Typical Extreme Posture Analysis of Symmetrical Stewart Platform

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Abstract. The Jacobian condition number of parallel manipulator will increase obviously when closing with singular point. In this paper, it can be found that, for the parallel manipulator without singular point in the whole workspace, the condition number is largest on the state of extreme posture. Based on 16 kinds of typical extreme postures, the singularity of symmetrical Stewart platform was analyzed, the bifurcation behavior of singular point was described by configuration curves. And from this paper, it can be found that the Stewart platform is very prone to Hunt singularity.

Introduction

Singularity is the inherent characteristics of many space mechanisms, which is also an important feature of parallel manipulator. When the manipulator is in singular position, the Jacobian matrix becomes a singular matrix, and the manipulator will receive the extra degree of freedom or become just the uncontrollable part for the loss of freedom, which can cause serious problems. So the singular position should be avoided as the manipulator works. However, the complicated nature of the kinematics of parallel manipulators makes it difficult to analyze.

Many researchers have studied on the singularity problem of parallel manipulators. Due to the complexity of configuration space, the singularity of the parallel manipulator shows different types. The studies about the classification and characteristics between different types have been developed by many scholars[1,2]. Moreover, the solution methods of singularity have been investigated to some extent, mainly focused on algebra, screw theory, line geometry methods and intelligent algorithm[3]. In fact, the singularity analysis of the parallel mechanism aimed at how to avoid the manipulator works in the singular position. Researchers have studied on the strategy to avoid the singularity, such as adding redundant actuation[4], path planning[5]. Although many scholars have researched much on the singularity in parallel manipulator based on different point of view, works on singularity analysis based on extreme postures have been mentioned rarely.

In this paper, the singularity of symmetrical Stewart platform based on typical extreme postures is analyzed. By solving the singular link length of extreme posture, the bifurcation behavior of singular point is described by configuration bifurcation curves. It can be found that Hunt singularity easily take place very much.

Posture with Maximal Condition Number

The Stewart platform is shown schematically in Figure 1. The condition number of Jacobian is bigger near the singular area, and the condition number will increase obviously when closing with singular point. So, for various set of link length, we can solve the posture of the Stewart platform with numerical iterative method, and obtain its condition number of Jacobian. Then, the set of link length with maximal condition number can be found out. If the link travel is divided into $n$ parts, there are $6(n+1)$ sets of link length. The greater $n$ the better description of link space, but computing cost will increase exponentially with the increase of $n$. 
Here, example manipulator’s minimum link length is 1m, and the maximum link length is 1.4m. Results show that no matter what the value of \( n \) is, the maximal condition number is the same, which corresponds to the three sets of link length as shown in Table 1. Because of manipulator’s symmetry, this three state characteristic is the same as. It can be found that, for the parallel manipulator without singular point in the whole workspace, the condition number is largest on the state of maximum or minimum length of link. We also know that symmetrical Stewart platform has 16 kinds of typical extreme postures whose link length is maximum or minimum. In order to further understand the characteristic of this type of parallel mechanism, we need to discuss the singular link length of typical extreme posture.

Table 1. Link lengths with maximum condition number [m].

<table>
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<th>No.</th>
<th>( l_1 )</th>
<th>( l_2 )</th>
<th>( l_3 )</th>
<th>( l_4 )</th>
<th>( l_5 )</th>
<th>( l_6 )</th>
</tr>
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<td>1</td>
<td>1</td>
<td>1.4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
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<td>1</td>
<td>1.4</td>
<td>1</td>
<td>1</td>
<td>1.4</td>
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</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>1.4</td>
<td>1</td>
<td>1</td>
<td>1.4</td>
</tr>
</tbody>
</table>

Typical Extreme Posture Analysis

Singular Link Length

Above shows that on the state of a certain extreme posture, the condition number is largest, and the kinematic characteristic is poor. In order to ensure better characteristic in the whole workspace, we can make all extreme postures do not occur singularity by controlling the links’ length. Therefore, in this section we will further study the singular points of all typical extreme postures.

Extreme posture is the one with maximum or minimum length links. But here, in order to control the links’ length, we assume that the maximum length of link is unknown. Then homotopy iterative method is adopted to solve the maximum length value with which all extreme posture do not occur singularity[6]. The singularity equation of extreme posture is as Eq. 1.

\[
\begin{align*}
F(x, L_i) &= 0 \\
x &= [x_0, y_0, z_0, x_1, y_1, z_1, x_2, y_2, z_2]^T, \quad i = 2, \cdots, 16.
\end{align*}
\] (1)

On initial situation, we suppose that the manipulator is on the state of the 1st typical extreme posture whose link lengths are all minimal, and whose length vector of link is \( L_1 = [l_{min}, l_{min}, l_{min}, l_{min}, l_{min}, l_{min}]^T \). To other each extreme posture, letting the length of stretched-out link increase constantly from the minimum value, we select the length variable of links as singularity parameter. If there is more than one stretched-out link, such as the 3rd to 16th extreme posture, we increase all the length of stretched-out link synchronously. This is because, if the length is not increased synchronously, the
Condition numbers manipulator is not on the state of extreme posture need to be discussed by us. The link length vectors of the 2nd to 16th extreme posture are as Eq. 2.

\[
\begin{align*}
L_2 &= [\mu, l_{\text{min}}, l_{\text{min}}, l_{\text{min}}, l_{\text{min}}, l_{\text{min}}]^T \\
L_3 &= [\mu, \mu, l_{\text{min}}, l_{\text{min}}, l_{\text{min}}, l_{\text{min}}]^T \\
L_4 &= [\mu, l_{\text{min}}, \mu, l_{\text{min}}, l_{\text{min}}, l_{\text{min}}]^T \\
L_5 &= [\mu, l_{\text{min}}, l_{\text{min}}, \mu, l_{\text{min}}, l_{\text{min}}]^T \\
L_6 &= [\mu, l_{\text{min}}, l_{\text{min}}, l_{\text{min}}, \mu, l_{\text{min}}]^T \\
L_7 &= [\mu, \mu, \mu, l_{\text{min}}, l_{\text{min}}, l_{\text{min}}]^T \\
L_8 &= [\mu, \mu, \mu, \mu, l_{\text{min}}, l_{\text{min}}]^T \\
L_9 &= [\mu, l_{\text{min}}, \mu, \mu, \mu, l_{\text{min}}]^T \\
L_{10} &= [\mu, l_{\text{min}}, \mu, \mu, \mu, \mu]^T \\
L_{11} &= [l_{\text{min}}, l_{\text{min}}, \mu, \mu, \mu, \mu]^T \\
L_{12} &= [l_{\text{min}}, \mu, l_{\text{min}}, \mu, \mu, \mu]^T \\
L_{13} &= [l_{\text{min}}, \mu, \mu, l_{\text{min}}, \mu, \mu]^T \\
L_{14} &= [l_{\text{min}}, \mu, \mu, \mu, l_{\text{min}}, \mu]^T \\
L_{15} &= [l_{\text{min}}, \mu, \mu, \mu, \mu, \mu]^T \\
L_{16} &= [\mu, \mu, \mu, \mu, \mu, \mu]^T
\end{align*}
\]

(2)

The link lengths corresponding to the 2nd to 15th singular points are obtained and showed in Table 2. The 16th kind of extreme configuration hasn’t singular point.

It can be found that the value of singular point of the 5th extreme posture is the minimum. Under this kind of extreme posture, when the rotation angle limit of joints and the interference between any two links aren’t considered, the manipulator can’t work normally unless the length of links is less than 1.4878m. So, in order to stay away from the singular area, the maximum length of links should be less than this value. Thus, the maximum length value with which all extreme posture do not occur singularity is obtained, which ensures better characteristic in the manipulator’s whole workspace.

To above manipulator, when \( \mu = 1.4 \text{m} \), the condition numbers of Jacobian matrix with 16 kinds of typical extreme posture are drawn in Figure 2. As above example, the condition number of the 5th extreme posture is maximal.

![Figure 2. Condition numbers of Jacobian matrix.](image)

### Configuration Bifurcation

The configuration curves of the 5th extreme posture are shown as Figure 3. The singular point on these curves, that is \( \mu = 1.4878 \text{m} \), is also the configuration bifurcation point where two configurations
superpose one another. It can be seen, when the length of links decreases constantly from the singular point, the movable platform can give out the output with two kinds of configurations corresponding to the configuration curves in Figure 3, which results in configuration bifurcation.

![Configuration curves of component x0](image)

![Configuration curves of component y0](image)

![Configuration curves of component z0](image)

![Configuration curves of component x](image)

![Configuration curves of component y](image)

![Configuration curves of component z](image)

**Figure 3. Configuration curves of the 5th extreme pose.**

**Hunt Singularity**

As can be seen from Table 2, the 5th extreme posture most likely occur configuration bifurcation, followed by the 14th extreme posture. Figure 4 shows the states of singular point about the two kinds of extreme posture. Figure 5 shows the Hunt singularity about 3-6Stewart whose six lines along six links intersect in the formation of a public line, that was presented by Hunt in 1983, and has become a typical singularity because of its intuitive geometry and clear physical concept. It can be found, the 5th and 14th extreme postures actually take place Hunt singularity. In this case, no matter how much force each link provides, the parallel manipulator will not overcome the torque around the public line induced by load or gravity, and will lose control. From this example, we can see that Hunt singularity take place very easily.
Summary
Because the Jacobian condition number is largest on the extreme posture, we analyzed the singularity of symmetrical Stewart platform based on typical extreme postures. With the presented analysis method in this paper, we can design a parallel manipulator without singular point in the whole workspace by restricting the length of links.

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References