1. INTRODUCTION

Under the background of continuously developing global economic culture, the application of English is also in gradual enhancement. As a result, there’s greater need for English training, especially in China. Huge development room and market are waiting for English training institutes. According to incomplete statistics, there are at least 5,000 English training institutions in China, which are still increasing. A large number of English practitioners are paying great attention to the study of English training teaching. Scientific study of English teaching can direct the reasonable development of English in a better way, and continuously enhance English system. However, the study of English teaching evaluation is still completed by interview investigation and documentary method at present. No study has ever been conducted on the quantitative analysis and prediction of English teaching evaluation. This paper can fill up the gap in the study.

ABSTRACT: With the rapid development of English teaching study, the establishment of evaluation system for English teaching has become particularly important. This paper studied the evaluation on English teaching by means of multi-level fuzzy theory. At first, it established a model based on multi-level fuzzy theory to evaluate English teaching according to the current development situation of English teaching evaluation. And then, this paper made quantitative analysis of the teaching evaluation result obtained by model calculation, so as to obtain an intuitive and scientific evaluation result.

Keywords: English teaching; teaching evaluation; multi-level fuzzy theory; fuzzy comprehensive evaluation; weight vector
study of this area\textsuperscript{[2]} and provide a scientific research method for the development and study of English training teaching. In addition, it is beneficial for the cultural and economic communication among countries, providing assistance to the development of English career in China.

2 MULTI-LEVEL FUZZY COMPREHENSIVE EVALUATION

Domain of discourse \( U \) covering all the influence factors is constituted of \( k \) layers (\( k \geq 2 \)). The first layer contains \( m \) influence factors \( U = (U_1^1 \ U_2^1 \ldots \ U_m^1) \). Evaluation set can be expressed as \( V = (v_1 \ v_2 \ldots \ v_m) \).

Therefore, the function of multi-level fuzzy comprehensive evaluation can be represented as:

\[
B = A \circ R = A \circ \begin{pmatrix}
(A_{11} \circ R_{111}) & (A_{11} \circ R_{112}) & \cdots & (A_{11} \circ R_{11r}) \\
A_{12} \circ R_{121} & (A_{12} \circ R_{122}) & \cdots & (A_{12} \circ R_{12r}) \\
\vdots & \vdots & \ddots & \vdots \\
A_{1p} \circ R_{1p1} & A_{1p} \circ R_{1p2} & \cdots & (A_{1p} \circ R_{1pr}) \\
\end{pmatrix}
\]

In the function: \( A \) refers to the weight vectors contained in each layer. The subscript \( x \) in the formula means the weight vector of \( A \) in the \((x+1)\)-th layer. \( R \) refers to the fuzzy relation matrix in the \(k\)-th layer of the functional matrix.

Multi-level fuzzy comprehensive evaluation starts from the \(k\)-th layer. Calculation is completed from lower layers to upper layers until the final evaluation set \( B \) is reached. The calculation steps are as follows:

Step 1: The followings can be obtained by calculation starting from the lowest layer:

\[
B_{11} = A_{11} \circ R_{111} \\
B_{12} = A_{12} \circ R_{112} \\
\vdots \\
B_{1s} = A_{1s} \circ R_{11s} \\
B_{mq1} = A_{mq1} \circ R_{mq1} \\
B_{mq2} = A_{mq2} \circ R_{mq2} \\
\vdots \\
B_{mqp} = A_{mqp} \circ R_{mqp}
\]

Step 2: Set \( R_{i} = \begin{pmatrix} B_{i1} \\
B_{i2} \\
\vdots \\
B_{im} \end{pmatrix} \)

Step 3: Obtain the followings from calculation of the second layer:

\[
B_{i} = A_{i} \circ R_{i} \\
B_{m} = A_{m} \circ R_{m}
\]

Step 4: Set \( R = \begin{pmatrix} B_{i} \\
\vdots \\
B_{m} \end{pmatrix} \)

Obtain the final calculation. Evaluation set is expressed as: \( B = A \circ R \).

2.1 Specification of model
Evaluation is assessed by three indexes: teaching results, teaching method and the student feedback. The correlativity in between is shown in Figure 1.

2.2 Fuzzy comprehensive evaluation method
In real life, assessment can be influenced by many factors at the same time. Therefore, comprehensive evaluation is needed in assessment. However, due to the existence of some fuzzy influence factors, fuzzy mathematics can be used to evaluate the fuzzy relation.

(1) Determination of evaluation set
Use \( X \) to represent all the evaluations that may appear; and use \( n \) to represent the number. All evaluations can constitute an evaluation set:

\[
X = \{x_1, x_2, \ldots, x_n \}
\]

In this formula, each \( x_i, i = (1, 2, \ldots, n) \) refers to all kinds of results that may appear in evaluation results. Obtain the evaluations of all factors to find the most
reasonable judgment results which can bring the most scientific evaluation result based on fuzzy comprehensive evaluation.

(2) Determination of factor set
U refers to factor set as follows:
\[ U = \{u_1, u_2, \ldots , u_n\} \]

In this formula, each \( u_i \) refers to the influence factor that may affect the results.

(3) Single-factor fuzzy evaluation
There are \( i \) factors \( u_i(1,2,\ldots ,m) \) in factor set that need corresponding judgment. In evaluation set \( u_i(1,2,\ldots ,m) \), there are \( j \) factors of which the degree of membership \( x_j \) is \( r_j(j=1,2,\ldots ,n) \). The influence judgment set of the \( i \)-th factor \( u_i \) is
\[ r_i = (r_{i1}, r_{i2}, \ldots , r_{in}) \]

(4) Fuzzy comprehensive evaluation
In judgment matrix \( R_i \): The \( i \)-th row of \( R_i \) reflects the influence degree that the \( i \)-th factor leaves on the subordination of the evaluation object. The \( j \)-th line of \( R_i \) refers to the influence that all factors leave on the subordination degree of the evaluation in \( j \)-th evaluation set. When there are weight set \( W \) and evaluation matrix \( R_i \), use fuzzy comprehensive evaluation to evaluate:
\[ A_i = W \bullet R_i = (w_{i1}, w_{i2}, \ldots , w_{in}) \bullet (r_{i1}, r_{i2}, \ldots , r_{in}) = (a_{i1}, a_{i2}, \ldots , a_{in}) \]
\[ A = W \bullet R = (W_1, W_2, \ldots , W_n) \bullet [A_1, A_2, \ldots , A_i] = (a_1, a_2, \ldots , a_n) \]

(5) Establishment of weight set
Weight set \( W \) is composed of weight value \( w_j \) which is determined by influence of factors \( u_j \). Use Analytic Hierarchy Process to find the weights that different factors have to different indexes:
\[ W = \{W_1, W_2, \ldots , W_i\} \]
\[ W_i = \{W_{i1}, W_{i2}, \ldots , W_{in}\} \]

The restraint condition of the weight of each hierarchy is as follows:
\[ \sum_{j=1}^{n} w_j = 1, \quad w_j \geq 0 \]

2.3 Modeling
The factors which can impact the final teaching effect during English teaching process cover several aspects. There are three measurable indexes of the multi-level fuzzy comprehensive theory for English in mathematics evaluation system. In the three measurable indexes, there are nine secondary measurable indexes which are test performance, English application ability, attendance, emphasis, individualized teaching, edutainment, interest in English, assessment on teacher and the English proficiency improvement. The relations between different indexes are fuzzy and can leave different degrees of influence on teaching evaluation [3].

Classify the scale of influence to construct subordinating degree function \( F(x) \) of each index. Combine weight vector and subordinating degree matrix to obtain fuzzy comprehensive evaluation matrix, and thus measuring the influence.

Fuzzy comprehensive evaluation method:
For \( Y = \{A, B, C, D, E\} \) in this paper. A level means the influence degree is very high; B level means the influence degree is high; C level means the influence degree is somewhat high; D level means the influence degree is low; E level means the influence degree is very low.

In order to obtain the value of subordinating degree corresponding to index \( x_i \), we conduct evaluation on the hierarchies between \( x_i \). The classification of hierarchy is as follows: Set \( A=10, B=7, C=5, D=3 \) and \( E=1 \). Thus, the hierarchy of factor \( x \) is shown as follows:
\[ M = 10A(x) + 7B(x) + 5C(x) + 3D(x) + E(x) \]

Fuzzy evaluation is \( R = \{r_{11}, r_{12}, \ldots , r_{1m}\} \in F(X \times Y) \).

For \( m \) factors, there are \( m \) fuzzy evaluation \( R_1, R_2, \ldots , R_m \) which can be expressed in the form of matrix.

3 MODEL SOLUTION

3.1 Determination of weight based on analytic hierarchy process
1) Construct judgment matrix
A proper decision matrix of foundation shall be established for each hierarchy. Decide the weight relation between indexes and establish the judgment matrix between factor relations.

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Obtain the comparative matrix by listing the importance degree between each factor in every hierarchy and obtain the comparative matrix \( A = (a_{ij})_{n \times n} \) and \( a_{ij} = 1 \), meaning the comparative result of factors is 1.

\[
r_i = \sum_{j=1}^{n} a_{ij} \quad i = 1, 2, 3, \ldots, n
\]

Comparative matrix can be deformed into indirect judgment matrix through mathematical variation. The variation method is shown as follows:

\[
d_{ij} = \begin{cases} \frac{r_i - r_j}{r_{\text{max}} - r_{\text{min}}} (b_m - 1) + 1 & r_i - r_j \geq 0 \\ \frac{r_i - r_j}{r_{\text{max}} - r_{\text{min}}} (b_m - 1) + 1 & r_i - r_j < 0 \end{cases}
\]

The obtained indirect judgment matrix contains the following properties:

\[
\begin{align*}
\frac{1}{b_m} \leq d_{ij} & \leq 1 & d_{ij} < 1 \\
1 \leq d_{ij} & \leq b_m & d_{ij} \geq 1
\end{align*}
\]

Which means the numerical range of \( d_{ij} \) is the scale of \( 1 - b_m \):

\[
d_{ij} = \frac{1}{d_{ij}}
\]

Thus, the indirect matrix after variation still contains the mutually reciprocal property between matrix and symmetry element:

\[
\begin{align*}
&\text{When there is } b_m = 9, \ 9 \text{ refers to the scale.} \\
&\text{2) Calculate relative weight}
\end{align*}
\]

This paper applied the square root method to solve the weights of \( n \) elements \( A_1, A_2, \ldots, A_n \) in multi-index in \( C_k \). The characteristic root to solve \( A \) is \( AW = \lambda_{\text{max}} \cdot W \). Process the characteristic root \( W \) by normalization and use it as the ranking weight of elements \( A_1, A_2, \ldots, A_n \) in index \( C_k \). There is equation to realize the sole existence of \( \lambda_{\text{max}} \), thus \( W \) can be expressed by positive component. \( W \) exists and there is only one of it \([4]\).

\[
CI = \frac{\lambda_{\text{max}} - n}{n - 1}
\]

When \( CI > 0 \), compare the value of \( CI \) with randomness index \( RI \).

When the consistency ratio of randomness is:

\[
CR = CI / RI \leq 0.1
\]

There’s high consistency in the matrix.

When the consistency ratio of randomness is:

\[
CR = CI / RI > 0.1
\]

There’s low consistency in the matrix and further modification is needed until satisfying consistency is reached.

For matrixes from the first order to the tenth order, the indexes to judge matrix consistency are shown in Table 1.

### Table 1. Index table to judge the consistency of matrixes from the first order to the tenth order.

<table>
<thead>
<tr>
<th>Order</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>RI</td>
<td>0</td>
<td>0</td>
<td>0.58</td>
<td>0.90</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
<td>1.49</td>
</tr>
</tbody>
</table>

### Table 2. The method of nine marks.

<table>
<thead>
<tr>
<th>Meaning</th>
<th>( u_i ) is as important as ( u_j )</th>
<th>( u_i ) is slightly more important than ( u_j )</th>
<th>( u_i ) is important</th>
<th>( u_i ) is much more important than ( u_j )</th>
<th>( u_i ) is extremely more important than ( u_j )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_{ij} ) value</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

\( u_{ij} \) values 2, 4, 6 and 8 are between the adjacent two judgment dimensions.

### Table 3. Value of consistency index \( RI \).

<table>
<thead>
<tr>
<th>Matrix order</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>RI value</td>
<td>0</td>
<td>0.58</td>
<td>0.90</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
</tr>
</tbody>
</table>

3.2 Detailed calculation

Evaluation was made on the English teaching of some school by issuing questionnaires. Quantitative scores of the nine secondary measurable indexes were obtained: test performance, English application ability, attendance, emphasis, individualized teaching, education, interest in English, assessment on teacher and English proficiency improvement. After data processing, judgment matrix for primary indexes was
established according to the method of nine marks. The method of nine marks is shown in Table 3.

Judgment matrix for primary indexes: \[ A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 7 & 3 \\ 8 & 1 & 7 \\ 3 & 1 & 6 \\ 1 & 3 & 1 \end{bmatrix} \]

The maximum eigenvalue of matrix \( A \) is \( \lambda_{\text{max}} \).

The corresponding eigenvector is \( u = (u_1, u_2, \ldots, u_n)^T \).

The maximum eigenvalue of \( A \): \( \lambda_{\text{max}} = 2.8889 \)

Weight vector: \( U = \begin{bmatrix} 0.0901 \\ 0.5655 \\ 0.3444 \end{bmatrix} \)

Based on the above, the weight vectors of each index corresponding to the secondary influence factors can be calculated as follows.

Teaching effect: judgment vector \( A_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 3 & 1 \\ 8 & 3 & 7 \end{bmatrix} \)

Maximum eigenvalue: \( \lambda_{\text{max}} = 2.8885 \)

Weight vector: \( U_1 = \begin{bmatrix} 0.2005 \\ 0.5845 \\ 0.2150 \end{bmatrix} \)

Teaching method: The judgment vector is

\[ A_2 = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 1 & 6 \\ 3 & 1 & 7 \end{bmatrix} \]

Maximum eigenvalue: \( \lambda_{\text{max}} = 3.0030 \)

Weight vector is \( U_2 = \begin{bmatrix} 0.0954 \\ 0.3048 \\ 0.5998 \end{bmatrix} \)

Student feedback: The judgment vector is

\[ A_3 = \begin{bmatrix} 1 & 4 & 6 \\ 1 & 2 \\ 1 & 6 & 2 \end{bmatrix} \]

Maximum eigenvalue: \( \lambda_{\text{max}} = 2.9950 \)

Consistency check:
Verification principle: The index of consistency check is \( CI = \frac{\lambda_{\text{max}} - n}{n-1} \) (n refers to the matrix order, where n equals 3); consistency ratio is \( CR = \frac{CI}{RI} \).

When there is \( CR < 0.1 \), the matrix contains consistency.

Results of consistency check are shown in Table 4.

<table>
<thead>
<tr>
<th>( \lambda ) value</th>
<th>( \lambda_{1\text{max}} )</th>
<th>( \lambda_{2\text{max}} )</th>
<th>( \lambda_{3\text{max}} )</th>
<th>( \lambda_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( CI )</td>
<td>0.00069</td>
<td>0.0480</td>
<td>0.001</td>
<td>0.0488</td>
</tr>
<tr>
<td>( RI )</td>
<td>0.58</td>
<td>0.58</td>
<td>0.58</td>
<td>0.58</td>
</tr>
<tr>
<td>( CR )</td>
<td>0.0012</td>
<td>0.0820</td>
<td>0.0016</td>
<td>0.0870</td>
</tr>
<tr>
<td>Consistency</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

3.3 Establishment of comprehensive assessment matrix

(1) Membership degree function
This paper used relevant data for comparison and analysis to obtain membership degree functions. For example, if the degree of teaching result was high, the function vertex would move to A; if the degree of teaching result was low, the function vertex would move to E. As shown in Figure 2, the starting points and end points of the function were set to be fixed, making the area sum of the image enclosed by membership degree functions and the coordinate axes as 1, in order to determine the membership degree function [6].

Figure 2. Function value of membership degree.

Vertex:
The functional images of all membership degrees as shown in Figure 3 were obtained by comparison.

(2) Fuzzy comprehensive assessment matrix
Weight vector \( U_i \) was obtained according to each
secondary index. Fuzzy comprehensive assessment matrix $R_i$ could be calculated by the membership matrix $V_i$. According to fuzzy matrix model which was $R_i = U_i \circ V_i$, could be translated into operation of a normal matrix product shown below so as to simplify the algorithm [7]:

$$R_i = U_i^T \ast V_i$$

We can obtain: the teaching result: $R_1 = \begin{pmatrix} 0.0555 \\ 0.1702 \\ 0.2648 \\ 0.3294 \\ 0.1801 \end{pmatrix}$; the teaching method: $R_2 = \begin{pmatrix} 0.0758 \\ 0.2312 \end{pmatrix}$; the student feedback: $R_3 = \begin{pmatrix} 0.0703 \\ 0.2982 \end{pmatrix}$.

Fuzzy comprehensive assessment model:

One-level fuzzy comprehensive assessment matrix

$$R = \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix}$$

was established according to $R_1, R_2, R_3$.

The grade obtained by multiplying the weight vector $U$ which is calculated from first-level index and the matrix $R$ which is achieved from one-level fuzzy comprehensive assessment was shown as follows:

$$M = 10 \times 0.1390 + 7 \times 0.2680 + 5 \times 0.3165 + 3 \times 0.2998 + 0.060 = 5.8079$$

4 CONCLUSIONS

This paper established an English teaching evaluation system based on multi-level fuzzy comprehensive theory according to analysis of the features contained in English teaching evaluation process. Analysis and fuzzy assessment were given to English teaching evaluation through multi-level fuzzy comprehensive theory. Besides, quantitative analysis was conducted on the result of English teaching evaluation, so as to obtain scientific assessment on English teaching evaluation. Verification was made on the calculation of concrete case and analysis of the result. Therefore, the model has been verified while its feasibility has been proved which can provide experiences and references to the establishment of other English teaching evaluations.

REFERENCES


