’s of great theoretical interest and industrial significance to study on the fundamental diagram of bicycle traffic flow. According to the operating characteristic, this paper proposes adopting logistic model to modeling the fundamental bicycle traffic flow and the method of model parameters calibration based on the maximum likelihood algorithm. This model was verified by the measured data from three sections of Hangzhou city and estimated the capacity was 2243 bikes/h/m. The fitting precision of this model was significantly improved compared to the traditional model. Based on the study mentioned above, it will provide data supporting to the application of fundamental diagram in the bicycle traffic flow field.

Introduction

Fundamental diagram (FD) is the relationship between density and volume of traffic, and is the important foundation for traffic state identification, capacity estimation, and determination the phase regime of traffic flow. FD is also an important feature which distinguishes traffic flow from other fluids. Therefore, FD is of significance for both motorized vehicle traffic flow and non-motorized traffic flow. Greenshields [1] firstly presented a linear speed-density relationship and parabolic density-volume relationship using the measured field data. After around 80 years’ development of FDs, there are numerous studies of FDs. The famous FD models includes Greenberg model [2], Underwood model [3], Newell model [4], Pipes model [5], and Qu models [6, 7].

Most of the FD models are single-regime models which could fit the empirical data samples well for motorized vehicle traffic flow. However, most of the FD models were presented for motorized vehicle traffic flow. There was almost no FD model proposed for non-motorized vehicles. In recent years, bicycle traffic (including electric bicycles) has been become one of the main trip modes in developing countries, especially in Southeast Asian countries such as China, Vietnam, and Indian. The electric bicycle (E-bike) has quickly become one of the main non-motorized travel modes in China [8-11]. With the rapid increase of bicycles, it is very important to study the traffic flow characteristics of bicycles. Traditional motorized vehicle-oriented FD models cannot adapt to the operating characteristics of bicycle traffic flow. Therefore, the purpose of this paper is to use logistic model for modeling the FD of bicycle traffic flow, and proposed a FD model for estimating the capacity of bicycle flow.

Model

Logistic Model of Bicycle Fundamental Diagram

Logistic curve is a model with many equations and wildly used in population ecology field. Wang et al. [12] presented a model that describes traffic flow speed-density relation. Compared the imitative
effect on logistic models of three or four parameters and five parameters, he used five parameters to model the motor vehicle flow. Concrete equation as follows:

\[ v(k) = v_b + \frac{v_f - v_b}{1 + \exp \left[ \frac{(k-k_c)/\theta_1}{\theta_2} \right]} \]  \hspace{1cm} (1)

where, \( v \) is the speed of bicycle flow, \( k \) is the density of bicycle flow, \( v_f \) is the free flow speed, \( v_b \) is the speed when traffic is on the condition of stop and go, \( k_t \) is the density from free flow to congested flow, \( \theta_1 \) is the parameter which determining the slope of the logistic curve, and \( \theta_2 \) is the parameter which determining the symmetry of the logistic curve.

Based on the basic relationship of volume-speed-density model,

\[ q = kv. \]  \hspace{1cm} (2)

The relationship of density-volume is presented as follow:

\[ q = k \left( v_b + \frac{v_f - v_b}{1 + \exp \left[ \frac{(k-k_c)/\theta_1}{\theta_2} \right]} \right) \]  \hspace{1cm} (3)

where, \( q \) is the volume of bicycle flow.

Considering the effect of E-bikes on bicycle flow, the volume of E-bikes should be converted to bicycle volume using the bicycle equivalent unit for the E-bike,

\[ q = q_b + \beta q_{eb} \]  \hspace{1cm} (4)

where, \( q_b \) is the volume of bicycles, \( q_{eb} \) is the volume of E-bikes, and \( \beta \) is the bicycle equivalent unit for the E-bike which can be set as 0.66 [8].

Based on this model, we can establish the fundamental diagram of traffic flow under the operation of hybrid bicycle. Applying this model can fit the operation characteristic of bicycle better.

**Parameter Calibration**

For Equation (4), the formula is nonlinear, and the traditional method of least squares cannot be used in parameter calibration directly. Therefore, a nonlinear least squares fitting method is proposed for calibrating the model parameters. For equation \( q=f(k) \), given \( f() \) and observation vector \( q \), the main steps of the LM algorithm are as follows:

Step 1: Set initial value \( k_0 \) and stop control constant \( \varepsilon \), and calculate \( \varepsilon_0 = \|q - f(k_0)\| \).

Step 2: Calculate the Jacobi Matrix \( J_k \), and \( \vec{N}_i = J_i^T J_i + \lambda_i I \), then establish the equation \( \vec{N}_i \delta_i = J_i^T \varepsilon_i \).

Step 3: Solve the above equation for \( \delta_i \).

1. If \( \|q - f(k_i + \delta_i)\| < \varepsilon_i \), then \( k_{i+1} = k_i + \delta_i \), else if \( \|\delta_i\| < \varepsilon \), stop iteration, output; else \( \lambda_i = \lambda_i / \nu \), return to Step 2.

2. If \( \|q - f(k_i + \delta_i)\| \geq \varepsilon_i \), then \( \lambda_{i+1} = \nu \lambda_i \), solve the function again and get \( \delta_i \), return to Step 1.

Using these three steps, we can complete the calibration and obtain the parameters that minimize the model error.
Date Collection

Field data collection is of significance for FD model estimation and calibration. In this paper, three bicycle path sections in Hangzhou, China, were selected for data collection. Every bicycle path was one-way, and located in the middle of road links, far away from intersections, which will minimize the effects of traffic signals and pedestrians on bicycle traffic. Camera was set up on the side of the bicycle path to take videos. The filed data was conducted on weekdays during peak and non-peak hours. Using video-processing technology, the volume and speed can be easily recorded automatically.

Table 1 provides the width of bicycle path, percentage of E-bike and percentage of male cyclist from these three surveyed sections. Traffic flow data from these three sections will use to fit and capacity estimate on fundamental diagram of bicycle traffic flow as below.

<table>
<thead>
<tr>
<th>Section name</th>
<th>Width of bicycle path</th>
<th>Percentage of E-bike</th>
<th>Percentage of male cyclist</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jiaogong Road</td>
<td>2.4 m</td>
<td>65.5%</td>
<td>62.7%</td>
</tr>
<tr>
<td>Hushu Road</td>
<td>3.0m</td>
<td>57.9%</td>
<td>67.3%</td>
</tr>
<tr>
<td>Wener Road</td>
<td>3.5 m</td>
<td>59.8%</td>
<td>66.3%</td>
</tr>
<tr>
<td>Overall</td>
<td>-</td>
<td>61.0%</td>
<td>65.5%</td>
</tr>
</tbody>
</table>

Results of Estimation

In order to analyze the advantage of logistic model further, this paper will analyze it by comparing to Greenshields model, Greenberg model and Underwood model individual. The above three model equations as follow:

\[ q = kv_f (1 - k/k_f) \]  \hspace{1cm} (5) \\

\[ q = kv_m \log (k_f/k) \]  \hspace{1cm} (6) \\

\[ q = kv_f \exp (-k/k_m) \]  \hspace{1cm} (7)

Calibrating the above four models’ parameter by applying the maximum likelihood algorithm and draw the relationship diagram between model estimation and field observed data. Table1-3 presents the fundamental diagram which was drawn by field observed data from three sections and the curve which is fitting by the four models. The gray circles are field observed data. The black, red and green solid lines represent the Greenshields model, Greenberg model and Underwood model respectively. The blue dotted line represents the outcomes of logistic model fitting. The logistic model has higher fitting precision to these three different widths of paths from the tables.
In order to analyze the fitting precision of model further, Table 2 gives fitting errors and estimated bicycle capacity of different models. Paper presents the average error of logistic model which gets higher fitting precision than others is around 15%. Meanwhile, the average estimated capacity of these three sections is about 2243 bikes/h/m, which is more correspond to actual fact.
Table 2. Results of density-volume relationships.

<table>
<thead>
<tr>
<th></th>
<th>Jiaogong Road</th>
<th>Hushu Road</th>
<th>Wener Road</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Capacity</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Greenshields</td>
<td>2340</td>
<td>2476</td>
<td>2432</td>
<td>2412</td>
</tr>
<tr>
<td>Greenberg</td>
<td>2530</td>
<td>2432</td>
<td>2412</td>
<td>2473</td>
</tr>
<tr>
<td>Underwood</td>
<td>2101</td>
<td>2098</td>
<td>2023</td>
<td>2062</td>
</tr>
<tr>
<td>Logistic model</td>
<td>2312</td>
<td>2322</td>
<td>2203</td>
<td>2243</td>
</tr>
<tr>
<td><strong>Fitting error</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Greenshields</td>
<td>19.41%</td>
<td>17.62%</td>
<td>18.34%</td>
<td>18.33%</td>
</tr>
<tr>
<td>Greenberg</td>
<td>34.12%</td>
<td>31.32%</td>
<td>28.23%</td>
<td>30.21%</td>
</tr>
<tr>
<td>Underwood</td>
<td>19.32%</td>
<td>17.23%</td>
<td>19.12%</td>
<td>18.54%</td>
</tr>
<tr>
<td>Logistic model</td>
<td>15.32%</td>
<td>14.43%</td>
<td>16.12%</td>
<td>15.12%</td>
</tr>
</tbody>
</table>

**Summary**

The study on the fundamental diagram of traffic flow focuses on the vehicle flow, and few of research addressed the non-motor vehicle. From the perspective of establishing the fundamental diagram of non-motor vehicle traffic flow which was based on logistic model, this paper verified the feasibility of fundamental diagram which was applied to the non-motor vehicle traffic flow. Following conclusions were obtained in this paper:

1. The speed-density relation model of bicycle traffic flow corresponds to the fundamental diagram, but the data points are more discrete compared to the vehicle data.
2. The outcome of fitting error based on the logistic model is significantly rising than the classic model.
3. The capacity and block density of bicycle path which were calibrated based on the logistic model are more correspond to actual fact.

**Acknowledgement**

This work was partly supported by the Zhejiang Provincial Natural Science Foundation (No. LQ17E080001), the China Postdoctoral Science Foundation, and the Key Research and Development Program of Zhejiang (No. 2018C01007).

**References**


