Analysis of Parabolic Trajectory Guidance Law for Airborne Missile

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Abstract. Air supremacy plays a decisive role on the outcome of the war. All the countries in the world are developing the research of airborne missile. In view of this demand, we first analyze the pure tracking method, the parallel approach, the three point method and the proportional navigation method and point out the defects of the conventional trajectory method, then introduce a compound guidance law of parabolic trajectory including initial guidance, middle guidance and terminal guidance and establish parabolic trajectory system. Through the analysis of the kill probability of the conventional trajectory and the parabolic trajectory at different distances, a result is obtained: the parabolic trajectory can effectively increase the attack range of the middle-short-range missile, and can be used in the missile’s technical upgrade.

Introduction

Modern warfare is a high technology war under the condition of information technology. The contention of air supremacy has become the key to win the war. The airborne missile plays in the important position and function of the long-range precision strike and the control of the air. It is playing a more and more important role in modern warfare[1,2].

Trajectory shaping design is a key technology, it relates to the overall performance of missile. For the airborne missile, trajectory shaping technology is mainly through the optimization of the flight plan or guidance algorithm, and designs to reduce the resistance consumption of airborne missile flight in the process, etc., so as to achieve the improvement of ballistic performance indicators and increase the range of the missile. Reasonable trajectory shaping technology can not only increase the flight distance of the missile effectively, but also reduce the attack time and improve the terminal velocity, to obtain more favorable conditions for the end of the missile and expand the operational envelope of the missile.

Introduction of Missile Guidance Law

Conventional Guidance Law

There are a lot of guidance laws used in various types of missiles, such as pure tracking method, the parallel approach, the three point method, proportional navigation method. The following is a brief introduction for the guidance laws.

(1) Pure tracking method

Pure pursuit method is that the direction of the missile's movement speed always points to the target in the process of flying. The advantage of this system is that it is easy to realize the guidance system engineering. But the drawback is that trajectory bends serious and needs larger normal overload when the missile against target or attacking close range high speed target.
(2) The parallel approach

In the process of approaching to the target, the target aspect angle is remained constant[3]. The advantage of the parallel approach is that it does not require too much normal overload, and the time required for a missile to fly in space until it hits the target is shorter, but this guidance law is difficult to achieve.

(3) The three point method

The three point method is to guide the missile in the process of flying to the target, so that the missile, the target and the guidance station is always in the same line[4]. The method is easy to implement and has good anti jamming performances. But the drawback is that trajectory bends serious and needs larger normal overload when the missile against target, the trajectory sinks seriously when the missile attacking the low altitude target, and it will lead to a large error when the target is maneuvering.

(4) Proportional navigation method

Proportional navigation method is often used in homing missiles[5]. It refers to the process of the missile to the target, the changing rate of missile velocity is proportional to the changing rate of the target line of sight, but the range is limited.

**Parabolic Guidance Law**

The optimal trajectory in the vertical plane is a parabolic, based on the theory of optimal trajectory shaping technology. Based on the geometric features of the optimal trajectory, the trajectory is segmented by the method of the guidance law which is easy to be realized, and the approximate optimal trajectory shaping can be achieved indirectly[6,7]. The airborne missile has the following four kinds of parabolic flight trajectory.

(1)The characteristics of the forward trajectory are climbing rapidly and taxiing slowly. Its climbing overload is restricted by the missile body structure. If the climb overload resistance is excessive, loss will also become large.

(2)The characteristics of “π” type trajectory are climbing fast, longer smooth flight, and dives to target finally. With the change of speed and overload, the optimum flight height of the missile will be changed continuously. As the end missile flight speed is low, the general long-range missile should not dive.

(3)Backward trajectory has the characteristics of climbing slowly. The highest point of the missile is closer to the target, and it dives to target finally. Front of the trajectory flight speed is large, but long term low altitude air resistance will lead to a great loss.

(4)It is proposed that the optimal trajectory of an airborne missile with finite thrust in the vertical plane has parabolic features, based on optimal trajectory shaping technique. With the premise of the project can be realized by the guidance algorithm, a design idea of subsection trajectory shaping is presented, based on trajectory geometry characteristics. The characteristics of the parabolic trajectory are climbing quickly, and the highest point of the missile is near the middle point of the ballistic trajectory. Climbing fast, the part of the kinetic energy is converted to potential energy, and more total energy can be obtained at the end of the active segment. When the velocity is larger, the missile is always in the higher airspace. The above four kinds of trajectory diagram is shown in figure 1 below.
Due to the long flight of the missile, the guidance information acquisition method is different in different flight phases, the guidance law of medium and long range airborne missile is generally adopted in the three stage guidance of initial guidance and middle guidance and terminal guidance.

### The Initial Guidance Law

The initial guidance uses program command control, to the \( t_0 \) (loader correction instruction time) time to control the body to a specified angle, using the following formula:

\[
\dot{\theta}_i = \frac{\pi}{2(t_0 - 0.6)} \theta_{\text{opt}} \sin\left(\frac{\pi(t - 0.6)}{2(t_0 - 0.6)}\right), \quad \dot{\phi}_c = \frac{\pi}{2(t_0 - 0.6)} \phi_{\text{opt}} \sin\left(\frac{\pi(t - 0.6)}{2(t_0 - 0.6)}\right)
\]  

(1)

where \( t_0 \) is the total initial guidance time, \( t \) is the current time, \( \theta_i \) and \( \phi_{ic} \) are missile trajectory inclination and angle. The launch coordinates of the missile is \( (X_s, Y_s, Z_s) \), the target coordinates is \( (X_t, Y_t, Z_t) \), the distance between the missile and the target is \( R_{ts} \).

Using the expected hit point \( L(X_t, Y_t, Z_t) \) of the missile and the target point to calculate the line of sight angle, the projectile longitudinal line of sight angle and lateral line of sight are expressed by the following equation:

\[
p_s = \arcsin\left(\frac{Y_s - Y_t}{R_{ts}}\right), \quad q_s = -\arctan\left(\frac{Z_s - Z_t}{X_s - X_t}\right).
\]  

(2)

In order to enable the missile to fly to a higher airspace at a projectile angle, the desired trajectory inclination is a preset initial guidance pre-trajectory inclination angle formed by presetting a projectile angle in the direction of the eye:

\[
\theta_{\text{opt}} = k_1 \cdot \theta_\alpha + \arctan\left(\frac{Y_t - Y_s}{\sqrt{(X_t - X_s)^2 + (Z_t - Z_s)^2}}\right), \quad k_1 = 2.5 \times \frac{Y_{\text{max}} - Y_s}{Y_{\text{max}}}
\]  

(3)

\( k_1 \) is the preset projection angle magnification coefficient, due to pneumatic control force constraints, \( Y_{\text{max}} = 25km \). The preset high throwing throw angle can be estimated by the following formula:

\[
\theta_\alpha = \begin{cases} 
0 & t_{up} \leq 30 \\
\frac{\pi}{6} \left(1 - e^{-0.02(t_{up} - 30)}\right) & t_{up} > 30
\end{cases}
\]  

(4)
where \( V_{sx}, V_{sz}, V_{tx}, V_{tz} \) are the components of the missile and target velocities in the \( X \) and \( Z \) axes of the transmitted coordinate system respectively. \( t_{up} \) is predictive flight time for missile preset angle guidance; when \( t_{up} \leq 30 \), indicating a closer projectile distance, do not need to use high throw to complete the attack.

The launch of the missile is generally facing the target, and the missile flight time depends largely on the horizontal distance of the projectile axis, position can be estimated using the following formula:

\[
\begin{align*}
X_L &= X_t + t_{exp} \times V_t \cos \theta_t \cos \phi_t \\
Y_L &= Y_t + t_{exp} \times V_t \sin \theta_t \\
Z_L &= Z_t - t_{exp} \times V_t \cos \theta_t \cos \phi_t
\end{align*}
\]

\( t_{exp} \) is the predict flight time of missiles, \( t_{exp} = \frac{|X_L - X_s|}{V_{st} - V_{ts}} \) \hspace{1cm} (6)

At the end of the initial guidance, the desired yaw attitude angle is taken as the projectile lateral line of sight direction:

\[
\phi_{C_{opt}} = -\arctan \left( \frac{Z_L - Z_s}{X_L - X_s} \right)
\] \hspace{1cm} (7)

**Middle Guidance Law**

According to the longitudinal proportional guidance, in order to suppress the rotation of line of sight, ballistic trajectory angle change rate \( \dot{\theta}_{d1} = K_z \dot{\theta}_s \). Because the missile adds a preset projection angle in line of sight angle in the initial guidance, in order to make the ballistic trajectory into a terminal state \( \theta_f = \theta_z \), you should make the ballistic trajectory angle rotation, the rotation angular velocity is:

\[
\dot{\theta}_{d2} = K \frac{\theta_f - \theta_z}{t_D}, \quad t_D = D = \frac{R_s \cos \theta_z - L_d \cos \theta_t}{V_s \cos \theta_t - V_t \cos \theta_t}
\] \hspace{1cm} (8)

Among, \( t_D \) is the forecast flight time of middle guidance, D is connection line of starting point and end point in middle guidance. The length change rate of connection line is \( \dot{D} \), \( L_d \) is the distance between the missile and the target when the terminal guidance seeker catch the target. So the missile guidance law in the vertical plane is:

\[
\dot{\theta} = \dot{\theta}_{d1} + \dot{\theta}_{d2} = K_z \dot{\theta}_s + K \frac{\theta_f - \theta_z}{t_D}
\] \hspace{1cm} (9)

Adopt proportional navigation method in lateral: \( \dot{\phi}_c = K_c \dot{\theta}_s \) \hspace{1cm} (10)

The solution to the above guidance coefficients is the optimization problem with initial and terminal constraint. \( \dot{\theta}_{d1} \) is used to suppress the rotation of sight of line, the requirements for \( \dot{\theta}_{d1} \) and \( \dot{\theta}_{d2} \) are contradictory. In the literature, the complete solution procedure is given by using energy optimal theory; the guidance law is:

\[
\dot{\theta} = 4 \dot{\theta}_s + 2 \frac{\theta_f - \theta_z}{t_D}, \quad \phi_c = 3 \dot{\theta}_s
\] \hspace{1cm} (11)
The required overload for middle guidance ballistic trajectory system is:

\[ n_{hZ} = \frac{V \cdot \dot{\theta} + \cos \theta_j \cdot n_{jZ}}{g}, \quad n_{jZ} = \frac{V \cdot \cos \theta_j \cdot \dot{\phi}}{g} \]  \hspace{1cm} (12)

**Terminal Guidance Law**

For terminal guidance, the guidance information is provided by the guidance of the missile seeker. At this stage, the frequently-used guidance is proportional guidance, overall consideration the target line of sight, the speed of the line of sight and the change of the relative distance, the overload for terminal guidance ballistic trajectory system is:

\[ n_{hM} = 57.3 \times (A \cdot |\Delta R| \dot{p}_s + B \cdot n_{x1} q_s), \quad n_{jM} = 57.3 \times (A \cdot |\Delta R| \dot{q}_s + B \cdot n_{x1} p_s) \] \hspace{1cm} (13)

\( n_{hM}, n_{jM} \) are the required overload for terminal guidance ballistic trajectory system; \( n_{x1} \) is the axial overload for missile system; \( \Delta R \) is the change rate of the relative distance of missile and target; \( A, B \) is the proportional navigation coefficient.

**The Results and Analysis for Trajectory Simulation**

In this paper, Matlab is used to construct the simulation system. The missile is assumed as a rigid body, and the simulation algorithm is based on the six degree of freedom missile motion equation and the instantaneous equilibrium hypothesis.

(1) Figure 2 is the curve of proportional navigation and parabolic guidance law of longitudinal plane trajectory and velocity, the missile attacks the target 100 km away, the target is at the altitude of 5 km to the missile and with uniform linear flight, speed of 400 m/s. For the simulation results, at the time the missile hit the target, the attack speed of proportional navigation at the end is obviously smaller than that by using parabolic trajectory at the end of the missile speed. The attack time did not increase for curve flight attack, but due to high altitude missile in a larger flight speed, the attack time is relatively reduced.

(2) Simulation experiments for different enemy aircraft longitudinal initial distance, the contrasts for conventional ballistic and parabolic trajectory simulation results are shown in Table 1.
Table 1. The simulation results for different attack distance.

<table>
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<th>20</th>
<th>28</th>
<th>36</th>
<th>44</th>
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<tr>
<td>Proportional navigation</td>
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<td>0.11</td>
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</table>

(3) The height of missile is 10km, speed is 500m/s; target initial height is 8km, located at 120km. Target is escaping in the negative direction along the Z axis, with equal overload climb flight, the acceleration is 0.5m/s², and the speed is 300m/s.

Figure 3. Target equal overload climb flight trajectory

The simulation results are shown in figure 3: miss distance is 0.1464m; hit speed is 434.16m/s; peak altitude is 30142.86m; hit time flight path angle is 3.8623 degrees, deflection angle is 53.2569 degrees; missile flight time is 135.426s.

Summary

In this paper, according to the contrasts of various guide methods for air-to-air missile, parabolic guidance method is more suitable for air-to-air missile guidance. The air-to-air missile with parabola trajectory attacks long-range targets requires for a short time, at the end of a large speed, high precision, and overload changes smoothly, good ballistic characteristics, has a good effect on the attack on medium and long range target. For the air-to-air missile that uses rocket engine, the achievement of parabolic trajectory compound guidance law technology is simple, there is no need to change the missile a lot; it has great significance to the air-to-air missile range.

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