Prices of Platforms’ Differentiated Competition under Different Levels of Agents’ Information: Based on the Model of Stackelberg Pricing Game

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ABSTRACT: We study the competitive prices of duopoly two-sided platforms with differentiation of cross-group network externality strengths under two different levels of agents’ information which are informed and uninformed through the Stackelberg pricing games model based on standard Hotelling model and the assumptions that the agents of platform form responsive expectations if they are informed and form passive expectations if they are uninformed. The results include the structures of equilibrium prices, the comparison of equilibrium prices between two platforms and the responses of equilibrium prices to the strengths of cross-group network externality and horizontal differentiation. We also find that there are significant differences in pricing strategies between not only the two sequential pricing platforms but also the one platform under two different levels of agents’ information through the comparisons among the results.

1 GENERAL INSTRUCTION

In markets with two-sided network effects, the value that an agent derives from joining a platform is determined by the number of agents on the other side. Examples include B2C e-commercial platforms like Taobao or Jingdong, payment systems like Visa or AliPay, Web search platforms like Baidu or Google and operation systems like Windows, etc. Because of this cross-group network externality, the number of agents that join each side ultimately depends on the prices charged to both sides. (Rochet & Tirole 2003, 2006).

Therefore, the agents’ information about the prices or demand of the other side will influence their decisions. Agents can take all prices into account when forming expectations, namely keep the expectations consistent with the real demand if the agents are informed for the prices or demand of the other side. However, some agents may not take all prices into account when forming expectations if they are uninformed. For instance, they may not investigate the prices of the other side or have sufficient information about aggregate demand on each side to compute each demand’s responsiveness to price changes. Instead, agents typically rely on external information to form expectations about the total number of developers that join a given platform. Expectations formed in this way usually do not respond to changes in platform prices. (Gabszewicz & Wauthy 2013, Hagiu & Halaburda 2014) In conclusion, agents’ responses to the price changes and the equilibrium prices of platforms will be different rely on different levels of agents’ information.

Meanwhile, differentiated competitions are very common in real two-sided market. The differentiations between competitive platforms will be in respect of location, preference, scale or quality, etc, and usually include more than one of them. For example, China Mobile and China Unicom are differentiate not only in the standard of 4G technology but also in user scale, Taobao and Jingdong are differentiate not only in the types but also in the qualities of commodity.

Then, we study the price strategies of two-sided platforms with horizontal differentiation and differentiation of cross-group network externality strengths under different levels of agent’ information, when they take Stackelberg pricing competition (also be called Bertrand-Stackelberg competition). We investigate the structures of equilibrium prices, the comparisons of equilibrium prices between two sequentially pricing platforms and the responses of equilibrium prices to the strengths of cross-group network externality and horizontal differentiation through the Stackelberg pricing games model based on standard Hotelling model and the assumptions that the agents of platform form responsive expectations if they are informed and form responsive expectations if they are uninformed.
The related literature can divide into two parts, one part is about the information or expectations of two-sided market. The other one is about the differentiated competitions of two-sided platforms, as following:

(1) Information or expectations of two-sided market. The majority of the existing literature on two-sided platform pricing typically assumes that agents on all sides have full information about all prices and are able to perfectly compute their impact on platform adoption. In other words, expectations adjust perfectly in response to changes in platform prices (e.g. Caillaud & Jullien 2003, Armstrong 2006, Armstrong & Wright 2007, Hagiu 2009, Choi 2010, Weyl 2010, Bedre-Defoliea & Calvano 2013, Reisinger 2014, Hagiu & Wright 2015).

Only two papers, which are Gabszewicz & Wauthy (2012) and Hagiu & Halaburda (2014), incorporated passive expectations to two-sided model under uninformed condition. They assumed that the agents of platform would keep their expectations fixed regardless the variation of the prices, but the responsive and passive were all rational and fulfilled in equilibrium. Passive expectations were first introduced in the economic literature on one-sided network effects by Katz & Shapiro (1985). This was also the first paper to explicitly distinguish passive expectations from responsive expectations. In particular, Katz & Shapiro (1985) study Cournot competition between n firms (technologies) with direct network effects. Gabszewicz & Wauthy (2013) firstly introduce the passive expectations into the researches about two-sided market and studied the output of monopoly and duopoly market with differentiation of expectations between two platforms. Hagiu & Halaburda (2014) have improved the above research, they systematically study the effects of agents’ information levels to equilibrium prices and profits in monopoly and duopoly market with Hotelling competition. Besides the comparisons of pure responsive expectations and passive expectations scenarios, they have considered the hybrid scenario of above two expectations and introduced the wary expectations under which agents may adjust expectations based on changes in their prices.

(2) Differentiated competitions of two-sided platforms. First, competition with horizontal differentiation. The majority of the existing literature on two-sided platform competition typically use standard Hotelling model to analyse the competition with horizontal differentiation (e.g., Armstrong 2006, Hagiu 2009, Reisinger 2014). However, some paper use the modification of Hotelling model to investigate other type of oligopoly market, such as partial multi-homing (Zhang & Li 2010, Ji 2011) and multi-oligopolies (Zhang, 2013).

Second, hybrid of horizontal differentiation and sequential differentiation. Diao et al. (2008) have introduced the Stackelberg pricing game model to standard Hotelling model to study the duopoly one-sided platforms with asymmetric direct network externalities. They found that the firms obtain more product differentiation in Stackelberg equilibrium comparing traditional Nash equilibrium and price leader enjoying higher price, share, profits in the market than the subsequent entrant. Xie & Li (2014) also use Hotelling model to study the duopoly with sequential entry. They investigate the optimal IT investment, namely optimal direct network externalities, and compare the optimal investment strategies of early entrant with that of the later entrant.

Third, vertical differentiation. Cao et al. (2008) have studied the duopoly one-sided platforms with differentiation of direct network externalities, they found that the network externality is not necessary to bring more profit for platforms in selecting the optimal pricing strategy, this depends on the strength of the network externality and the service quality/price ratio provided by platform. Diao et al. (2009) have study the optimal qualities of duopoly one-sided platforms with differentiation of direct network externalities through a three-stages game model.

Fourth, hybrid differentiation including all above. Ji & Wang (2014) have taken all three types of differentiation into consideration. They have studied the duopoly two-sided platforms with Stackelberg pricing game, horizontal differentiation and differentiation of cross-group network externalities network externalities. They found that the platform with higher cross-group network externality and later pricing would take more advantages than the other one.

In conclusion, the main contribution of this paper is to study the prices of asymmetric competition under uniformed agents and compare the prices with that of the informed agents. Though Hagiu & Halaburda (2014) have studied both situations which were informed and uninformed agents, they only have considered the symmetric competition, so can’t reflect the differences of competitive strategies between asymmetric platforms. Some paper (Cao et al. 2008, Diao et al. 2009, Ji & Wang 2014) have studied the asymmetric competition of platforms, nonetheless they have only considered the scenario of informed agents, in order that can’t investigate the difference of pricing strategies between the scenarios of informed and uninformed.
3 MODEL AND EQUILIBRIUM

3.1 Assumptions

For convenience, we label the two sides as “developers” (d) and “users” (u), (e.g., the APP developer and users of Android, or the sellers and buyer of B2C e-commerce platforms). $p_d$ and $p_u$ represent prices of developer and users respectively, $n_d$ and $n_u$ represent the demand of developer and users respectively, $n'_d$ and $n'_u$ are demand expectations of each sides respectively.

Assume that the agents of platform form responsive expectations about participation of the other side if they are informed and form passive expectations if they are uninformed. In other words, the responsive expectations means the expectations always match realized the participation of the other side: $n_d = n'_d$, $n_u = n'_u$; passive expectations means expectations are not adjusted regarding participation of the other side, in turn, the platform has no choice but to treat agents’ passive expectations ($n'_d$, $n'_u$) as fixed when it sets its prices. Expectations are fulfilled in equilibrium, because they are rational. (Gabszewicz & Wauthy 2013, Hagiu & Halaburda 2014).

Specifically, assume that a fraction $\lambda$ of users are informed, while the remaining fraction $1 - \lambda$ of users are uninformed and hold passive expectations. Throughout the paper we assume that all developers are informed, because the developers (or buyers) are usually proficient to collect and analyse information.

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From the perspective of developers who are profit-oriented, the two platforms are identical, or the sellers and buyer of B2C e-commerce platforms. Contrary, from the perspective of users with diversity of preference, the two platforms are horizontal differentiation, and the users are interested in joining at most one platform, i.e., they single-home (such market structure is called “competitive bottleneck”).

The sequence of Stackelberg pricing game between two platforms is as following: (1) Platform 1 decides the prices; (2) Platform 2 decides the prices after observing the prices of platform 1; (3) Agents decide whether to join the platform(s) or not after observing the prices of platform 1 and 2.

3.2 Process of Equilibrium

The process of solution is opposite to the game sequence (calculation is based on Matlab7 and we just list the main process as following rather than results of each step because of complexity):

(1) Demands.

Developers are differentiated by the fixed cost that they incur when joining each platform. The fixed cost per platform is the same regardless of whether a developer joins one or both platforms, i.e., there are no economies of scope in joining multiple platforms. So developer’s demand of platform $j$ is:

$$ n_{dj} = \alpha_j n_{uj} - p_{dj} $$  \hspace{1cm} (1)

Let us now turn to the case of symmetric competition side. Users are distributed along a Hotelling segment [0,1] with density 1 and transportation costs $t(t > 0)$. Assume that the distribution of users’ location is independent to the distribution of users’ information, based on the condition that marginal user is indifferent to join each platform, we have users’ demands:

$$ n_{u1} = \lambda \left( \frac{1}{2} + \frac{\alpha_j n_{u1} - \alpha_j n_{u2} - p_{u1} + p_{u2}}{2t} \right) $$

$$ + (1 - \lambda) \left( \frac{1}{2} + \frac{\alpha_j n_{u1} - \alpha_j n_{u2} - p_{u1} + p_{u2}}{2t} \right) $$

$$ n_{u2} = \lambda \left( \frac{1}{2} + \frac{\alpha_j n_{u2} - \alpha_j n_{u1} - p_{u2} + p_{u1}}{2t} \right) $$

$$ + (1 - \lambda) \left( \frac{1}{2} + \frac{\alpha_j n_{u2} - \alpha_j n_{u1} - p_{u2} + p_{u1}}{2t} \right) $$  \hspace{1cm} (2)

Solve with the equation (1) and (2) simultaneously, we have demand functions $n_{u1,j} = n_{u1}(p_{u1}, p_{u2}, p_{d1}, p_{d2}, n'_{d1}, n'_{d2})$ and

$$ n_{d1,j} = n_{d1}(p_{u1}, p_{u2}, p_{d1}, p_{d2}, n'_{d1}, n'_{d2}) $$

(2) Pricing of platform 2.

The profit function of platform 2 is

$$ \pi_2 = p_{u2} n_{u2} + p_{d2} n_{d2} $$

Substitute the results of last step into the function and solve the first order condition over $(p_{u2}, p_{d2})$, then we have

$$ p_{u2} = p_{u2}(p_{u1}, p_{d1}, n'_{d1}, n'_{d2}) $$

$$ p_{d2} = p_{d2}(p_{u1}, p_{d1}, n'_{d1}, n'_{d2}) $$

Meanwhile, we can have the second order condition of platform 2:

$$ 2t > \lambda (\alpha_1^2 + \alpha_2^2) \hspace{1cm} 8t > 4\lambda \alpha_1^2 + \alpha_2^2 (\lambda^2 + 2\lambda + 1) $$  \hspace{1cm} (3)

(3) Pricing of platform 1.

Substitute the results of last step into the demand functions, we have

$$ n_{u1}(p_{u1}, p_{d1}, n'_{d1}, n'_{d2}) $$

$$ n_{d1}(p_{u1}, p_{d1}, n'_{d1}, n'_{d2}) $$
Substitute them into profit function $\pi = p_u n_u + p_d n_d$ and solve the first order condition over $(p_u, p_d)$, we have

$$p_u = p_u(n_{d1}^*, n_{d2}^*) \quad \text{and} \quad p_d = p_d(n_{d1}^*, n_{d2}^*).$$

Meanwhile, we can have the second order condition of platform 1:

$$8t > 4\lambda \alpha_1^2 + \alpha_2^2 (\lambda^2 + 2\lambda + 1)$$
$$16t > \alpha_1^2 (\lambda^2 + 6\lambda + 1) + \alpha_2^2 (2\lambda^2 + 4\lambda + 2)$$

(4) Solve the expectations.

Substitute $p_u(n_{d1}^*, n_{d2}^*)$ and $p_d(n_{d1}^*, n_{d2}^*)$ into the demand functions $n_{d,j}(p_u, p_d, n_{d1}^*, n_{d2}^*)$, we have $n_{d,j} = n_{d,j}(n_{d1}^*, n_{d2}^*)$. Finally the expectation will be fulfilled: $n_{d1}^* = n_{d1}^*$, $n_{d2}^* = n_{d2}^*$ because of rationality. Let $n_{d,j} = n_{d,j}(n_{d1}^*, n_{d2}^*)$ and solve with them simultaneously, we have equilibrium demand of developers $n_{d,j}$ based on it we can solve equilibrium demand of users and equilibrium prices.

4 RESULTS

4.1 Equilibrium Prices

$\lambda = 0$ if all users form passive expectations $\lambda = 1$ if all users form responsive expectations based on assumptions above. Substitute the value of $\lambda$ into the results of last section, we have the final equilibrium prices, as are shown in Table 1:

<table>
<thead>
<tr>
<th>Table 1. Equilibrium Prices under Different Levels of Agents’ Information.</th>
<th>Uninformed $\lambda = 0$</th>
<th>Informed $\lambda = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_u^*$</td>
<td>$24t^2 - 3\alpha_1^2 t - 11\alpha_2^2 t + \alpha_1^2 \alpha_2^2 + \alpha_1^4$</td>
<td>$3t - \alpha_1^2 - 2\alpha_2^2$</td>
</tr>
<tr>
<td>$p_u^*$</td>
<td>$40t^2 - 8\alpha_1^2 t - 14\alpha_2^2 t + 2\alpha_1^2 \alpha_2^2 + \alpha_1^4$</td>
<td>$5t - 3\alpha_1^2 - 2\alpha_2^2$</td>
</tr>
<tr>
<td>$p_d^*$</td>
<td>$16t - 2\alpha_1^2 - 3\alpha_2^2$</td>
<td>0</td>
</tr>
<tr>
<td>$p_d^*$</td>
<td>$\alpha_1 (10t - 2\alpha_1^2 - \alpha_2^2)$</td>
<td>0</td>
</tr>
<tr>
<td>S.O.C.</td>
<td>$2t &gt; \alpha_1^2$</td>
<td>$2t &gt; \alpha_1^2 + \alpha_2^2$</td>
</tr>
</tbody>
</table>

4.2 Comparisons of Equilibrium Prices between Two Platforms

We can compare the equilibrium prices between price leader and follower based on the content of table 1, the results are shown in the Table 2:

<table>
<thead>
<tr>
<th>Table 2. Comparisons of Equilibrium Prices between Two Platforms under Different Levels of Agents’ Information.</th>
<th>Uninformed</th>
<th>Informed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Users if $t^2 - 2t + \alpha_1^2 &gt; 0$, otherwise price of follower is higher.</td>
<td>The price of leader is higher than follower if $8t^2 + 2\alpha_1^2 - 8\alpha_1^2 t + \alpha_1^4 &gt; 2(16t^2 - 2\alpha_1^2 - 3\alpha_2^2)$, otherwise price of follower is higher.</td>
<td>All prices 0</td>
</tr>
</tbody>
</table>

4.3 Responses of Equilibrium Prices to Cross-group Network Externality

We can have the responses of equilibrium prices $p_{u1}^*$, $p_{u2}^*$, $p_{d1}^*$ and $p_{d2}^*$ to the strengths of cross-group network externality $\alpha_1$ and $\alpha_2$ based on the prices’ partial derivative for $\alpha_1$ and $\alpha_2$, the results are shown in the Table 3.

4.4 Responses of Equilibrium Prices to Horizontal Differentiation

We can have the responses of equilibrium prices $p_{u1}^*$, $p_{u2}^*$, $p_{d1}^*$ and $p_{d2}^*$ to the strengths of horizontal differentiation between two platforms $t$ based on the prices’ partial derivative for $t$, the results are shown in the Table 4:

<table>
<thead>
<tr>
<th>Table 4. Responses of Equilibrium Prices to the Strength of Horizontal Differentiation between Two Platforms under Different Levels of Agents’ Information.</th>
<th>Uninformed</th>
<th>Informed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{u1}^*$</td>
<td>Positive direction.</td>
<td>Positive direction.</td>
</tr>
<tr>
<td>$p_{u2}^*$</td>
<td>Positive direction.</td>
<td>Positive direction.</td>
</tr>
<tr>
<td>$p_{d1}^*$</td>
<td>Positive direction if $6\alpha_1^2 &lt; 7\alpha_2^2$, otherwise negative direction.</td>
<td>0</td>
</tr>
<tr>
<td>$p_{d2}^*$</td>
<td>Positive direction if $6\alpha_1^2 &gt; 7\alpha_2^2$, otherwise negative direction.</td>
<td>0</td>
</tr>
</tbody>
</table>

*All results are based on the existence of equilibrium.
5 CONCLUSIONS AND PROSPECTS

We study the competitive prices of duopoly two-sided platforms with differentiation of cross-group network externality strengths under two different levels of agents’ information which are informed and uninformed through the Stackelberg pricing games model based on standard Hotelling model. The results include the structures of equilibrium prices, the comparison of equilibrium prices between two platforms and the responses of equilibrium prices to the strengths of cross-group network externality and horizontal differentiation. We also find that there are significant differences in pricing strategies between not only the two sequential pricing platforms but also the one platform under two different levels of agents’ information through the comparisons among the results. The details are shown in the last section.

The results of research can contribute to platforms with asymmetric competition for higher profit by optimal adjusting their price when the sequence of pricing, the strengths of cross-group network externality or horizontal differentiation is changing.

We can extend the research in two aspects: (1) Market structure: Extend to general multi-oligopolies with differentiation by Salop loop (Salop 1979, Gong & Li 2010, Zhang 2013); (2) Kinds of expectations: Besides responsive expectations and passive expectations, we can also consider some other expectations, e.g., adaptive expectation (Friedman 1968), wary expectation (Hagiu & Halaburda 2014) or etc.

Table 3. Responses of Equilibrium Prices to the Strengths of Cross-group Network Externality under Different Levels of Agents’ Information.

<table>
<thead>
<tr>
<th>( \alpha_1 )</th>
<th>Uninformed</th>
<th>Informed</th>
<th>Uninformed</th>
<th>Informed</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{u_1}^* )</td>
<td>Positive direction.</td>
<td>Negative direction.</td>
<td>Negative direction.</td>
<td>Negative direction.</td>
</tr>
<tr>
<td>( p_{u_2}^* )</td>
<td>Positive direction if ( 12x^2 + \alpha_1^4 - 7\alpha_2^2 t &lt; 0 ), otherwise negative direction.</td>
<td>Negative direction.</td>
<td>Negative direction.</td>
<td>Negative direction.</td>
</tr>
<tr>
<td>( p_{d_1}^* )</td>
<td>Positive direction if ( 2\alpha_1^2 \alpha_2^2 - 6\alpha_1^2 t + 25\alpha_2^2 t - 48r^2 - 3\alpha_1^4 &lt; 0 ), otherwise negative direction.</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( p_{d_2}^* )</td>
<td>Negative direction.</td>
<td>0</td>
<td>Positive direction.</td>
<td>0</td>
</tr>
</tbody>
</table>

*All results are based on the existence of equilibrium.

REFERENCES


