Joint Information and Energy Transfer in Fading Relay Systems

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Abstract. In this paper, we consider a three-point relay system, in which the relay has no fixed energy supplies and thus needs to replenish energy from RF signals transmitted by the source via wireless energy transfer (WET). We propose optimal cooperative transmission scheme when the source knows partial or full channel state information (CSI). When the source knows partial CSI, to maximize the ergodic throughput under the peak/total power constraints of source and causality constraints of relay, we formulate the joint optimization of power control and information/energy transfer scheduling as a non-convex optimization of functional. By introducing new variables and constraints into the problem, the problem is solved by combining the convex optimization method and linear search. When the source knows full CSI, the joint optimization of power control and information/energy transfer is addressed by utilizing convex optimization of functional.

Introduction

Wireless energy transfer (WET) is a promising technique to prolong the lifetime of an energy-constrained wireless network. In the wireless systems which support simultaneous wireless information and power transfer (SWIPT), both information transmission capacity and energy transfer efficiency should be taken into account.

To achieve SWIPT, some different types of receivers, such as time switching (TS) \cite{2}, power switching (PS) \cite{2}, and antenna switching (AS) were proposed to split the received signal in two parts, one for energy harvesting (EH) and the other for information decoding (ID). As an initial study on SWIPT in multiuser environment, \cite{2} and \cite{4} investigated the design of the transmit signals in the simplified scenarios with one ID receiver and one EH receiver.

Recently, some researchers have applied WET to relay networks \cite{5}-\cite{8}. \cite{5} and \cite{6} assumed both source and relay have fixed power supply and the destination has EH capability. Their goals were to achieve different tradeoffs for SWIPT by optimizing system parameters, such as transmit power, PS factor, precoders and so on. \cite{7}-\cite{8} focused on the outage probability and the throughput performance analysis of EH AF (or DF) relaying when TS or PS relaying protocol was adopted.

To the best of our knowledge, in the fading channels, when the relay is wireless powered by RF signals from the source, the joint power control and information/energy transfer scheduling to maximize the ergodic capacity based on full or partial CSI at source has not been investigated. We consider the relay system without direct link between the source and destination and treat both the partial and full CSI cases. (1) Assume that the source knows the CSI of all links. The source may select to transmit energy signals or information signals to the relay according to known CSI. By neglecting the causality constraints at the relay during the transmission, the throughput maximization problem is formulated as a convex problem. The optimal parameters in the optimal solutions are found by an algorithm based on bisection method. (2) Suppose that the source only knows the CSI of S-D link. The transmission process consists of two phases: the source first transmits information (or energy) signals to the relay, and then the relay forwards information signals to the destination using the harvested energy. During the period of source transmission, the source needs to decide when to switch between transmitting information and transferring energy based on the CSI of S-R link. We
propose to maximize the average throughput through joint power control and scheduling information/energy transfer, under the peak/total transmission power constraints of source and energy causality constraints of relay. The problem is formulated as a non-convex problem and solved by combining convex optimization and linear search.

System Model and Problem Formulation

System Model

We consider a three-node relay channel. A direct link from the source to destination does not exist. The relay node assists in the communication between the source and the destination by decoding, re-encoding and forwarding the signals from the source. We consider an orthogonal channel model, where the source and relay transmit in different time slots. We assume a flat Rayleigh-fading channel, i.e., the small-scale fading of all links in the system follows the Rayleigh distribution. The transmission distances in the S-R and R-D link are \((d_{sr}, d_{rd})\), respectively. The channel power gains in the S-R and R-D links \((\gamma_{sr}, \gamma_{rd})\) are modeled as exponentially distributed random variables with parameters \(L_{sr}, L_{rd}\). The relay node is assumed to have no fixed power supply and only can harvest energy from RF signals from the source. Due to the practical circuit constraints, the relay node cannot decode the information and harvest energy simultaneously, so the TS-based receiver architecture is adopted.

Problem Formulation

Full CSI: In this network, the channel gain vector is expressed as \(\gamma = \{\gamma_{sr}, \gamma_{rd}\}\). In each fading state, we define two indicator variables \(\{\rho(\gamma), \tilde{\rho}(\gamma)\} \in \{0,1\}\) to distinguish different transmission modes. Specifically, if the source transmits, the indicator variable \(\tilde{\rho}(\gamma) = 1\); if it transmits information to the relay, the transmit power is \(P_D(\gamma)\), the indicator variable \(\rho(\gamma) = 1\), and the rate \(r_s(\gamma) = \rho(\gamma) \log(1 + \frac{\gamma_{sr} P_D(\gamma)}{N_0})\); and if it transmits energy to the relay, the transmit power is \(P_E(\gamma)\), \(\rho(\gamma) = 0\). If the relay transmits, the transmit power is \(P_R(\gamma)\), \(\tilde{\rho}(\gamma) = 0\), and the rate \(r_r(\gamma) = (1 - \tilde{\rho}(\gamma)) \log(1 + \frac{\gamma_{rd} P_R(\gamma)}{N_0})\). Moreover, the transmit power at the source, and the received energy at the relay are given respectively by \(p(\gamma) = \rho(\gamma) P_D(\gamma) + (\tilde{\rho}(\gamma) - \rho(\gamma)) P_E(\gamma)\) and \(g(\gamma) = (\tilde{\rho}(\gamma) - \rho(\gamma)) \eta' \gamma_{sr} P_E(\gamma)\) where \(0 < \eta \leq 1\) denotes the energy conversion efficiency. To make the problem tractable, we only consider the total data and energy constraints, thus obtain the following formulation for an upper bound on the ergodic capacity.

\[
\max_{\{P_D(\gamma), P_E(\gamma), P_R(\gamma), \rho(\gamma), \tilde{\rho}(\gamma), p(\gamma)\}} R,
\]

s.t. \(R \leq \mathbb{E}_\gamma \{r_s(\gamma)\}, R \leq \mathbb{E}_\gamma \{r_r(\gamma)\}\),

\[
\mathbb{E}_\gamma \{p(\gamma)\} \leq P,
\]

\[
\mathbb{E}_\gamma \{r_s(\gamma)\} \leq \mathbb{E}_\gamma \{r_r(\gamma)\}, \mathbb{E}_\gamma \{P_R(\gamma)\} \leq \mathbb{E}_\gamma \{g(\gamma)\},
\]

\[
0 \leq P_D(\gamma), P_E(\gamma) \leq P, \quad P_R(\gamma) \geq 0,
\]
0 ≤ ρ(γ), \( \bar{ρ}(γ) \) ≤ 1, \( \bar{ρ}(γ) - ρ(γ) = \bar{ρ}(γ) \). \( (1d) \)

where \( P \) and \( P_p \) denotes the average power and peak power constraints, respectively.

**Partial CSI:** The formulation above assumes that the source knows full CSI \( γ \) so that it can schedule the transmission by the source or relay in each fading state. When the source only knows partial CSI \( γ_{sr} \) and has no feedback information from the relay, coordination between the source and relay cannot happen so that both the source and relay adapt the transmission parameters according to their own CSI. Consequently, the transmission process is divided into two phases. In the first phase, the source transmits information/energy signals to the relay with transmit power \( P_0(γ_{sr}) \) or \( P_E(γ_{sr}) \). In the second phase, the relay forwards information signals to the destination using the harvested energy with transmit power \( P_R(γ_{rd}) \). Assume the time allocated to the first phase and second phase are \( n_s \) and \( n_r \), respectively. \( R \) achieves the maximum value when \( n_s \bar{r}_s = n_r \bar{r}_r \), where \( \bar{r}_s = \mathbb{E}_{γ_{sr}} \{ r_s(γ_{sr}) \} \) and \( \bar{r}_r = \mathbb{E}_{γ_{rd}} \{ r_r(γ_{rd}) \} \). Therefore, the objective function in (1) becomes \( R = \frac{\bar{r}_s}{\bar{r}_s + \bar{r}_r} \). Let \( J = \frac{1}{R} \). When the source only knows partial CSI, the problem in (1) can be simplified as

\[
\min_{\{P_0(γ_{sr}), P_E(γ_{sr}), P_0(γ_{rd}), P_R(γ_{rd})\}} J = 1/\bar{r}_s + 1/\bar{r}_r
\]

\( (2) \)

\( s.t. \mathbb{E}_{γ_{sr}} \{ \rho P_0 + (1 - \rho) P_E \} \leq P, \)

\( (2a) \)

\( E_{γ_{rd}} \{ P_R \bar{r}_r \} \leq E_{γ_{sr}} \{ η(1 - \rho) γ_{sr} P_E \}, \)

\( (2b) \)

\( 0 \leq P_0, P_E \leq P_p, 0 \leq \rho \leq 1. \)

\( (2c) \)

**Information/Energy Transfer and Power Control**

**Solution to Full CSI**

In this subsection, we assume that \( γ_{rd} \) is fed back to the source via a low-rate feedback link by the relay so that the source knows the channel gain vector \( γ \) and can coordinate the transmission between the source and relay in each fading state. Firstly, we can prove that in (1), the optimal solution of \( P_E(γ) \) is \( P_E^*(γ) = P_p \).

The Lagrangian associated with the problem in (1) is

\[
\mathcal{L} = R - \mu(R - \mathbb{E}_γ \{ r_s \}) - \zeta(R - \mathbb{E}_γ \{ r_r \}) - \nu(\mathbb{E}_γ \{ P_0 \} - E(g)) - \lambda(\mathbb{E}_γ \{ p \} - P)
\]

\( (3) \)

where \( \mu \geq 0, \zeta \geq 0, \nu \geq 0, \lambda \geq 0 \) are Lagrange multipliers.

Utilizing the Euler-Lagrange equation, the optimal \( P_0(γ) \), \( P_R(γ) \) to maximize \( \mathcal{L} \) are given by

\[
P_0^*(γ) = \left[ C_d - \frac{1}{r_s} N_0 \right]^{P_p}, \quad P_R^*(γ) = \left[ C_r - \frac{N_0}{r_{rd}} \right]^{P_p}
\]

\( (4) \)
where $C_d = \frac{\mu}{\lambda \ln 2}$, $C_r = \frac{1 - \mu}{\nu \ln 2}$. Denote $C_e = \frac{\lambda}{\nu \eta}$. The following Lemma gives a clear relationship between $\rho^*(\gamma)$ and channel gain $\gamma_{sr}$.

**Lemma 1:** If $C_e < \frac{N_0}{C_d}$, $\rho^*(\gamma) \equiv 0$ for any $\gamma_{sr}$. Otherwise, there exists an SNR threshold $x_0$ so that when $\frac{\gamma_{sr}}{N_0} > x_0$, $\rho^*(\gamma) = 0$; else, $\rho^*(\gamma) = 1$.

Because the primal problem is convex, the optimal dual variables $\mu, \nu, \lambda$ can be obtained by subgradient-based method, such as ellipsoid method.

**Solution to the Partial CSI**

In (2), the optimal solution $P_L^*(\gamma_{sr}) = P_p$. Then, we solve for $P_R(\gamma_{sr})$ by fixing $P_D(\gamma_{sr}), \rho(\gamma_{sr})$. Finally, given $P_R(\gamma_{rd})$ in terms of $P_D(\gamma_{sr}), \rho(\gamma_{sr})$, we find the optimal $P_D(\gamma_{sr}), \rho(\gamma_{sr})$ by convex optimization and linear search.

To minimize $J$ over $P_R(\gamma_{rd})$, we first solve the following sub-problem:

$$
\max_{P_R(\gamma_{rd}) > 0} \bar{r}_r, \quad s.t. \quad E_{\gamma_{rd}} [P_R(\gamma_{rd})] \bar{r}_r \leq E_{\gamma_{sr}} [\eta(1 - \rho)\gamma_{sr} P_p] \bar{r}_r.
$$

Utilizing KKT condition, we get

$$
P_R^*(\gamma_{rd}) = \left[ C_r - \frac{1}{\gamma_{rd} / N_0} \right]^+ \tag{5}
$$

where $C_r = \frac{E_{\gamma_{sr}}[\eta(1 - \rho(\gamma_{sr}))\gamma_{sr} P_p]}{\mu \bar{r}_r \ln 2}$. Substituting $\bar{r}_r$ into (2), the primal problem in (2) is then equivalent to

$$
\min_{P_R(\gamma_{rd}) > 0} \frac{1}{\bar{r}_r} + \int_{f(\frac{x_0}{C_r \gamma_{rd}})}^\infty \frac{\ln 2}{\exp(-x) / x} \, dx, \quad s.t. (2a), (2c). \tag{6}
$$

The problem in (7) is not a convex problem since the second term of the objective function is not a convex function of $P_D$ and $\rho$. For any $t$, the equation $\int_{f(\frac{x_0}{C_r \gamma_{rd}})}^\infty \frac{\exp(-x) / x}{x} \, dx = t$ has only one solution $C_t = q(t)$. To solve (7), we introduce two new variables $\bar{r}_r, t$, as well as an inequality constraint (9b) and an equality constraint (9c) into (7) and reformulate it as

$$
\min_{t > 0} \ln \frac{2}{t + 1 / J_i(t)} \tag{8}
$$

where $J_i(t)$ is the maximum value of the objective function of the following problem:

$$
J_i(t) = \max_{P_R(\gamma_{rd}) > 0} \bar{r}_r, \quad s.t. \quad E_{\gamma_{sr}} [\hat{P}_D(\gamma_{sr}) + (1 - \rho(\gamma_{sr})) P_p] \leq P, \tag{9}
$$

(9a)
\[ \bar{r}_i \leq E_{\gamma_{sr}} \left\{ \rho(\gamma_{sr}) \log(1 + \frac{\gamma_{sr} \tilde{P}_D(\gamma_{sr})}{\rho(\gamma_{sr}) N_0}) \right\}, \quad (9b) \]

\[ C_s = \frac{E_{\gamma_{sr}} \left\{ \eta(1-\rho(\gamma_{sr})) \gamma_{sr} P_p \right\} }{\bar{r}_i} = q(t), \quad (9c) \]

where \( \tilde{P}_D(\gamma_{sr}) = \rho(\gamma_{sr}) P_p(\gamma_{sr}) \). For any given \( t \), (9) is a convex optimization problem. For given \( \lambda, \mu, \nu \), we maximize Lagrangian \( L \) by optimizing \( \tilde{P}_D(\gamma_{sr}), \rho(\gamma_{sr}), \bar{r}_i \). \( L \) is a convex functional of \( \tilde{P}_D(\gamma_{sr}), \rho(\gamma_{sr}) \) and \( \bar{r}_i \). Therefore, if \( \rho^*(\gamma_{sr}) \neq 0 \), by solving the Euler-Lagrange equation, we can get the closed-form expression for the optimal \( P_p^*(\gamma_{sr}) \),

\[ P_p^*(\gamma_{sr}) = [C_d - \frac{1}{\gamma_{sr} N_0}] \frac{\mu}{\lambda \ln 2} \]  

where \( C_d = \frac{\mu}{\lambda \ln 2} \). On the other hand, if \( \rho^*(\gamma_{sr}) = 0 \), \( P_p^*(\gamma_{sr}) = 0 \). \( -\frac{\partial L}{\partial \rho(\gamma_{sr})} \) is independent of \( \rho(\gamma_{sr}) \). Therefore, we can find a threshold \( x_0 \). When \( \frac{\gamma_{sr}}{N_0} > x_0 \), \( \rho^*(\gamma_{sr}) = 0 \); else, \( \rho^*(\gamma_{sr}) = 1 \). Finally, we will obtain the optimal \( C_d, x_0 \) which make the equalities in (9a) and (9c) hold. We propose an algorithm based on bisection method to solve the two nonlinear equations.

**Simulation Results**

In this subsection, we evaluate the performance of the three-point relay system when the source knows partial CSI and full CSI. The bandwidth is set to 10MHz. The AWGN at the receiver is assumed to have a white power spectral density of -150dBm/Hz. The energy harvesting efficiency is assumed to be \( \eta = 0.8 \). In Figure 1, we show the average throughput versus the average transmit power of source in two cases with different \( P_p \). We set \( d_{sr} = 2m, d_{sd} = 3m \). \( P \) is the average transmit power constraint. For a fair throughput comparison between full CSI case and partial CSI case, the average transmit power constraint of source in the full CSI case is made to equal \( \frac{P}{1 + n_e / n_c} \). \( P \) is set to \( 0 \sim 18dBm, 0 \sim 26dBm, 0 \sim 30dBm \) in the case of \( P_p = 0.1w, 0.5w, 1.2w \), respectively. It can be observed that the achievable rate is very close to the upper bound. When the average or peak power of source increases, the switching threshold \( x_0 \) decreases. As a result, the energy transmission process is shortened since the relay has more opportunities to harvest energy. In the low transmit power region (<15dBm), the scheme with full CSI only obtains slight capacity improvement compared to the scheme with partial CSI. With the increase of the transmit power, the capacity improvement due to the transmit node selection is more obvious. The position of the relay has an important influence on the average rate. In Figure 2, we set \( P_p = 1.2w, d_{sd} = 10m \). It is observed from Figure 2 that the system achieves the minimum throughput when \( d_{sr} \approx d_{sd} \). The throughput increases gradually when the relay approaches the source or the destination. This is because when the relay is close to the source or the destination, the source needs less time for transferring energy.
Conclusion

This paper studied the power allocation and cooperative transmission in a three-point relay system when the relay is wireless powered. Under a time-varying relay channel setup, we investigated joint optimization of information/energy scheduling and power control of source/relay when the source knows partial or full CSI. Simulation results reveal the impact of system parameters, such as average/peak transmit power and position of relay on the average throughput.

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References


