The Classification of Spiral Surfaces Based on Particle Forming Motion and Parametrically Accurate Modeling

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ABSTRACT

Spiral is widely used in engineering. Because of the complexity of its forming movements and diversity of forms, there is no unified mathematical model that can express the motion synthesis relationship of all spiral surface’s forming movements. The obstacles, to some extent, are just on the way of deep understanding and application of it. The authors analyze the spiral curves with different forming motions, categorizing the spiral surface into three kinds with different combinations of the forming movements of the particle, two rectilinear motions with a circular motion, two circular motions with a rectilinear motion, and two circular motions. The relatively unified mathematical models of spiral curves are built by the use of differential geometry theory of partial curves and surfaces, motion synthesis theory of kinematics and the modeling theory of complex surface. Accurate parametric modeling is carried out with Pro/TOOLKIT of Pro/E, thus intuitively revealing the characteristics of movement relationship between the movements and synthesis relations of spiral curve surface.1

KEY WORDS

Spiral Surface, Particle Forming Movement, Classification, Parametrically Accurate Modeling

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INTRODUCTION

Spiral surface can be seen as a particle in space moving in spiral manner, forming a spiral curve, and a spatial curve moves along the spiral curve, forming a spiral surface [1, 2]. At present, the researches of spiral surface are focused on the exploration of forming principle and the optimization of processing technology [3]. Although the study of forming principle of spiral surface has gained great progress, it is still in expertise to establish unified and more accurate mathematical model of spiral surface [4,5].

Parametric modeling technology is an important branch of CAD technology, covering computer technology, topology and differential geometry, and it is the main means of product design [6,7]. In recent years, based on the early MIT research findings, parametric modeling technology has got rapid development [8]. There are two main ways of parameterized modeling. One is to record every procedure, from building a single relational database, to modifying a certain dimensions for modifying a particular datum, to fulfill the parameterized modeling [9,10]. Another, by numerical computational method of the geometric constraint equations of differential geometry, parametric modeling is realized.

CLASSIFICATION OF SPIRAL SURFACES BASED ON THE PARTICLE FORMING MOVEMENTS

Spiral curve may be synthesized from several kinds of particle movements in the space, these movements comprise rectilinear motion and circular motion. The traditional methods of the classification for the spiral surfaces not fully summarize the entire spiral surfaces or are not suitable for concluding relatively unified mathematical models of the spiral surfaces, and it has certain limitations to reveal the new spiral surfaces or has certain restrictions to the visualization and parameterization of the spiral surfaces.

In the current study, we are going to divide the spiral curves into several classes according to the way in which the particle moves to form the curves. The spiral surface corresponding to the spiral curve with the particle forming movements of two rectilinear motions and a circular motion will be entitled in the short form of RRC, the forming movement of RRC class spiral curve: when the fixed point do circular motion about fixed line, at the same time, do straight line motion along the radius of gyration (or tangent direction) and do linear motion along the straight line is a spiral curve of RRC type(as shown in Fig.1a). Spiral surfaces of RRC class may even be divided into two sub-categories. When the unified model with rotation angle parameter θ=00 the surfaces will be RRC_1 for short and unified model with involute base circle radius parameter r0, and the length of variable-amplitude parameter of the variable-amplitude involute, d, will be RRC_2.Again, RRC_1 may still be divided into three sub-classes depending on the parameter m in the unified
model. When \( m=\omega \) it is expressed as \( \text{RRC}_{1\omega} \), similarly, we can obtain the \( \text{RRC}_{1\omega u} \) and \( \text{RRC}_{1\omega f} \), where \( u \) is an angular parameter, and \( a, f \) are constants.

The spiral surface corresponding to the spiral curve with which the forming movements are composed of two circular motions and a rectilinear motion will be \( \text{CCR} \) for short. The forming movements of spiral curve for \( \text{CCR} \) class are that the moving point does uniform circular motion meanwhile moving with an angular parameter \( u_2 \) around the original point in the moving coordinate system. At the same time the moving coordinate system does uniform circular motion with an angular parameter \( u_1 \) around the fixed line and does linear motion along a straight line (as shown in Fig. 1b). In this case, there exists: \( u_2/u_1=r_1/r_2 \), where \( r_1 \) is a radius with which the moving coordinate system does circular motion around the fixed straight line, and \( r_2 \) is a radius with which the moving point does the circular motion around the original point of the moving coordinate system. \( \text{CCR} \) may also be divided into two sub-categories. They are \( \text{CCR}_s \) with the unified mathematical model of the spiral curve obtaining the first syn-position symbol (subtraction sign) and \( \text{CCR}_p \) with the plus sign.

The spiral surface corresponding to the spiral curve with which the forming movement is two circular motions will be \( \text{CC} \) for short. The forming movements of spiral curve of \( \text{CC} \) class are that a point in the moving coordinate system \( O-X'Y'Z' \) does circular motion with angular parameter \( u_2 \) around \( Z' \) axis, and at the same time, the whole moving coordinate system does circular motion with angular parameter \( u_1 \) around \( X \) axis of the fixed coordinate system \( O-XYZ \) (as shown in Fig. 1c). Therefore, the synthesis movement of the two circular motions is just the forming movement of the spiral curves of \( \text{CC} \) class.

The classification of spiral surfaces is shown in Fig. 2.
ESTABLISHMENT OF PARAMETERIZED MODEL OF SPIRAL SURFACE

The accuracy of the spiral surface modeling system will directly affect the result accuracy of follow-up system, which in turn, affects the quality of the product processing. Establishment of a relatively unified and more accurate mathematical model and geometric model of spiral surface is one of the most key links to obtain the high accuracy and high quality of related spiral surface products.

According to the particle motion in the space forming the curve and the movement of the curve forming the surface, characteristics of particle movement determine the attribute features of space surface. In Euclidean space, particle can be described as \( P(x, y, z, t) \), where \( t \) is the time parameter, and \( x, y, z \) are coordinate parameters with respect to time \( t \). The construction of the parametric models of spiral surface is created in the following section. Initially we discuss the spiral surface with straight line segment and circle arc on cross-section.

Construction of Parametric Model of Spiral Surface of RRC_2 Class

The parametric solid model of spiral surface can be established by using the Program function. The process of creating three-dimensional parametric model in Pro/Engineer is as follows: create part, set the parameters, add relations, add the relationships, regenerate model. Where \( r_0, \theta, d \) are both constants, \( \theta \) is the angle between the tangent direction and X axis, \( 0 \leq \theta \leq \pi/2, \theta + u = \pi/2 \), \( r_0 \) is the base circle radius of involute, \( d \) is the length range of variable-amplitude of the involute, and \( u \) is angular variable parameter.

So we obtain the RRC_2 class unified model as formula (1).

The parameterized spiral curve and spiral surface of this kind unified model as shown in Fig. 3 with the following initials: \( r_0=80, d=10, c=50, k=20, n=400, q=5, x_1=1400, y_1=1600, r=2000 \).
\[ r_2 = \begin{bmatrix} \cos u & -\sin u & 0 \\ \sin u & \cos u & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r_0 + d \\ -r_0 u \\ (c + kp)u \end{bmatrix} = \begin{bmatrix} (r_0 + d)\cos u + r_0 u \sin u \\ (r_0 + d)\sin u - r_0 u \cos u \\ (c + kp)u \end{bmatrix} \] (1)

Similarly, we can Construction of parametric model of spiral surface of RRC_1 class and CCR class and CC class.
THE PARAMETERIZATION OF SPIRAL SURFACE

The parametric system of the spiral surface in discussion includes five kinds of spiral surfaces, RRC_1, RRC_2, CCR_s, CCR_p, and CC as shown in Fig. 4, each kind being further divided into spiral surface with straight line segment or circle arc on cross-section. Any one of the spiral surfaces may be clicked and the corresponding parameter input dialog box comes up as shown in Fig. 6. When the target parameters are entered, the regenerated parametric model is obtained.

• Composition of the resource file
This paper uses the dialog box to realize the user interaction, so only need to write the dialog box resource files. After the program is executed the main dialog box may appear in the form of Fig. 4.

• Editing of the source code programs
The implementation of Pro/E model change is realized by altering input parameters in the parameter input-dialog box, so transfer process of parameters is a key part of the program source code. The algorithm flow chart of the main program is shown in Fig. 5.

Figure 5. The algorithm flow chart of Pro/Toolkit for secondary development.
The implementation of Pro/E model change is realized by altering input parameters in the parameter input-dialog box, so transfer process of parameters is a key part of the program source code. The algorithm flow chart of the main program is shown in Fig. 5.

• Registration and running of the software
  The execution results of program as follows:
  • The main dialog of parametric design for spiral surface is shown in Fig. 4.
  • The parameter input dialog corresponding the types of spiral surface will pop up when selecting any one of icons, input the parameter values as shown in Fig. 6, Pro/E will generate corresponding model of the spiral surface as shown in Fig. 6.

CONCLUSIONS

Systematic classification of spiral surfaces has been done with three levels according to the combinations of forming movements of the spiral curves and surfaces with different parameters. Five kinds of spirals, RRC_1, RRC_2, CCR_s, CCR_p, CC were discussed in detail. Sub-classes of several types have even been got with different variable parameters presenting the forming movement combinations of the particle. The fixed scanning algorithm of spiral surface was used in the parametric modeling. Execution of the programs with concrete samples for the parametric model has shown that this algorithm is effective, convenient and accurate. A series of parametric models of spiral surfaces form into three-dimensional parametric model library with functions of ready access, updating data and extension. Under the environment of Pro/E, the formation of the spiral curve correspond to the parametric model directly from the mathematical model, the parametric model we constructed without any error in theory, achieving the accurate
models. The current study paved the way for the exploration of the characteristics of the spiral surfaces, in theoretical and technical support for the whole life cycle of the digital design of spiral surface products.

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