Magnetic Circuit Model and the Calculation of Qualified Rate for Magnetic Latching Relay in IEEC

JIAXIN YOU, HAO YU, HUIMIN LIANG, RUIMING YUAN
and WENWEN LI

ABSTRACT

The magnetic field simulation method for calculation of magnetic latching relay can reach high accuracy. However, it is not fit for the calculation of products qualified rate as its low computation speed. According to the analysis of the advantage and disadvantage of magnetic circuit method, this paper presented a magnetic circuit model considering the leakage flux based on the adjustment coefficient. Then, a computing method for qualified rate of relay virtual sample dynamic and static characteristics was given by stress-strength interference model. The computing example shows that the calculation results using the methods above can meet the engineering requirements, and the calculation efficiency is greatly improved, which lays the foundation for the quality consistency design and control of the electric energy meter.

1 INTRODUCTION

Load switch fault is one of the main faults in the failure of the intelligent electric energy meter (IEEM) according to incomplete statistics. Failure of the load switch usually leads to electric shock, fire and explosion. Performance index
error isn’t the reason which leads to the failure of load switch, but the poor quality consistency. The level of quality consistency was first determined by the design of quality and consistency. Qualified rate is an important index to evaluate the consistency of batch products. Therefore, it is very important to accurately calculate the qualified rate of load switch in order to design and control the quality consistency[1].

Load switch used in the IEEM can be divided into two kind: motor type and magnetic retaining type, compare with the motor type, magnetic structure is considered to be more concise for the magnetic retaining type, more convenient to be adjusted, and more reliability[2]. Compare with normal relay, magnetic relay also gain a higher conversion depth, saving more energy, smaller size, and lager input-output ratio.

Relay’s static and dynamic characteristic is the basis in calculating qualified rate. The method calculating static characteristic for electromagnetic system can be divided into two parts: magnetic field method and magnetic circuit method. The advantages of field method is its high accuracy, the disadvantage is its speed, meanwhile, using the mathematical model method of magnetic circuit is more simple and faster, although simply using this method will lead to lower accuracy, it can be easily improved by some means, which is another advantage of this method[3].

In this paper, an improved method based on the adjustment coefficient of magnetic circuit model is proposed, which can be used when magnetic pressure drop of iron core is not quite big. Based on it, using stress-intensity interference model to calculate the qualified rate in static and dynamic situations[4] can result in better applicability and controllability, in other words, this method can perfectly evaluate the quality of the product.

![Figure 1. Schematic diagram of the magnetic system of a certain type of relay.](image1)

![Figure 2. Sketch map of equivalent magnetic circuit model.](image2)
2 ESTABLISHMENT OF MAGNETIC CIRCUIT MODEL BASED ON ADJUSTMENT COEFFICIENT AND MAGNETIC LEAKAGE

As it is showed in Figure 1, this magnetic latching relay has two steady state, the armature group’s counterclockwise rotation cause air gap $\delta_1$ and $\delta_4$ become zero (state1), the armature group’s clockwise rotation cause air gap $\delta_2$ and $\delta_3$ become zero (state0). The armature itself closed in the polar yoke by the attractive force of permanent magnet flux. The conversion of two stable states will happen when the coil adopting different pulse. In Figure 1, the armature stay in the balance position, when a positive pulse passing through the coil, a clockwise electromagnetic flux $\Phi_c$ was generated, the direction of $\Phi_c$ is differ to the flux in $\delta_1$ and $\delta_4$, same to which in $\delta_2$ and $\delta_3$ caused by permanent magnet. This will cause the decrease of attractive in $\delta_1$ and $\delta_4$ and the increase of the attractive in $\delta_2$ and $\delta_3$, finally cause armature group rotates along clockwise direction; when a negative pulse passing through the coil, the armature group will rotate along counter clockwise direction and back to state1.

The magnetic leakage of this system mainly distributed on the region between central of iron core and the bottom of short armature, the magnetic circuit calculation become complicated due to the change of flux leakage by magnetic pressure. When the magnetic pressure drop of the iron core is not too large, a concentrated magnetic leakage can replace dispersive magnetic leakage which will greatly simplify the calculating process. Meanwhile, the calculation accuracy of magnetic circuit method is improved due to the consideration of magnetic field leakage[5], so it can meet the requirements of engineering. The equivalent magnetic circuit model considering the leakage magnetic field is showed in Figure 2.

In Figure2, $F_m$, $R_m$ represent permanent magnetic equivalent magnetic potential and the equivalent reluctance; $I$W represents coil’s magnetic potential; $R_s$ represents leakage reluctance, $k_1,k_2$ represents two adjustment coefficients of magnetic flux leakage; $\Phi_1$-$\Phi_5$ are the magnetic flux loop; $R_1$-$R_4$ mean the air reluctance formed by ends of armature and the yoke. $R_{a1}$, $R_{a2}$, $R_{b1}$, $R_{b2}$, $R_c$, $R_{d1}$, $R_{d2}$, $R_d$ are the reluctance of soft magnetic material in each segments. According to the magnetic circuit in Figure 2, the magnetic flux equations are written by the loop current method.
In matrix form

\[ [\phi_i(R_{11} + R_2 + k_1R_i) - \phi_2k_2R_i - \phi_1R_2 = 0] \]
\[ \phi_i(k_3R_2 + R_{b1} + R_{b2} + R_2) - \phi_2k_3R_2 - \phi_1R_2 = 0 \]
\[ -\phi_2R_{b1} + \phi_2k_3R_i + \phi_3R_{b2} = IW \]
\[ \phi_i(k_3R_2 + R_{b1} + R_{b2} + R_2) - \phi_2k_3R_2 - \phi_1R_2 = 0 \]
\[ \phi_i(R_1 + R_3 + R_{a1} + R_{a2} + R_3) - \phi_2R_3 - \phi_2R_{b1} + \phi_3R_{b2} = -F_m \]
\[ -\phi_3R_3 + R_m\phi_3 = -F_m \]

(1)

\[ \text{In matrix form} \]

\[ R\phi_b = F_b \]  \hspace{1cm} (2)

\( R \) represents the circuit reluctance matrix; \( \phi_b \) represents the loop flux matrix and \( F_b \) represents the loop magnetic potential vector matrix.

Air gap reluctance can be calculated by analytical method; The material of permanent magnet is NeFeB, its demagnetization curve is almost a straight line, so the equivalent magnetic potential and the equivalent reluctance can be described by the following equations[6].

\[ \begin{cases} 
F_m = Hc l_m \\
R_m = \frac{l_m}{\mu_m S_m}
\end{cases} \]  \hspace{1cm} (3)

where \( H_c, \mu_m, l_m, S_m \) respectively represents the permanent magnet coercivity, recovery permeability, permanent magnet length and cross-sectional area.

The reluctance of the soft magnetic material is solved by the following equation

\[ R = L/ (\mu S) \]  \hspace{1cm} (4)

where \( S, L \) respectively represents magnetizer’s cross-sectional area and length and \( \mu=B/H \) represents the magnetic permeability.

After obtaining the loop flux according to the equation (1), the working air gap magnetic flux can be determined, then calculate the attractive force responding to different \( \delta_t \) values by Maxwell equations, finally the determined attractive characteristic is shown in Figure 3.
In Figure 4, the contrast of static attractive curve obtained by magnetic circuit method and finite element method (FEM) is given. Through the contrast, the motion of the magnetic armature in prometaphase fits well to FEM, in the later period, the error is increased, but remain unchanged in trend.

According to magnetic circuit method, the time to calculate the attractive value for one certain working air gap is about 1s, meanwhile, 5min is required by ANSYS magnetic field simulation (16700 elements). It can be seen that the magnetic circuit method is much faster than the magnetic field simulation method, it can also meet the certain accuracy requirements, therefore, the following work is based on the magnetic circuit method to calculate the qualified rate of the product.
3 QUALIFIED RATE CALCULATION BASED ON STATIC CHARACTERISTICS

After the assembly of the relay is complete, whether it can act at its attraction voltage or return at under its release voltage mainly depended by the fit of attractive and counter attractive force[7], that means, when under the attraction voltage, if attractive force is smaller than the counter force, or, under the release voltage, if the counter force is smaller than attractive force, then this sample is unqualified. If the parts processing consistency is not good, the standard deviation of attractive and counter force distribution at a certain position is relatively large, and the qualified rate is poor[8]. Therefore, the qualified rate based on the static characteristics can reflect the quality of the product components and whether the tolerance range of parts is reasonable.

On Figure 5, it can be seen that as long as attractive force is bigger than counter force under the attraction voltage at point A, the armature can move correctly; as long as attractive force is smaller than counter force under the release voltage at point B and C, the armature is able to return[9]. Therefore, the qualified rate of static characteristics can be determined just by the fit of attractive and counter force in these three points-A, B, C.

For the qualified rate, Fig.6 shows a distribution of attractive force $F(x)$ and counter force $G(y)$ at one certain position, $x$ represents attractive force, and $y$ represents the counter force. Reference to stress-intensity-interference model, it can be known that the probability when event A (the attractive force $x$ is greater than a certain counter force $y_0$) and event B (the counter force lies within dy range) happen at the same time, is the qualified rate at $y_0[10]$. Since these two events is independent. According to the multiplication theorem of the independent event, the probability when event A and the event B happens at the same time is:

$$G(y_0)dy \int_{y_0}^{x} F(x)dx$$

Integrate all the possible values of the counter force, the overall qualified rate of this model is:

$$P(x > y) = 1 - \int_{-\infty}^{x} \int_{-\infty}^{y} F(x)dx G(y)dy$$

According to the product parts drawing sheet, generates 1000 sets of virtual sample based on normal distribution, calculated the static attractive force using the magnetic circuit model presented above, the counter force is calculated by the deformation energy method, in the end, get the distribution of attractive and counter force at point A, B, C, as shown in Figure 7~Figure 9.
Using the normal distribution formula to fit the shape and get the distribution of the attractive force \( F_A(x), F_B(x), F_C(x) \) and the counter force \( G_A(y), G_B(y), G_C(y) \) at points A, B and C:

\[
\begin{align*}
F_A(x) &= 1.8822 \cdot e^{-\frac{(x+0.681)^2}{0.089888}} \\
G_A(y) &= 3.9904 \cdot e^{-\frac{(y+1.25)^2}{0.02}} \\
F_B(x) &= 0.766666 \cdot e^{-\frac{(x+4.476)^2}{0.54155}} \\
G_B(y) &= 0.95465 \cdot e^{-\frac{(y+5.875)^2}{0.34945}} \\
F_C(x) &= 10.25819 \cdot e^{-\frac{(x+0.15655)^2}{0.00303}} \\
G_C(y) &= 6.4052 \cdot e^{-\frac{(y+0.2476)^2}{0.00776}}
\end{align*}
\]

Figure 7. Static attractive and counter force distribution at A point.

Figure 8. Static attractive and counter force distribution at B point.

Figure 9. Static attractive and counter force distribution at C point.

Figure 10. Distribution of index and dynamic characteristics.
Substitute the distribution expressions of static attractive force and counter force to equation (6) can the qualified rate of point A (99.36%), B (98.32%) and C (89.19%) is calculated. Overall-pass means these 3 points are qualified at the same time, so \( P = P_A P_B P_C = 87.13\% \).

4 QUALIFIED RATE CALCULATION BASED ON DYNAMIC CHARACTERISTICS

The manufacturers will measure the attraction voltage, release voltage, close time etc. at the end of the actual production process, if one beyond provisional scope, it is considered to be unqualified. Therefore, in the design phase of product quality consistency, it is also necessary to calculate the qualified rate based on dynamic characteristics.

The dynamic characteristics of the electromagnetic system are obtained by solving the differential equations of the dynamic characteristics for the electromagnetic system by the four-stage-Runge-Kutta method.

![Figure 11. Attraction voltage distribution.](image1)

![Figure 12. Release voltage distribution.](image2)

![Figure 13. Attractive time distribution.](image3)
\[
\begin{align*}
\frac{d\psi}{dt} &= U - iR \\
\frac{d\omega}{dt} &= \frac{M_i - M_f}{J} \\
\frac{d\alpha}{dt} &= \omega \\
\psi|_{t=0} = \psi_0, \omega|_{t=0} = \omega_0 = 0, \alpha|_{t=0} = \alpha_0
\end{align*}
\]  

(10)

In the equation, \(\psi\) represents flux of coil; \(U, i, R\) represent voltage, current, and resistance of coil; \(\alpha, \omega\) represent armature angular displacement and speed; \(J\) represents the moment of inertia; \(M_i, M_f\) represent moment of attractive and counter force; \(\psi_0, \alpha_0\) represent coil flux and armature displacement at time \(t=0\).

Based on the qualified rate of the dynamic characteristics and mainly assess whether the attractive time and attraction voltage can meet the standard given by manufacturers, in this case, only the index itself is in distribution type, the given standard is a certain value. The principle of qualified rate calculation was shown in Figure 10.

\(F(x)\) represents the distribution of the output parameters, \(x\) represents an output parameter value, and \(x_1, x_2\) represent the given index, in these case, the qualified rate of the output parameters is

\[
P(x_0 < x < x_1) = \int_{x_0}^{x_1} F(x)dx
\]  

(12)

Based on the technology of virtual sample and magnetic circuit calculation model, the dynamic characteristics of 1000 virtual samples is calculated, the distribution of the output parameters are showed in Figure 11~Figure 13.

Obviously, Figure 11 ~ Figure 13 still obey the normal distribution, the fitted normal distribution is respectively:

\[
F_{\psi}(x) = \frac{1}{\sqrt{2\pi \times 0.0789}} \exp\left(-\frac{(x-6.145)^2}{0.0125}\right)
\]  

(13)

\[
F_{\alpha}(x) = \frac{1}{\sqrt{2\pi \times 0.316}} \exp\left(-\frac{(x-4.476)^2}{0.01997}\right)
\]  

(14)

\[
F_{\psi}(x) = \frac{1}{\sqrt{2\pi \times 1.45}} \exp\left(-\frac{(x-54.687)^2}{4.205}\right)
\]  

(15)
Substituting equations (13) ~ (15) into equation (12) and obtain the qualified rate based on the dynamic characteristics corresponding to different output parameters, as shown in Table I.

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>Index</th>
<th>Qualified rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Attractive voltage</td>
<td>5.9V~6.1V</td>
<td>98.784%</td>
</tr>
<tr>
<td>Release voltage</td>
<td>5.0V~4.3V</td>
<td>99.125%</td>
</tr>
<tr>
<td>Attractive time</td>
<td>52ms~58ms</td>
<td>99.257%</td>
</tr>
</tbody>
</table>

5 CONCLUSIONS

(1) Established magnetic model based on the modified coefficient and magnetic flux leakage for electric energy meter. This method can be used to calculate the change static electromagnetic force when the magnetic pressure drop is not big on iron-core. The calculation accuracy can be guaranteed by this method and the calculation speed is much faster than the FEM.

(2) The method of calculating qualified rate of static characteristics and dynamic characteristics are given. For 1000 samples based on normal distribution, the static characteristics and dynamic characteristics is calculated and gain the distribution of different parameters, in the end calculate the qualified rate of static and dynamic characteristics. Finally an important content in design of the products’ quality consistency can be accomplished by this method.

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