An Adaptive Method to Improve the Iteration Efficiency for Non-binary LDPC Decoding

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Abstract. Non-binary low density parity check codes can achieve better err-correcting performance than binary LDPC codes when the code length is moderate at the cost of higher decoding complexity. Min-Max decoding algorithm is one of the low-complexity quasi-optimal iterative algorithms. However, a problem of the Min-Max decoding algorithm is iteration inefficient in the case of low SNR. In this paper, an adaptive method is proposed based on the Min-Max decoding algorithm. By comparing the correlation among each iteration output, setting an appropriate threshold to determine whether ahead of the end of iteration, which makes the adaptive Min-Max decoding algorithm very attractive for practical purposes.

Introduction

Because of the excellent error-correcting performance and high parallel decoding scheme, Low Density Parity Check (LDPC) codes have recently attracted tremendous research interest. LDPC codes have been lately selected by 3GPP RAN1 and are being New Radio (NR) long code-word lengths coding scheme for 5G standard.

However, the downside of LDPC coding defined on high order Galois fields is a highly increased implementation complexity, especially at the decoding side. Non-binary LDPC code are decoded using the Belief Propagation algorithm on their factor graph⁷⁻⁹. Because of the large number of multiplication operations increase the computational complexity, another algorithm⁴ reformulation uses logarithm domain with log-density or log-density-ratio representation of message. Logarithm domain algorithms require fewer quantization levels due to its lower sensitivity to quantization effects.

To copy with it, some algorithms with reduced complexity have been proposed, such as a logarithm domain implementation for the FFT-BP algorithm⁵. In this case, the elimination of multiplication operations is also possible; however, the addition and subtraction operations in the real domain FFT do not have straightforward equivalent in the logarithm domain. Therefore, the mixed-domain algorithm has been proposed, where the FFT operation is performed in the real domain and the VNs and CNs operate in the log-domain, so that only additions and subtractions are needed.

Alternatively, some suboptimal iterative decoding algorithm for non-binary LDPC codes have been proposed, such as extended min-sum (EMS) algorithm and min-mix (MM) algorithm⁶⁻⁹. However, MM algorithm uses comparison operation instead the addition operation in EMS algorithm node processing, eliminating a lot of real addition operation, decoding complexity is lower than EMS algorithm. Therefore, the MM algorithm is more suitable for hardware implementation.

In order to reduce the complexity of MM decoding further, this paper aim at a problem of the MM decoding algorithm that the iteration is inefficient in the case of low SNR, propose an adaptive method based on the MM decoding algorithm. By comparing the correlation among each iteration output, setting an appropriate threshold to determine whether ahead of the end of iteration, which makes the adaptive MM decoding algorithm very attractive for practical purposes.
The paper is organized as follows. In the next section we briefly review the MM algorithm for NB_LDPC codes. The adaptive MM decoding algorithm and the simulation results is presented in section III and section VI concludes the paper.

**Introduction of MM Algorithm**

In the log domain Min-Max algorithm, symbol reliability information available by log likelihood ratio (LLR) \( \gamma_n(a) = -\log(\Pr(x_n = a) / \Pr(x_n = s_n)) \), which \( s_n \) is the most likely element, \( a \) can be one of the \( q \) elements in the finite field, logarithmic likelihood ratio is all great than or equal to 0, and can be used to measure the distance between a symbol and the most probable symbol. the smaller the LLR, the smaller the distance, and the more reliable the corresponding message. It is convenient for calculation and presentation, we use a vector to organize the \( q \) LLRs, for which the following defined several vector:

- \( H \): The very sparse parity check matrix with \( M \) rows and \( N \) column;
- \( s_n \): The value of the maximum probability in \( x_n \);
- \( a \): One of \( q \) elements in a finite field;
- \( h_{m,n} \): Element values of the \( m^{th} \) row and the \( n^{th} \) column in a check matrix \( H \);
- \( v_{m,n} \): The LLR vector from the check node \( m \) pass to the variable node \( n \);
- \( u_{m,n} \): The LLR vector from the variable node \( n \) pass to the check node \( m \);
- \( M(n) \): Set of neighbor check nodes of variable nodes \( n \);
- \( N(m) \): Set of neighbor variable nodes of check nodes \( m \);
- \( L(m|\alpha_n = a) \): Set of sequences of finite field elements \( a_k (k \in N(m) \setminus n) \) such that \( \sum_{k \in N(m) \setminus n} h_{m,k} a_k = h_{m,n} a \).

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**Algorithm A: The Min-Max Decoding Algorithm**

**Initialization:**

\[
\gamma'_n(a) = \sum_{i=0}^{b-1} \frac{2 \gamma_{n,i}}{\sigma^2}
\]

\[
v_{m,n}(a) = \gamma'_n(a)
\]

**Iterations:**

- **Check node processing**
  \[
u_{m,n}(a) = \min_{a_k \in L(m|\alpha_n = a)} \left\{ \max_{k \in N(m) \setminus n} (v_{m,k}(a_k)) \right\}
  \]

- **Variable node processing**
  \[
v_{m,n}(a) = \gamma'_n(a) + \sum_{k \in M(a) \setminus m} u_{k,n}(a)
  \]
  \[
x = \min_{a \in GF(q)} v_{m,n}(a)
  \]
  \[
v_{m,n}(a) = v_{m,n}(a) - x
  \]

- **A posteriori information computation**
  \[
  \tilde{\gamma}_m(a) = \gamma'_m(a) + \sum_{k \in M(n)} u_{k,n}(a)
  \]

We will study this family of codes over the AWGN channel and BPSK signaling. The Min-Max
algorithm can be carried out according to Algorithm A after the multiplications/divisions by $h_{m,n}$ are computed by separate units\cite{5}. The complicated computation of the set $L(m|\alpha_n = a)$ can be avoided by using the forward-backward scheme\cite{4}.

**Improved MM Decoding Algorithm**

In Min-Max decoding algorithm the number of iterations is fixed. However, with the increase of the number of iterations especially at a small SNR, MM algorithm cannot decode correctly, hence reducing the iteration efficiency. In this section, we introduce an adaptive method to improve the iterative efficiency. By monitoring the output code-word at each iteration decoding, calculate the change rate of code-word, when the change rate is lower than the threshold preseted, stop the iteration directly, regard the last iteration decoding output as the output of this decoding. By introducing adaptive method, it has little effect on the performance of MM decoding algorithm. However, it can improve the iteration efficiency, further more reduce the decoding complexity and convenient for hardware implementation.

Figure 1 show the relationship between the SNR and iteration number with the Min-Max decoding for an example (384,1152) non-binary LDPC code over $GF(4)$. Figure 2 show the relationship between the threshold preseted and iteration number at a fixed SNR with the adaptive Min-Max decoding. An investigation shows that the decoding result is unsuccessful at a low SNR, yet at the cost of large number of iteration. It can't make use of enough decoding output information, and hence has to do a large redundancy repeat searching for the optimal solution, which reduces the efficiency of algorithm. The adaptive method can reduce the number of iterations so that improve the iteration efficiency, while achieving the same performance. However, because of the iteration efficiency is higher at a large SNR, the adaptive Min-Max decoding cannot improve the iteration efficiency much more and can lead to performance degradation.

In this paper, in the case of SNR below 2.2dB, the adaptive MM decoding algorithm is used, and the threshold $\eta$ is 0.0001; while the MM decoding algorithm is used in the case of SNR higher than 2.2dB. Decoding flow chart is shown in Figure 3.
Therefore, we compare the performance between Min-Max and Adaptive Min-Max Decoding Algorithm as show in Figure 4. In the condition of low SNR, AMM algorithm reduces the number of iterations, improves the iteration efficiency, performance and expense can be ignored.

**Conclusion**

Aim at the problem of the Min-Max decoding algorithm iteration inefficient in the case of low SNR, the adaptive method was introduced. By comparing the correlation among each iteration output, setting an appropriate threshold to determine whether ahead of the end of iteration, this paper propose an adaptive Min-Max decoding algorithm. Compared to the Min-Max decoding algorithm, this algorithm improves the iteration efficiency and reduce the decoding complexity, which is easy to implement on the hardware.

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