Comparative Analysis of Meshing Performance of Spiral Bevel Gears Generated by Duplex Helical and SGM Method

GUOBING HUANG, HONGZHI YAN, ZHIAN HU, MENG XIAO and TENGFEI ZHOU

ABSTRACT

In order to get a deeper understanding of duplex helical method, the mathematical model of spiral bevel gear pair by duplex helical method was established and two models of gear pairs generated by duplex helical method and SGM method respectively were built for meshing performance analysis. The duplex helical method for pinion generating was introduced in detail through the process of the pinion modeling. The meshing performance of the gear pairs modeled was analyzed by finite element analysis software. The results show that the meshing performance of the gear pair by SGM method is superior to that by duplex helical method. The contact pressure, contact force and transmission error of the gear pair by SGM method are preferable while severe edge contact happens to that by duplex helical method and its contact path is discontinuous. For duplex helical method, tooth modification should be done to get better meshing performance.

INTRODUCTION

Spiral bevel gear drive is an important transmission type that is widely used in cars, ships, aircrafts and other mechanical equipment for its high transmission performance. The contact area and transmission error of the gear drive play an

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important role on the working performance and life of the equipment. Thus, studying on the meshing performance of spiral bevel gear and to improve it are important.

Five-cut process and duplex helical method are two widely used cutting methods brought forward by Gleason. For five-cut process, the gear, cut by a roughing cutter after a finishing cutter, can be generated by generated method or formate method and the pinion, cut by an alternate blade roughing cutter after a finishing outer blade cutter and a finishing inner blade cutter, can be generated by tilting method or modified roll method. The concave side and the convex side are cut individually in five-cut process.

In 1987, Shchipelman [1] introduced the cutting principle of the five-cut process in detail for the first time. Litvin [2, 3] developed an approach to calculate the machine-tool setting of the pinion based on local synthesis theory. Xutang Wu [4], Cheng Chanqi [5], Xuezhu Dong [6] et al. did a serious of researches on the five-cut process including cutting principle, calculation of machine-tool settings of different kinds of machines and tooth contact analysis (TCA).

As for duplex helical method, the surfaces of the pinion or the gear are finished by an alternate blade cutter (with outside blade followed by inside blade and next outside blade and so on) at the same time [7]. The gear can be cut by generated or formate method and the pinion must be finished by duplex helical method. Wang Zhe, Liu Qingmin et al.[8-12] introduced the exact duplex helical method about the blank design, the determination of machine settings and calculating point. TSAY and LIN et al.[13, 14] established the mathematical model of the spiral bevel and hypoid gears using duplex helical method. ZHANG Yu, YAN Hongzhi et al. [15] established the mathematical model of spiral bevel and hypoid gears by half-formate duplex helical method and proposed a new method of TCA.

SPIRAL BEVEL GEAR MODELING

Coordinate systems and equations of pinion tooth surfaces

For duplex helical method, the gear can be produced by generating method or formate method. SGM and formate method are clearly described in [16] and is not presented in this paper.

In duplex helical method, the generating gear spins about its axis and moves along the axis at the same time. Figure 1 shows the generating coordinate systems of the pinion. The coordinate system \( \sigma_1(x_{01}, y_{01}, z_{01}) \) is fixed on the cutting machine with \( O_{01} \) located in the machine center and plane \( x_{01}O_{01}y_{01} \) lying on the machine plane. \( O_1 \) is the cross point of the pinion. \( O_{11} \) is the center of the cutter. \( E_{1j},j_{1i}, \delta_{M1,q1},S_{1j},X_{1j} \) and \( X_{B1} \) are the machine-tool settings and their meaning can be found in the succeeding TABLE II. Unit vector \( \vec{p}_1 \) represents the axis of the pinion. The coordinate system \( \sigma_2(x_{t1}, y_{t1}, z_{t1}) \) is a stationary system with \( z_{t1} \) axis coincident with the cutter axis.
As shown in Figure 1 a), the cross section of the cutter is a line segment. In \( \sigma_2 \), when the cutter rotates with an angle \( \theta_1 \), the point \( M \) on it can be expressed as:

\[
\overrightarrow{r}_c(u_1, \theta_1) = \left[ \begin{array}{c} (r_g + u_1 \sin \alpha_1) \cos \theta_1 \\ (r_g + u_1 \sin \alpha_1) \sin \theta_1 \\ -u_1 \cos \alpha_1 \end{array} \right] \tag{1}
\]

Where \( r_g = r_0 \pm W_{b1}/2 \) (\( r_0 \) is the nominal radius and \( W_{b1} \) is the point width of the cutter) is the radius of the tool nose; \( u_1 \) is the distance between point \( M \) and the tool nose; \( \alpha_1 = \alpha_{01} \) (for outside blade) or \( \alpha_1 = \alpha_{11} \) (for inside blade) is the blade angle.

The unit normal at \( M \) is represented by the equation:

\[
\overrightarrow{n}_c(u_1, \theta_1) = \frac{\frac{\partial \overrightarrow{r}_c}{\partial \theta} \times \frac{\partial \overrightarrow{r}_c}{\partial u_1}}{\sqrt{\left( \frac{\partial \overrightarrow{r}_c}{\partial \theta} \times \frac{\partial \overrightarrow{r}_c}{\partial u_1} \right)^2}} \tag{2}
\]

\( \overrightarrow{r}_c(u_1, \theta_1) \) and \( \overrightarrow{n}_c(u_1, \theta_1) \) can be expressed in \( \sigma_1 \) as:

\[
\overrightarrow{r}_c(u_1, \theta_1) = M_{ot} \cdot \overrightarrow{r}_c(u_1, \theta_1) \tag{3}
\]

\[
\overrightarrow{n}_c(u_1, \theta_1) = M_{ot} \cdot \overrightarrow{n}_c(u_1, \theta_1) \tag{4}
\]

Where matrix \( M_{ot} \) is the coordinate transformation matrix from \( \sigma_2 \) to \( \sigma_1 \).

When generating the pinion, the cradle rotates with respect to the axis \( z_{01} \). The moving surface of the spinning cutter profile is the generating surface. Assume that the cradle rotates \( \Delta q_1 \) from the initial position, it translates along its axis simultaneously because of the screw mechanism. The displacement is \( H_1 \Delta q_1 \) (\( H_1 \) is the first-order helical motion coefficient). The generating surface and its unit
normal vector are:

\[ \vec{r}_i(u_i, \theta_i, \Delta q_i) = \vec{r}_{i0}(u_i, \theta_i) + [0 \ 0 \ H_i \Delta q_i]^T \]  \hspace{1cm} (5)

\[ \vec{n}_i(u_i, \theta_i, \Delta q_i) = \vec{n}_{i0}(u_i, \theta_i) + [0 \ 0 \ H_i \Delta q_i]^T \]  \hspace{1cm} (6)

Where

\[ M_i = \begin{bmatrix} \cos \Delta q_i & -\sin \Delta q_i & 0 \\ \sin \Delta q_i & \cos \Delta q_i & 0 \\ 0 & 0 & 1 \end{bmatrix}. \]

The angle that the pinion goes through from the initial position is \( \varphi_i = i_{0i} \Delta q_i \) and \( i_{01} \) is the ratio of roll. Assume that the generating gear rotates about its axis \( z_{01} \) uniformly with unit angular velocity which means \( \Delta q_1=t \), the angular velocity of the work piece is:

\[ \frac{d\varphi_i}{dt} = i_{0i} \]  \hspace{1cm} (7)

The angular velocity and velocity of the generating gear relative to the work piece are:

\[ \vec{\omega}_{12} = \vec{k} - i_{0i} \vec{p}_i \]  \hspace{1cm} (8)

\[ \vec{v}_{12} = \vec{\omega}_{12} \times \vec{r}_i - i_{0i} \vec{p}_i \times \vec{m}_i \]  \hspace{1cm} (9)

Where \( \vec{k} \) is the unit vector of \( z_{01} \) axis, \( \vec{p}_i = -\cos \delta_{\alpha_{1i}} \vec{i} + \sin \delta_{\alpha_{1i}} \vec{k} \) and \( \vec{m}_i = \vec{O}_1 \vec{O}_{0i} \) (see Figure 1).

From the meshing equation \( \vec{v}_{12} \cdot \vec{n}_i = 0 \) and Eq. 5, the meshing equation can be written by the form as:

\[ f(u_i, \theta_i, \Delta q_i) = 0 \]  \hspace{1cm} (10)

The points on tooth surface must satisfy Eq. 10. Analytical solution of Eq. 10 cannot be found since there are three variables. However, it is possible to obtain the numerical solution in a certain range by using iteration algorithm.

**Establishment of 3-Dimensional Model of the Gear Pair**

Every set of numerical solution of Eq. 10 can help find a point by substituting them for the variables in Eq. 5. However, without knowing the ranges of \( u_i, \theta_i \) and \( \Delta q_1 \) beforehand, the point may be out of the tooth surface. Therefore, a changing coordinate system \( \sigma_3 \{O, L, R\} \) is established as Figure 2 shows to be the transitional coordinate system. From Figure 2, the following equations can be got:
Values of $L_1$ and $R_1$ are known from the blank parameters. Combining Eq. 10 to Eq. 12 and varying $L_1$ and $R_1$, the corresponding points on tooth surface can be calculated by programming.

Using the discrete points, a series of spline curves that are fitting the tooth surface can be got in the CAD software. Figure 3 shows the assembly model of the gear pair by duplex helical method. The major parameters of the gear pairs and the machine-tool settings are listed in TABLE I and TABLE II.

### TABLE I. MAJOR PARAMETERS OF THE GEAR PAIR.

<table>
<thead>
<tr>
<th>Cutting method</th>
<th>Pinion</th>
<th>Gear</th>
<th>Pinion</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of teeth</td>
<td>$z_1, z_2$</td>
<td>15</td>
<td>47</td>
<td>15</td>
</tr>
<tr>
<td>Module (mm)</td>
<td>$m_1, m_2$</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Shaft angle(deg)</td>
<td>$\Sigma$</td>
<td>90</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>Face width(mm)</td>
<td>$b_1, b_2$</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Pitch angle(deg)</td>
<td>$\gamma_1, \gamma_2$</td>
<td>17.7</td>
<td>72.3</td>
<td>17.7</td>
</tr>
<tr>
<td>Addendum(mm)</td>
<td>$h_{a1}, h_{a2}$</td>
<td>10.13</td>
<td>3.31</td>
<td>10.13</td>
</tr>
<tr>
<td>Dedendum(mm)</td>
<td>$h_{f1}, h_{f2}$</td>
<td>5.03</td>
<td>11.85</td>
<td>5.03</td>
</tr>
<tr>
<td>Hand of spiral</td>
<td>LH</td>
<td>RH</td>
<td>LH</td>
<td>RH</td>
</tr>
<tr>
<td>Spiral angle(deg)</td>
<td>$\beta_1, \beta_2$</td>
<td>35</td>
<td>35</td>
<td>35</td>
</tr>
</tbody>
</table>
TABLE II. MACHINE-TOOL SETTINGS AND INSTALLMENT SETTINGS.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Pinion</th>
<th>Gear</th>
<th>Pinion</th>
<th>Concave</th>
<th>Gear</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutting method</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cutter diameter(mm)</td>
<td>(D_{11}, D_{12})</td>
<td>306.47</td>
<td>304.8</td>
<td>304.8</td>
<td>304.72</td>
</tr>
<tr>
<td>Point width(mm)</td>
<td>(W_{b1}, W_{b2})</td>
<td>2.96</td>
<td>4.5</td>
<td>4.32</td>
<td>3.175</td>
</tr>
<tr>
<td>Radial distance(mm)</td>
<td>(S_{11}, S_{22})</td>
<td>151.79</td>
<td>152.04</td>
<td>154.97</td>
<td>147.37</td>
</tr>
<tr>
<td>Tilt angle(deg)</td>
<td>(i_{11}, i_{12})</td>
<td>16.04</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Swivel angle(deg)</td>
<td>(j_{11}, j_{12})</td>
<td>326.95</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Work offset(mm)</td>
<td>(E_{1}, E_{2})</td>
<td>1.959</td>
<td>0</td>
<td>-2.54</td>
<td>2.54</td>
</tr>
<tr>
<td>Machine root angle(deg)</td>
<td>(\delta_{M1}, \delta_{M2})</td>
<td>357.96</td>
<td>67.08</td>
<td>16.24</td>
<td>16.24</td>
</tr>
<tr>
<td>Machine center to cross point(mm)</td>
<td>(X_{1}, X_{2})</td>
<td>2.46</td>
<td>5.24</td>
<td>4.20</td>
<td>-4.34</td>
</tr>
<tr>
<td>Sliding base(mm)</td>
<td>(X_{B1}, X_{B2})</td>
<td>29.98</td>
<td>0</td>
<td>-1.18</td>
<td>1.21</td>
</tr>
<tr>
<td>Ratio of roll</td>
<td>(i_{01}, i_{02})</td>
<td>3.196</td>
<td>0</td>
<td>3.33</td>
<td>3.25</td>
</tr>
<tr>
<td>Helical motion-1(^{st}) order</td>
<td>(H_{1}, H_{2})</td>
<td>11.08</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

MESHING PERFORMANCE ANALYSIS

Finite Element Modeling

Spiral bevel gear has so complex geometrical structure that it is hard to directly build its finite element model in finite element analysis (FEA) software. Therefore, an import of the solid model into the FEA software and meshing it afterwards is adopted. The FEA of spiral bevel gear is non-linear, which often takes much more time of calculation than that of a linear one. In order to improve the calculating efficiency, a simplified FEA model of seven teeth, which ensures the efficiency and enough data at the same time, is used for the analysis as Figure 7 shows.

![Figure 7. Seven-teeth FEA model of spiral bevel gear.](image)

The pinion is set to be the driving gear and the gear to be the driven one as they are in most cases. For the pinion, the drive side is concave and the coast side is convex. The magnitude of the moment applied to the gear as the load is 300Nm. The gear and the pinion are only allowed rotating around their own axis.

16Cr3NiWMoVNB [17] is chosen to be the material of the spiral bevel gears because of its excellent combine property for gear pairs in high speed and heavy load conditions. Its material performance parameters are listed in TABLE III.
TABLE III. GEAR MATERIAL’S PROPERTIES.

<table>
<thead>
<tr>
<th>Material Type</th>
<th>Young’s Modulus (GPa)</th>
<th>Poisson Ratio</th>
<th>Density (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16Cr3NiWMoVNbE</td>
<td>185</td>
<td>0.3</td>
<td>7.85</td>
</tr>
</tbody>
</table>

The two models share the same structure and settings in FEA.

**Finite Element Analysis Results**

The simulation processes show that the gear pairs have very different meshing performances. The duplex helical one has one to three pairs of teeth are meshing at the same time while the SGM one has one to two pairs. The gear pair by duplex helical method has unfavorable meshing behaviors.

Figure 8. Contact pressure of the pinions.
Figure 8 a) and b) show the contact pressure contour plots of the fourth tooth surfaces of the pinions and Figure 9 a) and b) show the Mises stress. The maximum Mises stress of the pinion by duplex helical method is 694MPa and it is 2 times bigger than that of SGM. The contact pressure contour plots in temporal-order moments are drawn from bottom to top. In each figure, the first plot represents the moment that the tooth just begins meshing in and the last plot happens at the time the tooth is meshing out. Apparently, the contact area of the gear pair by duplex helical method is not a continuous one and the contact happens mostly at the root or the top of the gears which behave as severe edge contact. The maximum contact pressure is about 1500MPa. As for the gear pair by SGM method, the contact area is acceptable. The maximum contact pressure during meshing is about 314MPa, happening at the moment meshing-in which indicates a slight stress concentration on the tooth crest of the gear.

Figure 10 shows the contact forces of the contact teeth pairs. The curves from left to right represent the first contact teeth pair to the sixth pair respectively. The seventh curve is not plotted because it’s similar to the previous ones from the
periodicity. The dash lines in a) and b) are the composite forces of contact which show that the composite forces of the two models are similar and the maximum is about 2450N. From figure 10 a), each curve is made up of two parts: a major part and a minor part. The major one is ahead of the minor one and the maximum of the major one is approximately twice as big as that of the minor one. Between these two parts, the contact force is zero which means the tooth is not contacting and this mostly happens when the next contact teeth pair are meshing alone. Figure 10 b) shows that single tooth meshing and double teeth meshing are conducted alternatively.

From reference [18], contact ratio can be written as \( \varepsilon = T/T_1 \) as Figure 10 a) shows. Although the meshing performance is not favorable, the gear pair by duplex helical method has a higher contact ratio (2.47) than that of the gear pair by SGM method (1.35).

Figure 11 shows the contact area curves with respect to time. The figure indicates that the contact area of the gear pair by SGM method is larger than that by duplex helical method on the whole. The average contact area is 19.7 mm\(^2\) and 9.6 mm\(^2\) respectively. With smaller contact area, the contact pressure is much bigger as mentioned above.

The static transmission error is shown in Figure 12. At the beginning of the analysis, there is a gap between the first contact teeth pair and in order to eliminate that gap, the gear was given an initial angle and because the gaps of the two models are different, the initial angels of the gears are different which cause the different initial transmission error. The absolute values of the transmission errors of the gear pair by duplex helical method are much bigger than that of the gear pair by SGM method. But the peak-to-peak value is versa. The peak-to-peak values are about 6.2 arcsec and 4 arcsec respectively. The transmission errors of the gear pair by duplex helical method have values bigger than the initial transmission error, which means the gear rotates with an angle leading the theoretical angle and that is undesirable [16].
CONCLUSIONS

This paper presents the modeling of the spiral bevel gears generated by duplex helical method and gives the comparative analysis of the gear pairs by duplex helical method and SGM method. With two examples of the spiral bevel gear pairs with number of teeth 15×47 generated by duplex helical method and SGM method respectively, the following conclusions can be drawn:

1) The duplex helical method is feasible for spiral bevel gear modeling. The modeling steps are suitable for other spiral bevel gears generated by two-cut method.

2) For the duplex helical method, the maximum contact pressure is about 1500MPa and the maximum contact force of the contact teeth pair is 2450N. For the SGM method, the contact force is similar to duplex helical method because of the same load and similar dimensions but the maximum contact pressure (314MPa) is only about 20% of that of the duplex helical method.

3) Compared to SGM method, the contact performance of the duplex helical method is not favorable though it has higher contact ratio. Severe edge contact and undesirable contact area happened to duplex helical method. In order to get good meshing performance, tooth surface modification must be applied.

4) For both duplex helical method and SGM method, the maximum transmission error (peak-to-peak value) is less than 7 arcsec.

To sum up, the meshing performance of the gear pair by SGM method is better than that by the duplex helical method. Therefore, tooth surface modification is necessary for duplex helical method.

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