Texture Classification by Bit-plane Multifractal Spectrum and Bit-plane Barycentric Coordinates of Wavelet Coefficients Based on SVD

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Abstract A new texture classification method based on singular value decomposition (SVD) and wavelet transform is presented. Wavelet transform is employed on texture images having been preprocessed with SVD. The elements of the signature vector of an image are the fractal dimensions and barycentric coordinates of the bit planes of the wavelet coefficients in both the 3-Level high frequency domains and the third low frequency domain. The one-nearest-neighbor classifier with standard $L_1$-norm distance is utilized to perform supervised texture classification. Compared with some other classification methods, the method is experimentally proved more efficient and less time-consuming.

Introduction

Texture understanding and analysis play important roles in many aspects of computer vision and image processing. Despite lots of efforts in last decades, it still remains a challenging problem to model textures efficiently. Some available methods of texture classification and retrieval can be found in [1,2,3,4]. SVD is an important matrix theory and has been popularly employed in image processing, such as data compression [5], texture segmentation [6], and texture classification [7].

Specifically, wavelet-based methods have been intensively researched since wavelet analysis offers an efficient representation of multiresolutions and orientations of images [8,9,10,11]. Recently, models based on wavelet subband coefficients have also been used on texture classification. The existing models in literatures contain the Characteristic Generalized Gaussian Density (CGGD) model [12], the Bit-plane Probability (BP) model [13,14], the Refined Histogram [15], the Local Energy Histogram [16], and so on. Particularly, the Bit-plane Probability (BP) signature is a very competitive feature by modeling wavelet high-frequency subband coefficients via the Product Bernoulli Distributions (PBD). The Bit-plane Probability (BP) signature is built by the PBD model parameters. More specifically, the BP signature in conjunction with the use of weighted $L_1$-norm distance and the minimum distance classifier is presented concretely in paper [14].

In this paper, a new texture classification method based on SVD and wavelet transform is presented. At first, we reconstruct texture images with SVD by discarding the second half of the singular values. Then wavelet transform is employed on these reconstructed texture images. Meaningfully, we use a new texture classification method called BP-MFS, which is based on Bit-plane multi-fractal spectrum (MFS) [22] of wavelet coefficients. In detail, fractal dimensions of the bit planes of wavelet coefficients in high frequency domains and the lowest level's low frequency domain are computed and to be used as the elements of the signature vector of an image. What's more, to improve the higher accuracy of texture classifications, we use a new feature called BP-MFS-BCC [22], in which the signature vector of an image is constructed by both the fractal dimensions and barycentric coordinates (BCC) in three-dimensional coordinate system of the bit planes of the wavelet coefficients in both high frequency domains and the lowest level's low frequency domain. In paper [22], BP-MFS signature for texture classification has been tested by using one-nearest-neighbor (1NN) classifier and minimum distance classifier and the average classification accuracy rate with one-nearest-neighbor classifier is higher than that with minimum distance classifier. So 1NN with $L_1$-norm distance is utilized to perform supervised texture classifications in
this paper. The proposed method gives a satisfaction performance in comparison with some current state-of-art methods in supervised texture classification.

The rest of the paper is organized as follows. In Section 2, the details of the proposed texture classification method are presented. The experiments results and comparisons are shown in Section 3. Finally, the conclusion is given in Section 4.

Proposed Texture Classification Method

The BP-MFS-BCC Method

Due to the fractal dimension alone does not provide enough description of objects, multifractal spectrum (MFS) analysis, as an extension of classical fractal analysis, was proposed to describe complex patterns with objects of multiscales [17-19]. Precisely speaking, for a given objective function \( f(x,y) \) with domain \( S \subset \mathbb{R}^2 \), define a point categorization and split the domain \( D \) into \( K \) subdomain \( D_{ij}, j = 1, \ldots, K \). Then the vector \( d = (d_1, \ldots, d_K) \) is called the MFS of \( f(x,y) \), where \( d_j \) is the fractal dimension of \( D_{ij}, j = 1, \ldots, K \). Obviously, in MFS analysis, the two key roles are the objective function and the point categorization criteria. Since the characteristic of multi-scaling and triple direction decomposition, wavelet transform (WT) is widely used in representation of textures. Here, the high frequency subbands of the WT of an image are taken as objective functions, and every subband is divided into several binary bit plains using the wavelet coefficients.

Let \( I \) be a gray image, \( W_{i,1}, W_{i,2}, W_{i,3} \) be the 3 matrices of wavelet coefficients of high frequency in decomposition level \( i \). For simplicity, we only focus on the absolute values of all wavelet coefficients. According to computing algorithm of WT, amplitudes of wavelet coefficients increase exponentially with multiplication factor \( 1/\sqrt{2} \) as the scale \( i \) of wavelet decomposition increases. So we make a uniform measure for those coefficients by multiplying a factor \( 1/\sqrt{2}^i \), which keeps all these coefficients in \([0,255]\) as the original gray levels of \( I \) do. Thus each subband \( W_{ij} (j = 1, 2, 3) \) can be cut into 8 binary bit planes by quantization. As a logical image, each bit plane possesses fractal property in some sense and its fractal dimension is denoted by \( d_{ij}^{(k)}, k = 1, \ldots, 8 \). Hence the MFS of \( W_{ij} \) is

\[
\text{MFS}(W_{ij}) = (d_{ij}^{(1)}, \ldots, d_{ij}^{(8)}),
\]

And the MFS of the \( i \)-th level of WT is a vector of 24-dimension

\[
\text{MFS}(W_i) = (\text{MFS}(W_{i,1}), \text{MFS}(W_{i,2}), \text{MFS}(W_{i,3})).
\]

Assuming \( L \)-level wavelet decomposition is made on \( I \), the whole MFS of the \( 3L \) high frequency subbands is defined as a vector of dimension of \( 24L \),

\[
\text{MFS}(I,L) = (\text{MFS}(W_1), \ldots, \text{MFS}(W_L)).
\]

To emphasize the fact that all fractal dimensions are the dimension of bit planes of wavelet coefficients, above MFS is called MFS based on bit plane(BP-MFS),

\[
\text{BP-MFS}(I) = (\text{MFS}(W_1), \ldots, \text{MFS}(W_L)).
\]

In addition, to improve the higher accuracy of texture classifications, we proposed a new feature called BP-MFS-BCC based on the above new method BP-MFS. In this method, the signature vector of an image is constructed by both the fractal dimensions and barycentric coordinates in three dimensional coordinate system of the bit planes of the wavelet coefficients in both domains and the lowest level's low frequency domain. On account of the fractal dimension is on the closed interval \([0,2]\). To ensure our feature's conformity error, the three-dimensional coordinates of the bit planes are normalized on the same closed interval \([0,2]\). The barycentric coordinates of the bit planes of an image (with size of \([M,N]\)) are calculated by
\[ \bar{x} = \frac{2}{N \times \text{sum}} \sum_{k=1}^{8} \sum_{i=1}^{M} \sum_{j=1}^{N} x(i, j, k), \]
\[ \bar{y} = \frac{2}{M \times \text{sum}} \sum_{k=1}^{8} \sum_{i=1}^{M} \sum_{j=1}^{N} y(i, j, k), \]
\[ \bar{z} = \frac{2}{8} \sum_{k=1}^{8} \sum_{i=1}^{M} \sum_{j=1}^{N} z(i, j, k), \]
x(i, j, k), y(i, j, k), z(i, j, k) refer to the three-dimensional coordinates of each non-zero pixel in the 8 bit planes and num refers to the total number of the non-zero pixels in the 8 bit planes.

**Our Method**

To extract efficient features of texture images, this paper utilizes SVD theory to preprocess images as the first step. In detail, we reconstruct texture images with SVD by discarding some part of singular values, and then the feature extracting method of BP-MFS-BCC is employed on these reconstructed texture images. This preprocessing step aims to retain the most valuable information of images from the useful singular values and to reduce minimal interference from the unuseful singular values. In the next section, the selection of the valuable singular values will be determined by experience.

**Experimental Results**

To evaluate the performance of our signature for texture classification, we have tested it on texture images from the well-known and widely used Brodatz database [20] and UMD database [21]. In Brodatz database, a typical set of 80 grey 640 × 640 images is tested. Each image is divided into 16 nonoverlapping patches of 160 × 160, and thus total 1280 samples are available. 15 kinds of grey 960 × 1280 images are chosen from UMD database. Similarly, each image is divided into 16 nonoverlapping patches of size 240 × 320, and thus total 240 samples are gotten.

In the following experiments, we choose randomly half of the sample images in each class from the database as the training samples and the rest images are used for testing. In addition, one-nearest-neighbor (1NN) classifier is used in texture classification and such experiment is repeated 100 times to obtain the average results of classification performance. 3-level wavelet decomposition with the Daubechies 10 (db10) filter bank is utilized to obtain the signatures of the images.

**Selection of the Valuable Singular Values**

![Figure 1](image1.png)

Figure 1. Three ways to selecting singular values’ variation trends of classification accuracy rate on Brodatz.

![Figure 2](image2.png)

Figure 2. Three ways to selecting singular values’ variation trends of classification accuracy rate on UMD.
In this experiment, three ways to selecting singular values as image signature are tested and the results are shown in Figure 1 and Figure 2. In Figure 1 and Figure 2, the first plot describes variation trend of the classification accuracy rate when selecting each singular value as image signature, the second plot describes variation trend of the classification accuracy rate when selecting the top n singular values as image signature, and the third plot describes variation trend of the average classification accuracy rate when discarding the top n singular values as image signature. The plots in both Figure 1 and Figure 2 shows that, in texture classification, the smaller singular values play little role and the most valuable singular values focus on the first half part. Additionally, we choose 8 different parts of the singular values to reconstruct texture images and the average classification accuracy rates (ACAR) on both two databases are shown in Table 1. Figure 1, Figure 2 and Table 1 show that it is reasonable to reconstruct texture images with SVD by remaining the first half of the singular values.

Our Method on Texture Classification

We compare both BP-MFS signature and BP-MFS-BCC signature combined with SVD with BP method proposed in [14] and LEH method proposed in [16] on texture images from the Brodatz database and the UMD database. In these experiments, the number of the training samples in each texture image class is set as 8, half of total samples in each class. Besides the average classification accuracy rate (ACAR), the computational costs with different methods are also compared. The time for texture classification (TTC) is defined here as the costing time for the entire test samples being classified in once test. The ACAR and TTC of the four methods on above mentioned two data sets are listed in Table 2. As shown in Table 2, the proposed method gives a better performance.

Table 1. The ACAR of feature extraction with 8 different parts of the singular values to reconstruct texture images.

<table>
<thead>
<tr>
<th>Database</th>
<th>Signatures</th>
<th>8 different parts of the singular values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brodatz</td>
<td>[1,40]</td>
<td>[1,80] [1,120] [21,80] [21,100] [21,120] [81,120] [81,160]</td>
</tr>
<tr>
<td></td>
<td>BP-MFS</td>
<td>95.02% 95.92% 95.89% 95.34% 61.93% 60.89% 62.00% 24.40%</td>
</tr>
<tr>
<td></td>
<td>BP-MFS-BCC</td>
<td>95.48% 96.39% 96.06% 95.97% 75.67% 75.61% 75.51% 43.00%</td>
</tr>
<tr>
<td>UMD</td>
<td>[1,60]</td>
<td>[1,120] [1,180] [31,120] [31,150] [31,180] [121,180] [121,240]</td>
</tr>
<tr>
<td></td>
<td>BP-MFS</td>
<td>95.46% 96.76% 96.59% 83.24% 85.80% 86.23% 44.98% 41.67%</td>
</tr>
<tr>
<td></td>
<td>BP-MFS-BCC</td>
<td>96.39% 97.52% 96.88% 90.04% 91.29% 90.72% 58.63% 55.79%</td>
</tr>
</tbody>
</table>

Table 2. The ACAR and TTC(in seconds) of four methods tested on images from Brodatz database and UMD database.

<table>
<thead>
<tr>
<th>Database</th>
<th>BP-MFS-BCC</th>
<th>BP-MFS</th>
<th>BP</th>
<th>LEH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brodatz</td>
<td>ACAR</td>
<td>96.39%</td>
<td>95.92%</td>
<td>86.82%</td>
</tr>
<tr>
<td></td>
<td>TTC</td>
<td>1.08</td>
<td>1.05</td>
<td>0.14</td>
</tr>
<tr>
<td>UMD</td>
<td>ACAR</td>
<td>97.52%</td>
<td>96.76%</td>
<td>82.06%</td>
</tr>
<tr>
<td></td>
<td>TTC</td>
<td>0.05</td>
<td>0.03</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Summary

In this paper, we have presented a new method BP-MFS and BP-MFS-BCC combined with SVD for texture classification. It is based on SVD, wavelet transform and multifractal spectrum analysis. The one-nearest-neighbor classifier is utilized to perform supervised texture classification, where the standard L1-norm distance is a suitable measure for comparing the new signatures. We have tested our method on some texture images from the Brodatz database and the UMD database. The experiments have shown that our proposed method gives a satisfaction performance in supervised texture classification.
References


