Research on Calculation Methods for Ductility of RC Beams with Near-Surface Mounted CFRP Plate Reinforcement

Langni Deng, Jinchao Ma, Jun Ma, Risheng Luo and Xiaoxia Huang

ABSTRACT

Fiber-reinforced polymer (FRP) strengthening systems are widely accepted in engineering practice as means to enhance the flexural strength of RC members, but resulting in decreased ductility. To improve the flexural capacity of RC beams strengthened with CFRP plate, but also to ensure the enough ductility, a study was carried out. Putting forward the design method of CFRP embedded reinforcement beams based on ductility ability. The result of the study revealed that the method was reliable by the analysis of an example, and the method can be used in the design of reinforcement project.

INTRODUCTION

In the last 30 years, the fiber-reinforced polymer (FRP) strengthening technique has been widely used in civil engineering. Near-surface mounted CFRP is a new reinforced method and is widely used to enhance the mechanical properties of RC structure. Near-surface mounted CFRP is a reinforced method to advance on the surface of the components that need to reinforce the slot, and then put the CFRP strips into slots to improve the mechanical performance of the component. Compared with the surface stick method, near-surface mounted CFRP has the advantages of low dosage of materials, corrosion, resistance, applicable to the

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uneven surface of the component, and more effective component force, suffers accident load is not easy to damage, etc. (Li et al., 2005). But it also has some drawbacks; brittle failure is one of the greatest hindrances limiting FRP system from wider application. Brittle failure decrease the reliability of FRP strengthening system and limit their practical applications. Yu et al. (2016) indicated that the ability to enhance the ductility may have even greater significance than that to merely enhance the ultimate strength, because it can provide warning prior to the failure and showing progress of damage and relieve the upper limitations imposed on FRP and better utilizing the fiber strength. A variety of methods have been proposed to increase the ductility of FRP strengthened RC structures. Wu et al. (2010) developed a perforated slurry infiltrated fiber concrete with high ductility into the compression area of concrete, improving the ductility of CFRP strengthened components. The result of the experiments indicated that the above method can improve the ductility of the members, but decrease the bending stiffness and strength of beams. Zhou and Attard (2012) and Zhou et al. (2013) used carbon fiber-epoxy polyurea composite, combining with a polyphase hybridized interface, to avoid the dissipation of energy to achieve sustainable high strength and increase the ductility of the concrete. The proposed method was validated by experiments, but has a drawback of this method is not applicable in the NSW FRP strengthened system. Rasheed et al. (2010) developed tests on RC beams strengthened with NSW-CFRP. The extra transverse reinforcement was used to confine concrete and anchor longitudinal reinforcement, reducing the premature debonding. Experiments show that the member strength was fully utilized and the ductility was improved. Wu et al. (2007) set tests on RC beams strengthened with prestressed NSW-CFRP tendons. The experimental results demonstrated the flexural stiffness and ultimate load capacity of strengthened beams was largely enhanced by the prestressed NSW-CFRP.

The concept of structural ductility can be defined as an ability to sustain the applied loads beyond the elastic limit without significant loss of load-carrying capacity until failure (Grace et al. 1998; Pam et al. 2001). However, quantifying ductility is still a controversial issue because no absolute definition exists. Nevertheless, several methods exist for quantifying the ductile behavior of beams such as curvature ductility, displacement ductility, energy ductility, and deformability factors (Grace et al. 1998; CSA International 2000). AASHTO (2003) defines a ductility factor based on a deformation ratio employing the ultimate and yield deflections. CSA A23.3-04 (Cement Association of Canada 2006) states a general requirement to induce a ductile failure mode for reinforced concrete structures by controlling a ratio of c/d, where c is the distance from the extreme compression fiber to the neural axis, and d is the effective depth of the tension reinforcement. As concrete strength increases, its brittleness becomes notable and its ductility reduces remarkably, which is harm for anti-seismic (Seible et al. 1997; Xiao et al. 1997; Wang et al. 2005; Anggawidjaja et al. 2006; Wu et al. 2006; Binici et al. 2007; Wang et al. 2008).
Research that near-surface mounted CFRP can improve the shear strength and flexural capacity of reinforced concrete has been verified in many experiments, but researchers rarely consider near-surface mounted CFRP about improving the ductility or not systematic. Based on the results of the experiment, analyzing yield and ultimate curvatures, calculation methods for ductility of RC beams with near-surface mounted CFRP plate reinforcement was proposed.

ANALYSIS ABOUT CALCULATION METHOD OF DUCTILITY

Fundamental Assumptions

In order to simply the calculation, the author has done the following assumptions: all the equations in this article should conform the plane-section assumption, stress-strain diagram of elastic-plastic is adopted in stress-strain relationship of steel, stress-strain diagram of CFRP plate is linear elasticity, concrete tensile strength is ignored, “Rush” model is used in stress-strain diagram of concrete. Thus, the rising step of stress-strain diagram of concrete is curve, and the downward is straight line.

When

\[ \varepsilon \leq \varepsilon_0, \quad \sigma = f_c \left[ 2 \frac{\varepsilon}{\varepsilon_0} - \left( \frac{\varepsilon}{\varepsilon_0} \right)^2 \right] \]

(1),

and when

\[ \varepsilon_0 \leq \varepsilon \leq \varepsilon_{cu}, \quad \sigma = f_c \]

(2)

Resultant force of concrete in the compression zone is calculated from:

\[ F_c = \int_0^{\varepsilon_c} \sigma_c b dx = b \sigma_0 \left( \frac{\varepsilon_c x_c}{\varepsilon_0} - \frac{\varepsilon_c^2 x_c}{3 \varepsilon_0} \right) \varepsilon_c \quad (\varepsilon_c < \varepsilon_0) \]

(3)

\[ F_c = \int_0^{\varepsilon_c} \sigma_c b dx = b \sigma_0 \left( h_c - \frac{\varepsilon_c h}{3 \varepsilon_c} \right) (\varepsilon_c > \varepsilon_0) \]

(4)

Figure 1. Yield and limit strain diagram of the beam.
As it is showed in Figure 1, $\phi_f$ is calculated from equation 5:

$$\phi_f = \frac{f_y}{h_0 - x_c}$$  \hspace{1cm} (5)$$

According to Strain compatibility, $\varepsilon_b$ is calculated from equation 6:

$$\varepsilon_b = \frac{h_f - x_c}{x_c} \varepsilon_c$$  \hspace{1cm} (6)$$

According to stress equilibrium of section of yield, equation 7 is obtained:

$$F_c + \sigma_f A_f = f_y A_y + \sigma_f A_f$$  \hspace{1cm} (7)$$

Bring equation 3 into equation 7, equation 8 and equation 9 is obtained:

$$b \sigma_0 \left( \frac{\varepsilon_c x_c}{\varepsilon_0} - \frac{\varepsilon_f x_f}{3\varepsilon_0} \right) + \varepsilon_c E_s A_f = f_y A_y + \varepsilon_c E_f A_f$$  \hspace{1cm} (8)$$

$$x_c = \frac{\varepsilon_c}{\varepsilon_c + f_y / E_s} h_0$$  \hspace{1cm} (9)$$

Bring equation 9 into equation 5, equation 10 is obtained:

$$\phi_f = \frac{f_y / E_s}{h_0 (1 - \varepsilon_c / (\varepsilon_c + f_y / E_s))}$$  \hspace{1cm} (10)$$

Bring equation 9, equation 5 into equation 8, equation 11 is obtained:

$$A_f = \frac{-b \sigma_0 \varepsilon_c^3 + 3 \varepsilon_c \varepsilon_f^2 (b \sigma_0 h_0 + \varepsilon_0 E_s A_y) + 3 \varepsilon_0 \varepsilon_f (f_y - A_y) - 3 \varepsilon_0^2 \frac{f_y^2}{E_s}}{3 \varepsilon_0^2 E_f \left[ \frac{h_f}{h_0} - 1 \right] \varepsilon_c^2 + \left( \frac{2 h_f f_y}{h_0 E_f} - \frac{f_y}{E_s} \right) \varepsilon_c + \frac{f_y^2 h}{E_s h_0}}$$  \hspace{1cm} (11)$$
Sectional Curvature of Ultimate Limit State

\( \phi_u \) is calculated from equation 12:

\[
\phi_u = \varepsilon_{cu} / \chi_{cu}
\]  
(12)

According to internal force equilibrium, equation 13 is obtained:

\[
F_y + \varepsilon_{cu} E_y A_y = f_y A_y + \sigma_f A_f
\]  
(13)

Bring equation 4 into equation 13, equation 14 is obtained:

\[
b\sigma_0 (1- \frac{\varepsilon_0}{3\varepsilon_{cu}}) \chi_{cu} + f_y A_y = f_y A_f + \frac{h_f - \chi_{cu} E_y A_f}{\chi_{cu}}
\]  
(14)

Compile equation 14, the following equation is obtained:

\[
b\sigma_0 (1- \frac{\varepsilon_0}{3\varepsilon_{cu}}) \chi_{cu}^2 + \left( f_y A_y + \varepsilon_{cu} E_y A_f - f_y A_f \right) \chi_{cu} - \varepsilon_{cu} h_f E_f A_f = 0
\]

\( \chi_{cu} \) is solved from the above matrix equation:

\[
\chi_{cu} = \left( -B + \sqrt{B^2 - 4AC} \right) / 2A
\]  
(15)

where, 

\[
A = b\sigma_0 (1- \frac{\varepsilon_0}{3\varepsilon_{cu}}), \quad B = f_y A_y + \varepsilon_{cu} E_y A_f - f_y A_f, \quad C = -\varepsilon_{cu} h_f E_f A_f.
\]

Bring equation 15 into equation 12, \( \phi_u \) is obtained:

\[
\phi_u = \frac{\varepsilon_{cu}}{\chi_{cu}} = \frac{2A\varepsilon_{cu}}{-B + \sqrt{B^2 - 4AC}}
\]  
(16)

Curvature ductility factor is calculated from equation 17:

\[
\mu_d = \frac{\phi_u}{\phi_y} = \frac{2A\varepsilon_{cu} E_y h_j (1- \frac{\varepsilon_e}{\varepsilon_e + f_y / E_y})}{f_y (\sqrt{B^2 - 4AC} - B)}
\]  
(17)
Reinforcing Design Based on Given Curvature Ductility Factor

Experiments indicate that the failure models of concrete beams reinforced by CFRP can be classified into two types: (1) bending failure induced by rupture of CFRP. (2) bending failure by concrete crushing. Failure model 2 may have short progressive deboning processes, but also exhibit very limited ductility. So it should be avoided, and measure is to increase the amount of CFRP, making $\rho_f > \rho_{fb}$, where, $\rho_f$ means reinforcement ratio of CFRP, $\rho_{fb}$ means limits reinforcement ratio of CFRP.

Design steps of reinforced CFRP based on ductility are the following: firstly, $\varepsilon_c$ is calculated with the combination of equation 15, 17 and 11, given the known $\mu_y$, secondly, $A_f$ is calculated from equation 12, thirdly, calculate the strain of CFRP, with the combination of equation 11 and 14, given the known $\varepsilon_0$. If the result exceeds ultimate tensile strain of CFRP, $\varepsilon_{cu}$ and $x_{cu}$ should be recalculated. When CFRP is broken, equation 14 should be replaced by equation 18:

$$b\sigma_0(1-\frac{\varepsilon_0}{3\varepsilon_{cu}})x_{cu} + f_yA_y = f_fA_f + f_fA_f$$

(18)

According to Strain compatibility, equation 19 is obtained:

$$\frac{\varepsilon_{cu}}{x_{cu}} = \frac{\varepsilon_{fu}}{h_f - x_{cu}}$$

(19)

Bring equation 18 into equation 19, equation 20 is obtained:

$$\varepsilon_{cu} = \frac{b\sigma_0h_f\varepsilon_0 + 3(f_yA_y + f_uA_f - f_uA_y)\varepsilon_{fu}}{3(b\sigma_0h_f + f_yA_y - f_uA_y - f_uA_y)}$$

(20)

Bringing equation 20 into equation 12, $\phi_u$ is calculated, and then bringing $\phi_u$ into equation 17, $\phi_y$ is calculated. Bringing $\phi_y$ into equation 5 and combining with equation 9, $\varepsilon_c$ is calculated. Bringing $\varepsilon_c$ into equation 11, the amount of CFRP is calculated.
The following symbols are used in this paper: \( \sigma_0 \) = maximum compressive stress of the concrete; \( \varepsilon_0 \) = maximum compressive strain of the concrete; \( \varepsilon_c \) = compressive strain on the edge of the concrete; \( h_c \) = concrete depth of compression zone; \( h_0 \) = concrete effective height of section; \( E_s \) = elastic modulus of concrete; \( \varepsilon \) = sectional curvature; \( \varepsilon_u \) = ultimate curvature; \( \mu_\phi \) = curvature ductility factor; \( \mu_\phi \) = Ductility Coefficient; \( \varepsilon_{cu} \) = ultimate compression strain of Concrete; \( x_w \) = depth of compressive zone, when concrete is crushed; \( f_y \) = tensile strength of steel; \( f_c \) = compression strength of steel; \( A_t \) = area of tensile steel; \( A_c \) = area of compression steel; \( h_f \) = sectional depth of CFRP bar; \( \varepsilon_{fu} \) = ultimate tensile strain of CFRP; \( f_u \) = tensile strength of CFRP; \( \rho_f \) = reinforcement ratio of CFRP; \( \rho_f \) = limits reinforcement ratio of CFRP; \( A_{fy} \) = calculated value of reinforcement area of CFRP; \( A_{fy} \) = experimental value of reinforcement area of CFRP; \( A_f \) = area of CFRP.

Design of CFRP-Strengthened Beams

In order to investigate the applicability of the calculation method, five RC beams were constructed for two-point loading tests. Table 1 summaries the specimen program, where L1 and L2 acted as a control specimen, JGL 3, JGL 4 and JGL5 were strengthened with NSW-CFRP. The cross-sectional dimension of all specimen was 150mm(width) \( \times \) 250mm(depth) \( \times \) 2600mm(length). Concrete C30 is used. All specimens were reinforced with 2#6 compression steel bars. Three beams were provided 2#8 longitudinal tension steel bars, ratio of 0.32%. The other two beams were provided 2#12 longitudinal tension steel bars, ratio of 0.72%. In this experiment, pure bending region length is 600mm, and shear bending region length is 900mm. In order to prevent shear failure in the loading process of shear bending region. Steel stirrups, 6-mm-diameter bars were used to confine the concrete and spaced on center at 100mm in shear bending region, and 200mm in pure bending region. Concrete cover thickness is 30mm (distance from the edge of stirrup to concrete surface). Figure 1 shows test beam design and loading schemes. Table 1 shows design parameters and failure models of the experimental beams, Table 2 shows curvature ductility factor for experiment beams, Figure 2 shows deflection at mid span of the experimental beams.
Figure 2. Test beam design and loading schemes.

### TABLE 1. DESIGN PARAMETERS AND FAILURE MODELS OF EXPERIMENTAL BEAMS.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Reinforcement ratio(%)</th>
<th>CFRP width (mm)</th>
<th>Length of CFRP (mm)</th>
<th>Failure model</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1</td>
<td>0.32</td>
<td>-</td>
<td>-</td>
<td>bending failure</td>
</tr>
<tr>
<td>L2</td>
<td>0.72</td>
<td>-</td>
<td>-</td>
<td>bending failure</td>
</tr>
<tr>
<td>JGL3</td>
<td>0.32</td>
<td>15</td>
<td>2200</td>
<td>concrete crushing</td>
</tr>
<tr>
<td>JGL4</td>
<td>0.32</td>
<td>20</td>
<td>1600</td>
<td>Interfacial debonding</td>
</tr>
<tr>
<td>JGL5</td>
<td>0.72</td>
<td>20</td>
<td>2200</td>
<td>concrete crushing</td>
</tr>
</tbody>
</table>

### TABLE 2. CURVATURE DUCTILITY FACTOR FOR EXPERIMENT BEAMS.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>L1</th>
<th>L2</th>
<th>JGL3</th>
<th>JGL4</th>
<th>JGL5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_y$ ($10^{-6}$mm$^{-1}$)</td>
<td>9.5</td>
<td>11.9</td>
<td>11.5</td>
<td>11.9</td>
<td>13.2</td>
</tr>
<tr>
<td>$\phi_u$ ($10^{-6}$mm$^{-1}$)</td>
<td>99.1</td>
<td>95.2</td>
<td>88.6</td>
<td>30</td>
<td>75.7</td>
</tr>
<tr>
<td>$\mu_\phi$</td>
<td>10.4</td>
<td>8</td>
<td>7.70</td>
<td>2.5</td>
<td>5.7</td>
</tr>
</tbody>
</table>
TABLE 3. COMPARISON BETWEEN CALCULATED VALUE AND EXPERIMENTAL VALUE.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Value of $A_f$ ($mm^2$)</th>
<th>Value of $A_{fe}$ ($mm^2$)</th>
<th>Error(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>JGL3</td>
<td>39.4</td>
<td>42</td>
<td>6.2</td>
</tr>
<tr>
<td>JGL5</td>
<td>58.6</td>
<td>56</td>
<td>4.6</td>
</tr>
</tbody>
</table>

Comparison Between Calculated Value and Experimental Value

Table 3 shows comparison between calculated value and experimental value. Where, $A_{fe}$ means calculated value of reinforcement area of CFRP, $A_f$ means experimental value of reinforcement area of CFRP. The results show the error is small, thus the calculated method proposed in this article can be used in project.

CONCLUSIONS

A calculate method is developed to ensure the ductility of a RC beam strengthened with NSW-CFRP. Additionally, calculate example were conducted to validate the calculate method based on ductility. Referring to the calculate results, the method is proved reliable and the method can be used in the design of reinforcement project.
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CONFLICTING INTERESTS

The authors declare that there is no conflict of interest.

REFERENCES