Optimization Design of Parameters of Z-source Inverter based on the Small-signal Model using State-space Averaging Method

Houyun Liu¹, Tao Sun²*, Shiyng Hou², Zhicheng Guo², Ruimiao Wang¹ & Ruilin Xu¹
¹Electric Power Research Institute, State Grid Chongqing Electric Power Company, Chongqing, China
²State Key Laboratory of Power Transmission Equipment & System Security and New Technology, Chongqing University, Chongqing, China

ABSTRACT: Optimization of inverter’s parameters is one of the key issues in improving the inverter’s performance. Z-source inverter is a new kind of topological structure. The research on optimization of its impedance network’s Parameters is of great value in theory and application. First, the state-space averaging method is used to establish the small-signal model of impedance network capacitor voltage and obtain the transfer function from the direct duty cycle to the impedance network capacitor voltage. And the influence of inductor and capacitor’s parameters changes on dynamic performance of capacitor voltage’s step response is analyzed based on this. At the same time, the optimized principle of impedance network’s parameters is proposed. Finally, the validity of the proposed optimized principle is proved by the simulation results.

Keywords: Z-source inverter; state-space averaging method; small-signal model; parameter optimization

1 INTRODUCTION

Because of the prevalence of the intermittent problems, the volatility and the problem that output voltage level is low in new renewable energy, the traditional bridge inverters are unable to get good performance on these occasions because of inherent disadvantages. Z-source inverter has emerged as the solution for problems, and traditional inverters have been applied on these occasions. Z-source inverter allows the two switch tubes on the same bridge arm shoot, inverter bridge and impedance network coupling to provide a new mechanism to achieve up/down inverter, so it has broad application prospects in new energy generation, frequency control system and other occasions.

Compared with traditional bridge inverter, Z-source inverter is increased in an X-type impedance network. Therefore, it is very important to model Z-source impedance network accurately in system’s dynamic performance analysis, and domestic and foreign scholars have already proposed a variety of modeling methods on impedance network.

[3-6] had a study on Z-source inverter based on steady-state circuit model and constructed a simplified circuit model for Z-source inverter. The model can only analyze quiescent operating point of the system, it is unable to get the small-signal model of the system and cannot possess frequency domain characteristics and transfer function of the system variables to guide the design of the controller. [7] used the state-space averaging method for Z-source inverter modeling and designed the controller based on the model, but this model only analyzes the transfer function of the shoot through the duty cycle to capacitor voltage and has no comprehensive analysis of the effect on inductor parameters, and capacitor parameters may have on dynamic performance; [8] used the state-space averaging method and built a small-signal model for Z-source inverter with a continuous conduction mode. This model introduces dynamic characteristic of capacitor and inductor in the impedance network, expands the steady-state model and gives the small-signal equivalent circuit. This paper uses Bode diagram to analyze the impact of inductor and capacitor parameters on the system dynamic performance, while this model does not consider the parasitic parameters of inductor and capacitor within a certain frequency range, and the accuracy of the model is not enough, so it is unable to make the affect parameters possess the dynamic performance of DC link voltage; [9] respectively used the

*Corresponding author: suntao@cqu.edu.cn
small-signal analysis and the signal flow diagram analysis method to get transfer function of shoot through the duty cycle to the capacitor voltage and the input voltage to the capacitor voltage, and used the root locus analysis method to analyze the impact of shoot through duty cycle, inductor, capacitor and their parasitic parameters on capacitor voltage stability; [10] used the signal-flow diagram modeling method to obtain dynamic small-signal model of Z-source impedance network, considering parasitic parameters of components, and the transfer function of shoot through duty cycle to capacitor voltage reveals the existence of right-half-plane zero point and used the classical root locus method to analyze the influence of component parameters on the dynamic performance of the system; [11] respectively modeled the Z-source impedance network and the inverter bridge, obtained the state-space model of Z-source inverter and completed the design of a closed-loop controller; [12] given the mathematical model of three-phase inverter bridge described by both switching function and duty cycle and designed the control system guided by the mathematical model described by duty cycle.

Currently, the modeling and analysis of Z-source impedance network is always conducted to analyze network parameters on the system dynamic and static performance, so how to determine the selection principle of parameters and optimize impedance network parameters ranges based on the analysis about the intrinsic characteristics of the impedance network is still key issue to be solved.

For existing problems in Z-source inverter impedance network modeling studies, in this paper we use the state-space averaging method to establish a small-signal model of impedance network capacitor voltage for the transfer function of shoot through duty cycle, using classical control theory root locus method to analyze the effects that network inductor, capacitor and parasitic capacitor as well as shoot through duty cycle have on the capacitor voltage dynamic performance. Then we research on the optimization of the impedance network parameters in the model based on the verification of correctness of the small-signal model of capacitor voltage. By comparing the step response of Z-source inverter circuit simulation and the small-signal model of capacitor voltage when duty cycle is under mutation, we verified the correctness of the model; on this basis, inductor and capacitor parameters were optimized to provide a theoretical guidance for the design of the impedance network parameters, according to the effect that the impedance network parameters have on the capacitor voltage dynamic performance of the step response.

2 STATE-SPACE AVERAGING SMALL-SIGNAL MODEL OF Z-SOURCE INVERTER

The general topology of Z-source inverter is shown in Figure 1. This topology is built on the introduction of a X-type two-port network based on a conventional voltage-source inverter, and DC input power source and the main circuit of the inverter bridge coupled together through the network, thus ensuring that the Z-source inverter can operate under the condition the conventional voltage-source inverter operated in while working in shoot through a state the tradition inverter prohibited, on which state the inverter bridge provides a unique up/down function[13].

When building a small-signal model of impedance network, we need to consider the parasitic resistances $r_1$ and $r_2$ of the network inductor and the capacitor equivalent series resistances $R_1$ and $R_2$. For the convenience of analysis, the impedance network takes a symmetrical structure, so $C_1 = C_2 = C$, $L_1 = L_2 = L$, $r_1 = r_2 = r$, $R_1 = R_2 = R$. Choose the impedance network inductor current and capacitor voltage as the state-space average model state variables. When the diode $D_m$ is off, Z-source inverter will be in through state, and the state-space equation of direct connection can be shown as follows:

$$\begin{pmatrix}
\frac{di_1}{dt} \\
\frac{dV_{C_1}}{dt} \\
\frac{di_2}{dt} \\
\frac{dV_{C_2}}{dt}
\end{pmatrix} =
\begin{pmatrix}
r + R & 1 & 0 & 0 \\
L & -1 & 0 & 0 \\
0 & 0 & r + R & \frac{1}{L} \\
0 & 0 & -\frac{1}{C} & 0
\end{pmatrix}
\begin{pmatrix} i_{1r} \\
V_{C_1} \\
i_{2r} \\
V_{C_2}\end{pmatrix}
+ \begin{pmatrix} 0 \\
0 \\
0 \\
0
\end{pmatrix} V_m
+ \begin{pmatrix} 0 \\
0 \\
0 \\
0
\end{pmatrix} i_m
+ \begin{pmatrix} 0 \\
0 \\
0 \\
0
\end{pmatrix} i_m
$$

(1)

When the diode $D_m$ is conducted, Z-source inverter will be in a non-pass-through state. In this case, the inductor and capacitor of the impedance network simultaneously provide energy to the inverter side to achieve a boost function of the DC link, and the cor-
responding state-space equation can be shown as follows:

\[
\begin{align*}
\frac{di_i}{dt} &= \left( \begin{array}{ccc}
-r + R & 0 & -1/L \\
0 & 0 & 1/C \\
0 & -1/L & -r + R/L \\
\end{array} \right) i_i \\
\frac{dV_{C_i}}{dt} &= \left( \begin{array}{c}
1/L \\
0 \\
-1/C \\
\end{array} \right) V_{C_i} \\
\end{align*}
\]

(2)

According to the mind of state-space averaging, and in order to establish piecewise-linear equations based on the circuit continuum model with different switching states and get the state equation of the circuit in the whole switching cycle, we need to segment the above equation and make them be averaged. As the selected state variables, input variables and output variables of a straightforward state and non-through-state are the same, so the averaging equation can be obtained by the weighted average of the coefficient matrix. Here, we take shoot through duty cycle \( D_0 \), so the non-pass-through duty cycle is \( 1-D_0 \). And the state averaging equation can be obtained according to the state averaging method.

\[
\begin{align*}
\frac{di_i}{dt} &= \left( \begin{array}{ccc}
\frac{(r + R)}{L} & -\frac{D_0}{L} & 0 \\
-\frac{D_0}{L} & 0 & 0 \\
0 & 0 & \frac{(r + R)}{L} \\
\end{array} \right) i_i \\
\frac{dV_{C_i}}{dt} &= \left( \begin{array}{c}
\frac{1}{L} \\
0 \\
\frac{1}{C} \\
\end{array} \right) V_{C_i} \\
\end{align*}
\]

(3)

Based on the symmetry of the impedance network, we can obtain:

\[ V_{C_1} = V_{C_2} = V_C, \quad i_{L1} = i_{L2} = i_L \]

The original four-order equation reduces to two-order equation as follows:

\[
\begin{align*}
\frac{di_L}{dt} &= \left( \begin{array}{c}
\frac{(r + R)}{L} & -\frac{2D_0}{L} \\
0 & -\frac{2D_0}{L} \\
\end{array} \right) i_L \\
\frac{dV_c}{dt} &= \left( \begin{array}{c}
\frac{R}{L} \\
0 \\
\end{array} \right) V_c \\
\end{align*}
\]

(4)

According to the inductor volt-second balance and capacitor charge balance, we can get static working point for steady state:

\[
\begin{align*}
i_L &= \frac{1-D_0}{1-2D_0} i_w \\
V_c &= \frac{1-D_0}{1-2D_0} V_w + \frac{(1-D_0)(1-2D_0)R-(r+R)}{(1-2D_0)^2} i_w
\end{align*}
\]

(5)

In order to solve the dynamic transfer function of the system, we introduce the disturbance quantity of stable operating point to obtain the state variable which contains disturbance quantity: \( i_L + \hat{i}_L, V_C + \hat{V}_C, V_W + \hat{V}_W, i_w + \hat{i}_w \) and \( D + \hat{D} \).

Defined matrix:

\[ A_1 = \begin{bmatrix}
\frac{-(r + R)}{L} & 0 & 1/L \\
-1/C & 0 & 0 \\
\end{bmatrix} \]

\[ A_2 = \begin{bmatrix}
\frac{-(r + R)}{L} & 0 & -1/L \\
1/C & 0 & 0 \\
\end{bmatrix} \]

\[ B_1 = \begin{bmatrix}
\frac{1}{L} & \frac{R}{L} & 0 \\
0 & -1/C & 0 \\
\end{bmatrix} \]

\[ B_2 = \begin{bmatrix}
\frac{1}{L} & \frac{R}{L} & 0 \\
0 & -1/C & 0 \\
\end{bmatrix} \]

Make \( A = D_0A_1 + (1-D_0)A_2, B = D_0B_1 + (1-D_0)B_2 \), \( X = \begin{bmatrix} i_L \\ V_C \end{bmatrix}, U = \begin{bmatrix} V_w \\ i_w \end{bmatrix} \), we can get linear dynamic equation of small-signal model of impedance network.

\[
\begin{align*}
\frac{d\hat{i}_L}{dt} &= A_1 \hat{i}_L + B \hat{V}_w \\
\frac{d\hat{V}_C}{dt} &= A_2 \hat{V}_C + B \hat{V}_w \\
\end{align*}
\]

(6)

Take the Laplace transform of (6) at both ends, it is shown as follows:

\[
(sE - A) \hat{x}(s) = B \hat{u}(s) + \left[(A_1 - A_2)X + (B_1 - B_2)U\right] \hat{d}(s)
\]

(7)

Wherein, \( E \) is the second order unit matrix.
\[
\dot{x}(s) = \begin{bmatrix} \dot{i}_L(s) \\ \dot{V}_C(s) \end{bmatrix}, \quad \hat{d}(s) = \begin{bmatrix} \hat{V}_s(s) \\ \hat{i}_{\text{in}}(s) \end{bmatrix}
\]

By (7) we can get

\[
\begin{align*}
\dot{x}(s) & = \left( s - \frac{1 - 2D_0}{L} \right) i_L(s) + \left( \frac{1 - 2D_0}{L} \right) V_C(s) \left( 1 - \frac{1 - 2D_0}{L} \right) \hat{d}(s) \\
& = \left( \frac{1 - 2D_0}{L} \right) i_L(s) + \left( \frac{1 - 2D_0}{L} \right) \hat{V}_s(s) \left( 1 - \frac{1 - 2D_0}{L} \right) \hat{d}(s)
\end{align*}
\]

Where: \( Q = LCs^2 + (R + r)Cs + (1 - 2D_0)^2 \). Therefore, the small-signal inductor current of Z-source network is shown as follows:

\[
\dot{i}_L(s) = \frac{(1 - 2D_0) \cdot C \cdot s \cdot V_C(s) + (1 - 2D_0) \cdot (R \cdot C \cdot s + (1 - 2D_0)) \cdot \hat{i}_{\text{in}}(s)}{Q} \\
+ \frac{(2V_C - V_m - R_m) \cdot C \cdot s - (1 - 2D_0) \cdot (i_m - 2\hat{i}_m)}{Q} \hat{d}(s)
\]

The small-signal capacitor voltage of Z-source network is shown as follows:

\[
\dot{V}_C(s) = \frac{(1 - D_0) \cdot (1 - 2D_0) \cdot V_L(s) + (1 - D_0) \cdot (1 - 2D_0) \cdot R \cdot (s + r)}{Q} \hat{i}_{\text{in}}(s) \\
+ \frac{(2V_C - V_m - R_m) \cdot (s + r) \cdot (i_m - 2\hat{i}_m)}{Q} \hat{d}(s)
\]

In (10), make \( \dot{V}_C(s) = 0 \), \( \dot{i}_L(s) = 0 \), and the transfer function of capacitor voltage to shoot through the duty cycle of Z-source network can be shown as follows:

\[
G_{\text{in}}(s) = \frac{\dot{V}_C(s)}{\hat{d}(s)} = \frac{(1 - 2D_0) \cdot (2V_C - V_m - R_m) \cdot (s + r) \cdot (i_m - 2\hat{i}_m)}{LCs^2 + (R + r)Cs + (1 - 2D_0)^2}
\]

3 VERIFY THE SMALL-SIGNAL MODEL IMPEDANCE NETWORK

Z-source inverter works by adjusting the shoot through the duty cycle to achieve the DC link Boost, so it has some impacts on system performance. Meanwhile, in order to maintain a stable inverter bridge, DC link voltage needs to be achieved by controlling the Z-source capacitor voltage to be stable. By (11) we can see that the transfer function of capacitor voltage to shoot through the duty cycle has right half-plane zero, it leads to the phenomenon that the transient response of Z-source network will appear in a non-minimum phase. Therefore, we need to verify the correctness of the model at first in order to optimize the dynamic characteristics of the system by parameters.

In order to observe whether the results of both two are the same, we compare the voltage dynamic response of linear small-signal model and dynamic simulation results of the actual circuit capacitor voltage.

Simulation parameters of Z-source impedance network are shown as follows: \( L_1 = L_2 = 1mH \), \( C_1 = C_2 = 1000\mu F \), \( V_m = 100V \), \( f_s = 10kHz \), \( R_4 = 30 \Omega \).

We use simple boost control mode. In this process, the shoot through the duty cycle is changed from 0.1 to 0.11 at 0.1s. Figure 2 is the step response of simulation circuit of Z-source impedance network and small-signal model of capacitor voltage. As can be seen from the figure, small-signal model is under the disturbance \( \hat{d} = 0.01 \), its step response is consistent with the output of switch electronic circuit. It can show that the small-signal linear model of shoot through the duty cycle to capacitor voltage is valid. It can accurately reflect the internal dynamic characteristics of Z-source inverter impedance network.

In order to verify and explain the non-minimum phase characteristic caused by right-half-plane zero in (11) and analyze the effect that impedance network passive components parameters have on system dynamic performance, we conduct analysis through the capacitor voltage transient response curve when the duty cycle has a step change. It visually shows that system impedance parameters have effects on the dynamic performance of the system. Figure 3 is the impedance of the capacitor voltage network step response curve in the duty cycle mutated from 0.1 to
0.15 when t=0.1s. As can be seen from the figure, the capacitor voltage increases volatility as the inductance increases. At the same time, both the rise time and the settling time increase, the negative impulse caused by the non-minimum phase also will be intensified.

Similarly, Figure 4 is capacitor voltage step response curve when the duty cycle is mutated at t=0.1s and the need of different capacitor values in the ripple range is met. (11) shows that the system damping is increased with the increase of the capacitor value, but system zero has no relationship with capacitor value, which means that the change of capacitor value has no effect on the non-minimum phase characteristics of the system. It can be clearly seen from Figure 4 that, with the increasing value of capacitor, the rising time of the system is increased at the same time. The increase of system damping reduces the overshoot of the system to a certain extent.

![Figure 4. The step response of capacitor voltage when capacitor parameter varies.](image)

Figure 4. The step response of capacitor voltage when capacitor parameter varies.

4 THE IMPEDANCE NETWORK PARAMETER OPTIMIZATION

From the previous analysis, we know the selection of impedance network capacitor, and inductor parameters have a direct influence on the dynamic characteristics of the capacitor voltage. Therefore, in order to obtain the desired capacitor voltage dynamic characteristics and to ensure the stability of DC voltage, we can get a stable output waveform of the inverter bridge and we need to optimize the value of inductor and capacitor. However, selecting the optimal parameter values is determined based on several conditions of general restrictions, so we should consider the following criteria at first:

1. A suitable damping factor and an appropriate quality factor.
2. Consider both power density and economic condition and meet ripple requirement based on the components of inductor and capacitor as smallest as possible.
3. Make sure that the closed-loop system has sufficient bandwidth.
4. The resonant frequency of the system should try to stay away from the switching frequency of the inverter bridge.
5. Effect of non-minimum phase characteristic.

In addition to the above principles, we need to consider a non-work state of Z-source inverter caused by improper impedance network parameters, which is discontinuous inductor current mode. Therefore, we need to consider the basis of the above constraints on the parameters of the impedance network optimization research.

When we design the network parameters, the influence of Z-source inverter non-work state on the performance of the inverter output needs to be considered in the optimization of inductor parameters. And such non-work state can be avoided by selecting the inductor value appropriately. Therefore, we should first consider the critical inductor value of Z-source inverter discontinuous mode and provide a reference for selecting the optimal parameters.

To avoid discontinuous current mode, there is a need to meet:

\[
i_{L_{min}} = I_L - \frac{\Delta i_L}{2} \geq \frac{i_{L_{max}}}{2}
\]  

(12)

According to Z-source inverter operation principle, when in pass-through state, the diode is turned off, and the capacitor voltage \( u_C \) and inductor voltage \( u_L \) network are equal. At this point, the impedance network inductor current ripple \( \Delta i_L \) is:

\[
\Delta i_L = \int_0^{T_s} \frac{u_C}{L} dt = \frac{u_C \cdot D_0 \cdot T_s}{L}
\]

(13)

The maximum \( i_{L_{max}} \) of the inverter bridge input side DC link current and the valid value \( I_{I_{f}} \) of the inverter output side currents satisfy:

\[
i_{L_{max}} \geq \sqrt{2} I_{I_{f}}
\]

(14)

From the above equation, we can get that the inductor value range in continuous inductor current mode is:

\[
L \geq \frac{u_C \cdot D_0 \cdot T_s}{2 I_{I_{f}} - \sqrt{2} I_{I_{f}}}
\]

(15)

According to the parameters given in Section 3, combined with (15) we can obtain that the critical minimum parameter of inductor which meets the continuous inductor current is 300μH. Taking into account a certain margin when selecting actual inductance parameters, the increase of the inductance value will decrease the natural frequency of the conjugate
pole. So if the inductor value is too large, it will increase the negative impulse the non-minimum phase phenomena brings, and the selection of inductor parameter is 700μH.

Figure 5 shows that the capacitor parameters have effect on the system step response. The increasing capacitor value will increase the system damping, making the system response more stable. When the capacitor value is more than 1000μF, it will have little impact on overshoot system. Therefore, in order to take both system damping factor and quality factor into account, we select the capacitor parameters which are 1000μF. With these parameters, the system transfer function open loop gain is 20dB and the resonant frequency is 190Hz, which meet the system’s design specifications.

In order to verify the above capacitor, the inductor parameter is a relatively good choice. We select three sets of approaching data to compare based on this parameter. Three sets of control data values are shown as follows: 1) capacitor (500μF) and inductor (700μH); 2) capacitor (1000μF) and inductor (1000μH); 3) capacitor (1000μF) and inductor (700μH).

Based on these three sets of data, we can get the response curves of capacitance voltage under different parameters in Figure 5. As can be seen from the figure, we can find by comparison as follows: The response curve of capacitor (1000μF) and inductor (700μH) is relatively good, the overshoot is relatively small, and the adjustment time is relatively short. Therefore, the principle of optimization parameters selection provides a strong theoretical support.

5 CONCLUSION

In this paper, we construct a small-signal model-based Z source inverter based on the state-space averaging method, and obtain the transfer function of impedance network capacitor voltage to shoot through the duty cycle by the model. By comparing the output of the impedance network emulation circuit and the step response of the capacitor voltage small-signal model in case that the duty cycle changes suddenly, we verified the correctness of the model; on this basis, we defined the optimization of the impedance network inductor and the capacitor parameter selection principle according to the effect of impedance network passive components parameters changing on the dynamic performance of capacitor voltage step response; finally, the simulation results demonstrate that the principle of optimization parameters is feasible. And the conduction of the small-signal model and the optimal selection method in this paper will lay a theoretical foundation for further work to improve the performance of Z-source inverter.

REFERENCES