Image Retrieval Relevance Feedback Based on Grover Quantum Searching Algorithm

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Abstract. In order to improving the retrieval rate and speed for a large number of images, an algorithm of image retrieval relevance feedback based on Grover quantum search is proposed. Grover quantum searching algorithm solves the unsorted database search problem with computation complexity of $O(\sqrt{N/M})$, where the size of database is $N$ and there is $M$ target solutions. Making use of the image features re-weighted feedback to make up the deficiency of computer’s understanding of user’s needs, so that the system can effectively retrieve the image library and improve the retrieval precision. The experiment proves the feasibility and effectiveness of the algorithm.

Introduction

Content-based image retrieval refers to the use of image visual content, such as color, texture, shape, spatial layout, etc., from a large number of image data search for the users which satisfied their need. Due to the existence of the semantic gap [1], retrieval system cannot understand well what the users’ need which makes the retrieval accuracy is not ideal. So, the relevance feedback technology was applied to the field of image retrieval and achieved good results [2][3]. There’s a variety of technical support to achieve and SVM is more common used. However, the performance of SVM based RF system is not very effective as the average precision is 0.74[4] and 0.64[5]. We propose a feature re-weighted image retrieval relevance feedback algorithm based on Grover quantum search. From a length of $n$ disorderly number of digging out a search for a target, using the classical algorithm on average need to find $N/2$ times, the probability of success is 0.5, the search complexity is $O(N)$, and the search complexity of Grover search algorithm only $O(\sqrt{N})$, Since the Grover algorithm can bring secondary acceleration to the classical algorithm, the Grover algorithm is used to accelerate the traditional machine learning algorithm[6][7] in recent years. We combine the Grover quantum search algorithm with the feedback process, which avoids the influence of the initial retrieval on the user's cognition and the limitation of the feedback effect.

Grover Quantum Search Algorithm and RF Algorithm

In Grover quantum search algorithm [8], assume to search in the space which contains $N = 2^n$ elements. Element metrics can be stored in n qubits. Assume that the search question only has $M$ targets which $1 \leq M \leq N$. The initial state is a uniform superposition of $n$ quantum bits.
\[ |\phi\rangle = \frac{1}{\sqrt{N}} \sum_{i=0}^{N-1} |x_i\rangle \]  

(1)

The Grover iteration process can be divided into the following four steps:

1. Verify whether the search element is the required solution for these problems which are not solved by applying Oracle arithmetic operators.

\[ O = I - 2 \sum_{i=1}^{M} |r_i\rangle\langle r_i| \]  

(2)

2. Conduct the Hadamard gate transformation \( H^{\otimes n} \) towards the result of 1.

\[ H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \]  

(3)

3. The result of phase 2 is phase shifted, this results in a phase rotation of -1 for each of the basic quantum states except \( |0\rangle \). Namely:

\[ |x\rangle \rightarrow (-1)^{|\delta_x|} |x\rangle \]  

(4)

4. Conduct the Hadamard gate transformation towards the result of 3.

RF is a strategy for users to participate in the optimization of image query, it has high requirements toward the real-time and it has just a few markings samples [9]. Feature weight adjustment algorithm refers that the retrieval system analyzes the users return information based on the relevant feedback. The new weight plays an important role in offering different features to the images, and it can improve the weight of these features which have a close relationship with the image.

An Image Retrieval RF Algorithm Based on Grover Quantum Search Algorithm

Algorithm Description

Firstly, extract feature of each picture in the image library and get \( N \) kind of pictures’ characters, record it as \( F_m, j = 1, 2, \ldots, N \), the characteristics of the type M 

\[ F_m = \left( F_m^{(1)}, F_m^{(2)}, \ldots, F_m^{(n)} \right)^t \]

\( n_m \) refers to the dimension of feature \( F_m \). We take the weighted distance of Euclidean to measure the image distance, it can be recorded as 

\[ w_{ij}^{k} D_{ij} \]

\( i = 1, 2, \ldots, n, j = 1, 2, \ldots, n, i, j \) refers to the un-searched images and the images in the library. \( k = 1, 2, \ldots, n \). Represents the feedback \( k \) times. Also the weight of the measurement distance is adjusted by making the users to mark the irrelevant image, the formula [10]:

\[ w_{ij}^{k+1} = \delta_{ij} \ast \frac{\epsilon + \sigma_{x,ij}^k}{\epsilon + \sigma_{x,ij}^k} \]  

(5)

\( \sigma_{x,ij}^k \) refers the characteristic standard deviation of the \( i \) dimension got in the return image of the \( k \) circles’ feedback. \( \sigma_{x,ij}^k \) refers the characteristic standard deviation of the \( i \) dimension got in the relative images of the returned image of the \( k \) circles’ feedback.
\[
\delta^i = 1 - \frac{\sum_{i=1}^{k} |p^{i||p}|}{\sum_{i=1}^{k} |f^{i||f}|}
\]

\[
\sum_{i=1}^{k} |p^{i||p}| \text{ refers to the number of non-correlation images in the first to the k rounds of the non-correlation image falls within the range of the dimension value of the correlation image, } \\
\sum_{i=1}^{k} |f^{i||f}| \text{ refers to the number of non-correlation features appearing from 1 to } k.
\]

In Grover quantum search algorithm, the phase rotation angle is \(\pi\) which is fixed. When the number of the researched targets is higher than the half of the database records, the probability of success is rapidly decreased. When the number of targets exceeds half of the database, then the algorithm fails. In order to improve this deficiency, many scholars at domestic and abroad have made continuous exploration and research, two main improvements algorithm will be introduced as follows:

To improve the Grover algorithm based on \(\pi / 2\) phase rotation [11], the value of these two-phase rotation is \(\pi / 2\), and it stays in the opposite direction. The two phase shift operators are:

\[
U = I - (1 - \beta) \sum_{\tau=1}^{M} |\tau\rangle \langle \tau_\tau| \\
V = (1 + i) |\phi\rangle \langle \phi | - i I
\]

Assume \(M / N = \lambda\), the improved algorithm can be divided into two search processes:

1. When \(0 < \lambda \leq 1 / 3\), use basic Grover search.
2. When \(1 / 3 < \lambda < 1\), use the improved Grover search, it is divided into the following three steps:

   1. (9) makes \(|\phi\rangle\) the phase of the target state moves forward \(\pi / 2\) this can make people get the intermediate state of the system superposition state:

      \[
      |\phi\rangle = (I - (1 - \beta) \sum_{\tau=1}^{M} |\tau\rangle \langle \tau_\tau|) |\phi\rangle
      \]

   2. (10) the final state of the system superposition state:

      \[
      |\tilde{\phi}\rangle = ((1 + i) |\phi\rangle \langle \phi | - i I) |\phi\rangle
      \]

   3. Measure \(|\tilde{\phi}\rangle\), can get one solution of the searched question based on probability which is at least \(27 / 25\).

Based on the fixed phase rotation of the generalized Grover algorithm[12], it is no difficulty to find that the difference of the rotation phase value can generate different kinds of Grover algorithms. When the rotation phase is \(\pi\), it is equal to the basic Grover algorithm. When the rotation phase is \(\pi / 2\), it is equal to Yonnes' local diffusion algorithm. When the rotation phase is \(1.825\pi\), it is equivalent to Yonnes’ fixed phase algorithm. The success rate of these improved algorithms has been further improved, and its iteration step number will also be increased. According to Professor Long Guiliu's study [13], in the three-dimensional space, the iterative operator G can change
the values of phase rotation, its value can be calculated by using \( \arcsin(\sin(\alpha/2)\sin \theta) \). Therefore, in order to achieve the maximize probability of success, and the required number of iterations is as follows:

\[
R = \frac{\pi}{4 \arcsin(\sin(\alpha/2)\sin \theta)}
\]

As \( \sin \theta = \sqrt{M/N} \), when \( \theta \) is small, \( \sin \theta \approx \theta \), so when \( M \ll N \):

\[
R \approx \frac{\pi}{4 \sin(\alpha/2)\sqrt{M/N}} = O(\sqrt{M/N})
\]

According to (12) we know that the number of iterations of the algorithm derived from the generalized Grover algorithm is determined by \( \alpha \):

1. when \( \alpha = \pi \),

\[
R = \frac{\pi}{4 \arcsin(\sin(\alpha/2)\sin \theta)} = \frac{\pi}{4 \sqrt{M}}.
\]

2. when \( \alpha = \pi/2 \),

\[
R = \frac{\pi}{4 \arcsin(\sin(\alpha/2)\sin \theta)} = \frac{\pi}{2 \sqrt{2} \sqrt{M}}.
\]

3. when \( \alpha = 1.825 \pi \),

\[
R = \frac{\pi}{4 \arcsin(\sin(\alpha/2)\sin \theta)} = \frac{1.825 \pi}{2 \sin \theta}.
\]

After the Grover algorithm has been improved, its successful probability and the number of iterations can be seen from the following chart:

Figure 1. The success probability comparison after the Grove algorithm has been improved.
Algorithm Flow

(1) The user inputs the images that need to be searched, the system returns to the initial search results after the calculation;

(2) The user selects the relevant image feedback to the system based on the evaluation of the retrieval results;

(3) The system recalculates the weight of the features based on the feedback images, and then gets a series of feature weight which can be used to guide the new round of retrieval. The weighted distance between the images is calculated and it has been put in $\text{dist}[1,2,\ldots,N]$. Find the required $M$ pieces images by using Grover search algorithm. According to the Grover algorithm in the search list of the minimum value of the application[14], the operator is designed as follows:

$$
    f(y) = \begin{cases} 
        1, & \text{dist}[j] < \text{dist}[y] \\
        0, & \text{others} 
    \end{cases} \quad (16)
$$

$y$ refers to the subscripts which are generated randomly in the N sequence. After having initialized the register and set the minimum value as $\text{dist}[y]$, Mark all the subscripts which can satisfy $\text{dist}[j] < \text{dist}[y]$, apply the improved Grover search algorithm. Consider the $y'$ as the output, if $\text{dist}[y'] < \text{dist}[y]$, change the initial index to $y'$, return to $y$, and the search results of new images can be achieved.

Experimental Results

Experimental Environment

Our experiments were conducted in Corel standard test image library in which there are 1000 images. 10 types of images and each type includes 100 images. According to proposed algorithm, 20 images will be presented after one retrieval cycle. Matlab2014b is selected as Soft Development Platform and we set the initial value of feature weight in each dimension to 1. We use the color entropy, the gray histogram, the edge contour extraction based on the canny operator, totally 59 image feature vector.

We described retrieval accuracy as follows:

$$
    \text{retrieval accuracy} = \frac{\text{number of relevant images}}{\text{total of retrieval images}} \times 100\% \quad (17)
$$
**Analysis**

Figure 3 shows the first retrieval result, input image is marked by blue box and the red one is irrelevant image which will be presented as a feedback.

![Figure 3: The first retrieval result of horses.](image)

![Figure 4: The first feedback result of horses.](image)

![Figure 5: The second feedback result of horses.](image)

Table 1. Performance of three type of Grover(horses).

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Number of iterations</th>
<th>Overall time(s)</th>
<th>Number of feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Grover</td>
<td>4</td>
<td>0.06519</td>
<td>1</td>
</tr>
<tr>
<td>$\pi / 2$ Grover</td>
<td>6</td>
<td>0.07201</td>
<td>1</td>
</tr>
<tr>
<td>$1.825 \pi$ Grover</td>
<td>16</td>
<td>0.08271</td>
<td>1</td>
</tr>
<tr>
<td>Original Grover</td>
<td>4</td>
<td>0.06256</td>
<td>2</td>
</tr>
<tr>
<td>$\pi / 2$ Grover</td>
<td>6</td>
<td>0.07975</td>
<td>2</td>
</tr>
<tr>
<td>$1.825 \pi$ Grover</td>
<td>16</td>
<td>0.09745</td>
<td>2</td>
</tr>
</tbody>
</table>
Table 2. Comparison of retrieval accuracy (horses).

<table>
<thead>
<tr>
<th>Number of retrieval</th>
<th>Retrieval accuracy</th>
<th>Number of feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90%</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>95%</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>100%</td>
<td>2</td>
</tr>
</tbody>
</table>

As can be concluded from the 1-3, feedback algorithm contributes significantly to the improvement of retrieval accuracy. Irrelevant image is introduced as a feedback which prompts the recalculation of previously set feature weight of each dimension. All of those enable users to retrieve desired images effectively and accurately.

Conclusion from figure 1 to figure 3, table 1 and table 2 shows that the improved Grover has great ability to achieve a higher success rate but at the cost of increasing number of iterations; Original Grover fails to retrieve the images correctly at the time the number of those images are close to the image library in this test (please note that this image library means flowers or horses respectively, not the whole number of all the types) or just nearly 50% of it; In this case, as can be seen from experiments, our proposed algorithm performs well in the test without failure as original Grover do. Those improvements can be founded in experimental results not only in iterations, but also in the accuracy and success rate.

Table 3. Performance of proposed in 10 types of images.

<table>
<thead>
<tr>
<th>Images</th>
<th>First retrieval</th>
<th>First feedback</th>
<th>Second feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>African</td>
<td>50%</td>
<td>55%</td>
<td>70%</td>
</tr>
<tr>
<td>beaches</td>
<td>70%</td>
<td>76%</td>
<td>88%</td>
</tr>
<tr>
<td>monuments</td>
<td>39%</td>
<td>51%</td>
<td>68.5%</td>
</tr>
<tr>
<td>buses</td>
<td>76%</td>
<td>71%</td>
<td>90%</td>
</tr>
<tr>
<td>dinosaurs</td>
<td>89%</td>
<td>95%</td>
<td>98%</td>
</tr>
<tr>
<td>elephants</td>
<td>52%</td>
<td>69%</td>
<td>78%</td>
</tr>
<tr>
<td>flowers</td>
<td>81%</td>
<td>86%</td>
<td>93.3%</td>
</tr>
<tr>
<td>horses</td>
<td>64%</td>
<td>80%</td>
<td>93%</td>
</tr>
<tr>
<td>mountain</td>
<td>42.5%</td>
<td>60%</td>
<td>73.7%</td>
</tr>
<tr>
<td>food</td>
<td>34%</td>
<td>50%</td>
<td>63%</td>
</tr>
</tbody>
</table>

Significant improvement in Table 3 describes us that proposed algorithm do contribution to all the types of images in our experiments.

Summary

We combined Grover with a feedback mechanism which enables users take part in the retrieval. Feature weight is considered to be adjusted according to the feedback to select relevant images from irrelevant images, which provides system an opportunity to present retrieved images on the base of users’ feedback. Also, the compute complexity is decreased, $O(N)$ in traditional algorithm but $O(\sqrt{N/M})$ in Grover, which proves that the introduce of Grover do great help to speed up our image retrieval.

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References


