The Two Level Atom System Dissipation Influence to Single Photon Propagating Properties

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ABSTRACT

The studies of interactions of electromagnetic fields with two level atom systems is a main theme of quantum optics. Here, we shows how the inherent dissipation of atom influences the interaction between atom and single photon propagating in the waveguide. Based on a fully quantum mechanical theory in real space, we exactly calculated the transportation probability of a single photon for a two-level atom system next to the waveguide. We find that the existence of a dissipated atom can be utilized to act as the light router as well as the beam splitter, if properly adjust relationship of atom dissipation $\gamma_q$ and coupling strength $V$ between atom and single photon in waveguide.

INTRODUCTION

Single-photon routing is a well-studied but nonetheless hot topic in optical physics because it is related to various important applications, specifically the realizations of the optical quantum computation[1, 2], quantum information[3]. Recently, the Cooper pairs in the superconductor can be quantized as two level atom(TLA) system that couples strongly to a single photon to exhibit coherent

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behavior, which is a hot topic in quantum optics and generate the cavity quantum electrodynamic (cavity QED)\cite{4} in the past decades. In our paper, we briefly study the atom dissipation influence the single photon transport properties utilizing the full quantum mechanical theory in the real space.

![Figure 1](image.png)

**Figure 1.** Single photon propagating along the wave guide interacts with dissipated atom with the coupling strength $V$. Where, the green rectangle represents the channel; the red dot represents the dissipation atom aside the waveguide, which energy level presented nearby the atom.

**ATOM DISSIPATION INFLUENCE PHOTON PROPAGATING**

Following Ref. [5], the effective Hamiltonian describing a single photon propagating along a one-dimensional waveguide scattered by a single two-level atom can be expressed as:

$$H_{\text{eff}} = \int dx \{-iv_g C_{R}^\dagger(x) \frac{\partial}{\partial x} C_{R}(x) + iv_g C_{L}^\dagger(x) \frac{\partial}{\partial x} C_{L}(x)\} + (\Omega - i\gamma) a^\dagger a + \Omega a^\dagger a^\dagger + \{ \int dx \delta(x)[VC_{R}^\dagger(x)\sigma + VC_{L}^\dagger(x)\sigma] + \text{c.c.}\}$$

Here, $C_{R/L}^\dagger(x)$ represents the bosonic creation operator at the position $x$ of the photon traveling in the right/left direction. $\Omega_e - \Omega_a = \Omega$ is the atomic transition frequency, $a^\dagger a$ represents the ground/excited state frequency of the atom, and $\sigma^\dagger = a^\dagger a^\dagger$ is the creation (annihilation) ladder operator of the atom. $v_g$ is the group velocity of the incident photon in the waveguide, $\gamma$ is the dissipation rate of the excited atom, and $V$ is the efficient coupling strength between atom and waveguide photon. Obviously, the first and second parts in the above Hamiltonian describe the photon freely transporting along the waveguide and the bare atom, respectively. The third part describes the interaction between atom and single photon in waveguide.
The generic quantum state of the system can be expressed as

\[ |\Psi(t)\rangle = \int dx [\phi_R(x,t)C_R^+(x) + \phi_L(x,t)C_L^+(x)]|0,-> + e_q(t)\sigma^+ |0,-> \]  

which satisfies the Schrödinger equation:

\[ H |\Psi(t)\rangle = i\hbar \frac{\partial}{\partial t} |\Psi(t)\rangle \]  

Above, |0,-> depicts the state of the system without any propagating photon in the waveguide, and the atom in its ground state. \( \phi_{R/L}(x,t) \) is the single-photon wave function transporting in the mode, and \( e_q(t) \) the excitation amplitude of the atom.

The system wave function can be rewritten as

\[ |\Psi(t)\rangle = e^{i\varepsilon t} |\varepsilon\rangle \]  

with \( \varepsilon = \omega + \Omega_\gamma \), \( \gamma = kv_g \). Thus system eigenstate:

\[ |\varepsilon\rangle = \int dx [\phi_R(x)C_R^+(x) + \phi_L(x)C_L^+(x)]|0,-> + e_q\sigma^+ |0,-> \]  

Calculated the above expression by introducing the Heaviside step function \( \theta(x) \), and \( \phi_R(x) = e^{ikx}[\theta(-x) + t\theta(x)], \phi_L(x) = e^{-ikx}r\theta(-x) \), with t and r being the transmission and reflection amplitudes of the photon, respectively. We can straight derive the parameter of single photon propagating into the waveguide by assuming \( V = V^* \), and the external width \( \Gamma_1 = \frac{|\hbar|^2}{v_g} \) to simplify the calculation. Then we attain the transport parameters as follows:

\[ t = \frac{(i\gamma_q + \omega - \Omega)}{(i\gamma_q + \omega + \Omega)^2 + i\Gamma_1}, r = \frac{-i\Gamma_1}{(i\gamma_q + \omega - \Omega)^2 + i\Gamma_1}, e_q = \frac{V}{(i\gamma_q + \omega - \Omega)^2 + i\Gamma_1} \]  

The transmission coefficient, reflection coefficient as well as the atom excitation are given by \( |t|^2, |r|^2, |e_q|^2 \), and the spectrum of them are shown in Fig.2, and Fig. 3., respectively. As it shows, the width is monotonously increasing with \( \gamma_q \) the (atom dissipation). Otherwise, we also can concluded that:

1) when the dissipation of atom equals to zero, incident light and atom is at resonance, i.e. \( \gamma_q = 0, \omega = \Omega \), the transmission coefficient \( |t|^2 = 0 \), reflection coefficient \( |r|^2 = 1 \), and meets \( |t|^2 + |r|^2 = 1 \). While atom has the probability \( |e_q|^2 = 0.2 \) to be excited. At resonance, the light is fully reflected by the two level atom system which acts like a mirror.

2) when \( \gamma_q = \Gamma_1, \omega = \Omega \), the relationship \( |t|^2 + |r|^2 = 1 \) is still satisfied, while atom excitation probability decreases with atom dissipation increasing. The light is split two ways and depends on the coupling strength if the dissipation is fixed.
3) when $\gamma_q = 2\Gamma_1, \omega = \Omega, |t|^2 + |r|^2 \neq 1$, because the light radiates to the environment and the atom excitation is weak, for the weak interaction with single photon. Furthermore, transmission coefficient $|t|^2 = 1$ when the atom dissipation is big enough to neglect the coupling interaction.

![Figure 2](image1.png)

**Figure 2.** The transportation property of single photon propagating into the waveguide versus the different atom dissipation.

![Figure 3](image2.png)

**Figure 3.** The atom excitation versus the different atom dissipation.

**CONCLUSION**

In the superconducting Cooper pairs which can simplified to TLA system, the coupling strength of photon and qubit can be expressed as $V = (2\pi \hbar / \omega_k)^2 \Omega D \cdot \bar{e}_k^{\gamma}$ (in Ref[5]). If properly adjust relationship of the dissipation $\gamma_q$ and coupling strength $V$ between atom and single photon propagating in the waveguide, the light
can be routed by the atom. For example, adjusts the dipole unit vector of Cooper pair and the light polarization, the two level system can act as beam splitter. The Cooper pair box is class of particularly versatile qubit, and can be largely tuned both the physics and the mesoscopic scale. And we specular the future clever designs will bring a wide application in the quantum computing and information process devices.

REFERENCE